Sales-Based Compensation and Collusion with Heterogeneous Firms

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Abstract

Pricing and output decisions are often delegated to managers compensated on the basis of sales. Prior literature has shown that when firms are homogeneous, the delegation of pricing or output decisions to managers, compensated on the basis of sales, does not facilitate collusion. We show that when firms are heterogeneous, either in marginal cost or product quality, sales-based compensation can facilitate collusion under both price and quantity competition. As a result, compensating managers on the basis of sales can increase firm profits and reduce consumer welfare. Additionally, we find that owners can strategically design managerial compensation structures to incentivize collusion between rival managers.

Keywords: Collusion, Sales-based Compensation, Heterogeneous Firms, Manager Collusion, Strategic Delegation

JEL Codes: L21, D43, L13

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1 Introduction

The compensation structures of managers tasked with setting prices or choosing output levels often depend on both sales and profits.[1] This paper studies the possibility of collusion between managers compensated, at least partly, on the basis on sales. Prior literature (Lambertini and Trombetta, 2002) has shown that sales-based compensation has no effect on the sustainability of collusion between managers of homogeneous firms. We find that sales-based compensation can facilitate collusion when firms are heterogeneous.

We analyze two types of firm heterogeneity: differences in marginal cost and differences in product quality. Our first finding is that sales-based compensation can facilitate collusion between managers when marginal costs are asymmetric. Cost asymmetries typically hinder the sustainability of collusion.[2] However, we find that, when manager compensation is based on revenue, the anti-collusive effect of cost asymmetries is diminished. Under sales-based compensation, managers seek to maximize revenue—a quantity which does not depend on the cost of production. Thus, managers compensated on the basis of sales disregard any efficiency or inefficiency in marginal cost (relative to their rival) when setting prices, because their pay is not tied to their costs of production. This effectively eliminates asymmetries between firms and facilitates collusion.

We find a similar result when firms differ in product quality. Differences in product quality are known to reduce the sustainability of collusion (Häckner, 1994).[3] However, we find that when manager compensation is based on sales, the effect of quality asymmetries on the sustainability of collusion is diminished. Sales based compensation structures can potentially correct for asymmetries in product quality and align the interests of managers.

Next, we examine the impact of collusion between managers, compensated on the basis of sales, on firm profit and consumer welfare. In a static (i.e., one-period) setting, sales-based compensation reduces firm profits, because sales-based compensation distorts managers' pricing decisions. Intuitively, this result occurs because managers, compensated on the basis of sales, have incentives to expand output or reduce price in order to increase their pay, at the expense of firm profit. However, in a dynamic (i.e., repeated game)

[1] An FW Cook Report (see *2019 Annual Incentive Plan Report. FW Cook. October 2019.* https://www.fwcook.com/content/documents/publications/10-17-19_FWC_2019_Incentive_Plan.pdf and Groysberg et al. (2021)) found that 91% and 49% of companies tie executive compensation to profits and sales respectively.

[2] In some settings (e.g., homogenous product competition (Ivaldi et al., 2003)), asymmetries hinder collusion because low cost firms earn relatively large profits in the Nash equilibrium and are therefore difficult to discipline. In other settings (e.g., Vasconcelos (2005) and Villar et al. (2012)), asymmetries hinder collusion because high cost firms do not earn sufficient collusive profits to ensure they do not wish to cheat on collusion. While the details depend on the particular setting, the anti-collusive effect of asymmetries in marginal cost is more general.

[3] The effect of differences in product quality on the sustainability of collusion is similar to that of cost asymmetries. In some settings (e.g., Häckner (1994)), this result occurs because high quality firms earn large profits from defection. In other settings (e.g., differentiated price or quantity competition as examined in this study), this result occurs because low quality firms earn relatively small profits from collusion. While the details depend on the particular setting, the anti-collusive effect of differences in product quality is more general.

setting, sales-based compensation can, as we show, facilitate collusion between managers. Collusion results in elevated prices which enhances firm profit. We show that the later effect can dominate and, as a result, sales-based compensation can increase firm profit.

We present an analogous finding regarding consumer welfare. In a static setting, sales-based compensation enhances consumer welfare because managers have incentives to expand output or reduce price in order to increase their sales (and thus, their pay). However, in a dynamic setting, sales-based compensation can reduce consumer welfare because sales-based compensation may facilitate collusion. We find that the collusive effect of sales-based compensation can dominate and, as a result, sales-based compensation can increase prices and reduce consumer welfare.

Finally, we explore the possibility of owners strategically designing managers' compensation structures in order to facilitate or incentivize collusion between rival managers. Specifically, we study a model of strategic delegation where owners simultaneously determine their respective manager's compensation structure prior to managers engaging in an infinitely repeated pricing game. We find that owners can, by selecting sales-based compensation structures that enhance the sustainability of collusion between managers and are not tied too strongly to sales levels, induce manager collusion and enhance their profits.

Manager delegation and sales-based managerial compensation have been studied extensively in prior literature (Fershtman and Judd, 1987; Sklivas, 1987; Fershtman and Judd, 1986; Fershtman, 1985; Baumol, 1958; Vickers, 1985). Our study is most closely related to prior literature analyzing the impact of sales-based compensation on the sustainability of collusion.[4] Bian, Lai and Hua (2013) and Wang and Wang (2021) study the collusive effects of sales-based compensation in a vertical setting. Bian, Lai and Hua (2013) show that compensating downstream firms' managers on the basis of sales reduces the sustainability of collusion between upstream firms when downstream firms choose quantities. When firms choose prices, compensating downstream firms' managers on the basis of sales can enhance or reduce the sustainability of collusion between upstream firms (Wang and Wang, 2021; Bian, Lai and Hua, 2013). In our analysis, sales-based compensation increases the sustainability of collusion between managers in a horizontal setting.

The remainder of the paper is structured as follows. In Section 2, we introduce the basic model. For comparison, Section 3 analyzes the case of homogeneous firms. Section 4 analyzes the effect of sales-based compensation when firms are heterogenous in marginal cost and/or product quality. In Section 5, we study the impact of sales-based compensation and collusion on firm profit and consumer welfare. A model of strategic delegation is presented and analyzed in Section 6. Section 7 concludes. Proofs are presented in the

[4]A distinct literature explores the impact of compensation structures which depend on a rivals' profits (also known as relative performance evaluation or RPE) on the sustainability of collusion. See, for example, Matsumura and Matsushima (2012), de Lamirande, Guigou and Lovat (2005) and Delbono and Lambertini (2020). Spagnolo (2005) shows how delegating pricing/output decisions to managers with a preference for smooth profits (across time) can facilitate collusion.

appendix.

2 Base Model

Two firms (denoted firm 1 and firm 2) produce differentiated products at constant marginal costs $c_1 > 0$ and $c_2 > 0$. Following Singh and Vives (1984), we assume the demands for the firms' products result from the utility-maximizing choice of a representative consumer. The representative consumer's utility when she consumes q_i units of good $i = 1, 2$ is

$$U(q_1, q_2) = q_1 + (1 + a)\, q_2 - \frac{1}{2}\left(q_1^2 + q_2^2 + 2bq_1q_2\right) + m$$

where m denotes the numeraire good. The parameter $b \in (0, 1)$ measures the degree of substitutability between products (i.e., b is a measure of inverse product differentiation). $b = 0$ represents the case of completely independent goods, and $b = 1$ represents the case of no horizontal differentiation. a denotes the degree of quality or demand asymmetry between the two goods.[5] Without loss of generality, we restrict $a \geq 0$ which implies firm 2 sells a higher quality product than firm 1. When $a = 0$, the two products have the same quality. We assume $c_1 < 1$ and $c_2 < 1 + a$. Let c_1 and $c_2 - a$ denote the quality-adjusted marginal cost of firm 1 and firm 2, respectively. The quality-adjusted marginal cost is a measure of firm efficiency that accounts for differences in product quality.

When prices are such that there is positive demand for both products, this utility function results in the following demand functions:

$$D_1(p_1, p_2) = \frac{1}{1 - b^2}\left[1 - b - ba - p_1 + bp_2\right] \tag{1}$$

and

$$D_2(p_1, p_2) = \frac{1}{1 - b^2}\left[1 - b + a - p_2 + bp_1\right]. \tag{2}$$

If the difference between prices is sufficiently large, the higher priced product receives 0 demand. Specifically, $D_1(p_1, p_2) = 1 - p_1$ and $D_2(p_1, p_2) = 0$ if $1 - b + a + bp_1 < p_2$ and $D_1(p_1, p_2) = 0$ and $D_2(p_1, p_2) = 1 + a - p_2$ if $1 - b - ba + bp_2 < p_1$. In the main text, we assume managers compete in prices. Our main results also hold under differentiated quantity competition. See the online appendix (Lee and Turner, 2023) for details.

Firm i's profit is $\pi_i(p_1, p_2) = D_i(p_1, p_2)\,(p_i - c_i)$. Firm i's sales (or revenue) is $s_i(p_1, p_2) = D_i(p_1, p_2)p_i$. Pricing decisions are made by managers who seek to maximize their compensation. Manager i's compensa-

[5] Throughout the text, we refer to a as the degree of asymmetry in product quality between the two products. However, a also captures factors such as differences in advertising or product reputation.

tion, denoted $M_i(p_1, p_2)$, is a weighted sum of profits and sales:[6]

$$M_i(p_1, p_2) = (1 - \theta_i)\, \pi_i(p_1, p_2) + \theta_i s_i(p_1, p_2) \tag{3}$$

where θ_i denotes the weight placed on sales.[7] When $\theta_i = 0$, manager i maximizes firm profit. When $\theta_i = 1$, manager i's compensation depends only on sales. We assume $\theta_i \in [0, 1]$.[8]

Note that manager i's payoff can be re-written as

$$\begin{aligned} M_i(p_1, p_2) &= (1 - \theta_i)\, D_i(p_1, p_2)\, (p_i - c_i) + \theta_i D_i(p_1, p_2) p_i \\ &= D_i(p_1, p_2)\, (p_i - (1 - \theta_i)\, c_i)\,. \end{aligned}$$

A sales weight of θ_i gives managers an incentive to set prices as would a profit-maximizing firm with a marginal cost of $\tilde{c}_i = (1 - \theta_i)\, c_i \leq c_i$. We refer to $\tilde{c}_i$ as a manager's perceived marginal cost. Thus, managerial compensation structures that place a greater weight on sales (i.e., a higher θ_i) have an effect which is equivalent to a reduction in marginal cost. Analogously, $\tilde{c}_2 - a$ denotes firm 2's perceived, quality-adjusted marginal cost, and $\tilde{c}_1$ denotes firm 1's perceived, quality-adjusted marginal cost. Let $X(\theta_1, \theta_2) = |\tilde{c}_2 - \tilde{c}_1 - a| = |c_2\, (1 - \theta_2) - c_1\, (1 - \theta_1) - a|$ denote the perceived asymmetry between managers.[9]

Manager pay in Equation (3) is a function of both profit and sales. Manager pay may be increasing in profit for at least two reasons. First, managers may hold company stock or be paid in stock options. If the value of these assets is positively correlated with firm profit, managers have incentives to maximize profits. Second, if managers generating larger profits are less likely to be fired, then managers will aim to increase firm profits (Sklivas, 1987). Manager pay in Equation (3) may be increasing in sales for at least two reasons. First, manager compensation structures often include sales bonuses.[10] Second, managers may place an intrinsic value on sales volume, regardless of actual compensation (Baumol, 1958; Koplin, 1963; Vickers,

[6] This managerial compensation structure has been explored extensively in prior literature on strategic delegation (e.g., Vickers (1985); Sklivas (1987); Fershtman and Judd (1987); Fershtman (1985)). Fershtman (1985) showed that this objective function can result from bargaining between a manager that wishes to maximize sales and a manager that wishes to maximize profit. In Subsection A.1 of Lee and Turner (2023), we show that the managerial compensation structure in Equation (3) is equivalent to a pay structure that compensates managers on the basis of profits plus the quantity sold times a positive constant (i.e., $\pi_i(p_1, p_2) + \alpha_i D_i(p_1, p_2)$ where $\alpha_i > 0$). Lambertini and Trombetta (2002) (in Appendix A) showed the equivalence of these contract types for symmetric firms.

[7] Manager pay may also consist of a fixed payment. However, a fixed payment does not affect pricing or output decisions and, therefore, does not affect a manager's willingness to collude. For simplicity, any fixed payments are normalized to 0.

[8] We do not consider negative weights on sales or profits. A managerial contract which incentivizes firms to restrict sales is difficult to implement in practice, likely to raise antitrust concerns, and inconsistent with empirical evidence (see Kopel and Pezzino (2018) for a review). If managerial compensation is decreasing in sales, managers have incentives to reduce output or increase price even in the absence of collusion.

[9] The asymmetry between firm 2's quality-adjusted marginal cost and firm 1's quality-adjusted marginal cost is $X(0, 0) = |c_2 - c_1 - a|$. $X(0, 0)$ is closely related to the cost per unit of quality supplied (i.e., the difference between a firm's marginal cost and its inverse demand intercept) as defined in Zanchettin (2006).

[10] Owners may employ sales-based compensation due to the need to incentivize managers to control costs (see Fershtman and Judd (1986) and Szymanski (1994)) or because accurate measures of economic profit may be unavailable and therefore non-contractable.

1985; Hay and Morris, 1979).[11] In summary, the weight θ_i placed on sales in a manager's objective depends on the specific terms of a manager's compensation structure, the likelihood of termination and managers' intrinsic preferences for high sales levels.[12]

Empirical studies of executive incentives plans support the pay structure in Equation (3). An FW Cook report[13] on executive compensation found that 76% of companies use at least two financial measures in their annual employee incentive plans. Additionally, profits and sales are the two most commonly used and heavily weighted financial measures of employee performance with 91% of companies compensating executives, at least party, on the basis of profits and 49% compensating executives on the basis of sales.

We consider an infinitely repeated game where managers seek to maximize the discounted present value of their compensation. Managers have a common discount factor δ. Manager compensation structures are public information[14] and exogenous. In Section 6, we consider an extension of the model introduced in this section wherein owners endogenously choose θ_1 and θ_2 prior to price competition (or collusion) between managers. Following prior literature,[15] manager compensation structures (i.e., θ_1 and θ_2) are fixed for all periods (i.e., contracts cannot be renegotiated). This formulation reflects two considerations. First, managerial compensation contracts are often long term.[16] Second, implicit factors which determine manager incentives (such as the social prestige associated with high sales or the likelihood of termination if profits are low) cannot be easily adjusted by employers.

Following Lambertini and Trombetta (2002), Matsumura and Matsushima (2012), and Bian, Lai and Hua (2013), we assume managers collude by choosing prices to maximize their joint pay (i.e., full collusion): $\max_{p_1,p_2} M_1(p_1,p_2) + M_2(p_1,p_2)$.[17] Collusion is sustained through grim trigger strategies. If any manager does not set the price that maximizes their joint payoff, then both managers revert to Nash equilibrium play

[11] This may occur due to the social prestige or long-term career benefits associated with employment at large (i.e., high sales) firms. For example, if high sales levels indicate managerial skill, then managers may have an incentive to increase sales in order to improve their reputations as effective managers. As Vickers (1985) writes, "[s]everal theories of the firm postulate that the interests of managers - their incomes, status, power, security, etc. lie partly with sales and growth rather than purely with profits."

[12] θ_i in Equation (3) reflects a combination of a manager's wage structure and their intrinsic preferences. For ease of exposition, we refer to a manager's payoff function in Equation (3) as the manager's "compensation" or "pay" throughout the paper, recognizing that other factors not explicitly included in a manager's wage structure can impact their objective and incentives.

[13] See *2019 Annual Incentive Plan Report. FW Cook. October 2019.* https://www.fwcook.com/content/documents/publications/10-17-19_FWC_2019_Incentive_Plan.pdf and Groysberg et al. (2021). The reported focused on the largest 250 companies in the S&P 500 by market cap.

[14] This reflects regulatory requirements which require companies to disclose detailed information regarding executive compensation (Bloomfield, 2021) and is consistent with prior literature (Lambertini and Trombetta, 2002; Wang and Wang, 2021).

[15] Dockner and Löffler (2015) and Mujumdar and Pal (2007) also assume contracts, based on profits and sales, cannot be renegotiated. Matsumura and Matsushima (2012) make a similar assumption in their analysis of collusion when manager compensation depends on relative profits.

[16] As Fershtman and Judd (1987) write, "[w]e view the manager's contracts as being infrequently altered and in force for a substantial amount of time."

[17] In Subsection A.8 of Lee and Turner (2023), we consider an alternative assumption that collusive prices are determined by Nash bargaining between managers, rather than joint profit maximization. Results suggest that sales-based compensation can also facilitate collusion, enhance owner profit, and reduce consumer surplus when collusive prices are determined by Nash bargaining. Additionally, the set of θ_1 and θ_2 values that facilitate collusion under Nash bargaining closely resembles the set of values that facilitate collusion under joint profit maximization (see Condition 1).

for all subsequent periods. Collusion is sustainable if

$$\frac{M_i^C}{1-\delta} \geq M_i^D + \frac{\delta}{1-\delta} M_i^N \tag{4}$$

holds for $i = 1, 2$ where M_i^C denotes manager pay during collusion, M_i^D denotes manager pay during defection, and M_i^N denotes manager pay during Nash competition. Inequality (4) holds when managers are sufficiently patient (i.e., δ is sufficiently close to 1). Let $\delta_i^*(\theta_1, \theta_2)$ (hereafter, manager i's critical discount factor) denote the smallest discount factor such that Inequality (4) holds for manager i when sales weights are θ_1 and θ_2. The critical discount factor $\delta^*(\theta_1, \theta_2) = \max\{\delta_1^*(\theta_1, \theta_2), \delta_2^*(\theta_1, \theta_2)\}$ is the smallest discount factor such that Inequality (4) holds for both managers. Throughout the paper, we assume

$$(1-\tilde{c}_1)\left(1 - \frac{(4-b^2)\sqrt{b^2+8} - b^3}{8}\right) < \tilde{c}_2 - \tilde{c}_1 - a < (1-\tilde{c}_1)\left(1 - \frac{(4-b^2)\sqrt{b^2+8} + b^3}{16-6b^2}\right). \tag{5}$$

When the perceived asymmetry between managers is large and, as a result, Inequality (5) is violated, collusion is unsustainable for all discount factors and, thus, collusion never occurs.[18] Note that $\delta^*(0,0)$ (resp. $\delta^*(1,1)$) represents the critical discount factor when manager compensation depends entirely on firm profit (resp. sales).

During collusion, managers maximize joint compensation. Manager 1 sets price $p_1^C = \frac{1+\tilde{c}_1}{2}$ and manager 2 sets price $p_2^C = \frac{1+\tilde{c}_2+a}{2}$.[19] Collusive manager payoffs are

$$M_1^C = \frac{1}{1-b^2}\left(\frac{1-\tilde{c}_1}{2}\right)\left(\frac{1-\tilde{c}_1 - b(1-\tilde{c}_2+a)}{2}\right)$$

and

$$M_2^C = \frac{1}{1-b^2}\left(\frac{1-\tilde{c}_2+a}{2}\right)\left(\frac{1-\tilde{c}_2+a - b(1-\tilde{c}_1)}{2}\right).$$

When defecting, a manager chooses the price that maximizes their compensation given their rival charges their collusive price. Defection payoffs, and therefore the critical discount factor, depend crucially on the degree of product differentiation, determined by the parameter b. If products are sufficiently homogeneous, a manager serves all demand when defecting. Otherwise, a defecting manager serves only a portion of total demand.

[18]As the focus of the current study is the potential for sales-based compensation to facilitate collusion, we restrict attention to circumstances under which this outcome is possible. Formally, Inequality (5) implies $\delta^*(\theta_1, \theta_2) < 1$. See Subsection A.4 in the online appendix (Lee and Turner, 2023) for a proof.

[19]Note that both firms produce during collusion due to the presence of horizontal product differentiation between the two firms' products and Equation (5) which ensures the degree of perceived asymmetry between managers is sufficiently moderate. If firms sold a homogenous product, joint profit/payoff maximization would dictate that the entirely of output should be produced by the more efficient firm.

Manager 1's defection payoffs are[20]

$$M_1^D = \begin{cases} \frac{(2(1-\tilde{c}_1)-b(1-\tilde{c}_2+a))^2}{16(1-b^2)} & \text{if } \frac{-(1-\tilde{c}_1)+\sqrt{(1-\tilde{c}_1)^2+2(1-\tilde{c}_2+a)^2}}{1-\tilde{c}_2+a} > b \\ \left(\frac{1-\tilde{c}_2+a}{2b}\right)\left(\frac{2b-a+\tilde{c}_2-1}{2b} - \tilde{c}_1\right) & \text{if } \frac{-(1-\tilde{c}_1)+\sqrt{(1-\tilde{c}_1)^2+2(1-\tilde{c}_2+a)^2}}{1-\tilde{c}_2+a} \leq b \end{cases}. \tag{6}$$

Manager 2's defection payoffs are

$$M_2^D = \begin{cases} \frac{(2(1-\tilde{c}_2+a)-b(1-\tilde{c}_1))^2}{16(1-b^2)} & \text{if } \frac{-(1-\tilde{c}_2+a)+\sqrt{(1-\tilde{c}_2+a)^2+2(1-\tilde{c}_1)^2}}{1-\tilde{c}_1} > b \\ \left(\frac{1-\tilde{c}_1}{2b}\right)\left(\frac{2b+2ab+\tilde{c}_1-1}{2b} - \tilde{c}_2\right) & \text{if } \frac{-(1-\tilde{c}_2+a)+\sqrt{(1-\tilde{c}_2+a)^2+2(1-\tilde{c}_1)^2}}{1-\tilde{c}_1} \leq b \end{cases}. \tag{7}$$

Nash equilibrium payoffs are

$$M_1^N = \frac{\left(\left(2-b^2\right)\left(1-\tilde{c}_1\right) - b\left(1-\tilde{c}_2+a\right)\right)^2}{\left(4-b^2\right)^2\left(1-b^2\right)}$$

and

$$M_2^N = \frac{\left(\left(2-b^2\right)\left(1-\tilde{c}_2+a\right) - b\left(1-\tilde{c}_1\right)\right)^2}{\left(4-b^2\right)^2\left(1-b^2\right)}.$$

Manager 1's critical discount factor is $\delta_1^*(\theta_1,\theta_2) = \frac{M_1^D-M_1^C}{M_1^D-M_1^N}$, and manager 2's critical discount factor is $\delta_2^*(\theta_1,\theta_2) = \frac{M_2^D-M_2^C}{M_2^D-M_2^N}$.

Proposition 1. *i)* $\delta_1^*(\theta_1,\theta_2) = \delta_2^*(\theta_1,\theta_2)$ *when* $\tilde{c}_2 - a = \tilde{c}_1$,

ii) $\delta_1^*(\theta_1,\theta_2) < \delta_2^*(\theta_1,\theta_2)$ *when* $\tilde{c}_2 - a > \tilde{c}_1$, *and*

iii) $\delta_1^*(\theta_1,\theta_2) > \delta_2^*(\theta_1,\theta_2)$ *when* $\tilde{c}_2 - a < \tilde{c}_1$.

Proposition 1 establishes that the critical discount factor is determined by the critical discount factor of the manager with the higher perceived, quality-adjusted marginal cost. Put differently, the less efficient (in terms of perceived, quality-adjusted marginal cost) manager has weaker incentives to collude and, thus, a higher critical discount factor. To understand this result, recall that managers maximize their joint pay during collusion. Thus, joint payoff maximization results in managers setting prices such that the majority of output/demand is produced by the firm with the lower perceived, quality-adjusted marginal cost of production. Thus, the less efficient manager produces only a relatively small fraction of total output during collusion. As a result, the less efficient manager earns a relatively low payoff from collusion and, thus, has a relatively strong incentive to deviate and undercut its rival.[21] Thus, the critical discount factor of the

[20] When manager payoffs are symmetric (i.e., $\tilde{c}_1 = \tilde{c}_2 - a$), the threshold value for b in Equation (6) and (7) is $\sqrt{3}-1 \approx .73$, which is consistent with prior literature (Majerus, 1988; Albæk and Lambertini, 1998).

[21] In other settings (e.g., homogenous product competition (Ivaldi et al., 2003)), low cost firms have the strongest incentive to defect from collusion. Analogously, the high quality firm may have the strongest incentive to deviate from collusion under

manager with the higher perceived, quality-adjusted marginal cost is greater than the critical discount factor of the manager with the lower perceived, quality-adjusted marginal cost.

3 Homogeneous Firms

Before analyzing the effect of sales-based compensation on the sustainability of collusion when firms are heterogeneous, we first present the case of homogeneous firms.[22] Firms are homogeneous when $c_1 = c_2 \equiv c$ and $a = 0$.

Proposition 2. *Suppose* $c_1 = c_2$ *and* $a = 0$. *Then,*

(i) $\delta^*(\theta_1, \theta_2) = \delta^*(0, 0)$ *if* $\theta_1 = \theta_2$, *and*

(ii) $\delta^*(\theta_1, \theta_2) > \delta^*(0, 0)$ *if* $\theta_1 \neq \theta_2$.

Proposition 2 part (i) reports that the critical discount factor under sales-based compensation is the same as the critical discount factor under strictly profit-based compensation when compensation structures are identical (i.e., $\theta_1 = \theta_2$).[23] Thus, sales-based compensation has no impact on the sustainability of collusion. Proposition 1 in Lambertini and Trombetta (2002) provides an analogous result under homogeneous product Cournot competition: when managerial sales weights are symmetric, the sustainability of collusion is unaffected by sales-based compensation structures. This result occurs because sales-based compensation has an effect which is equivalent to a reduction in marginal cost. However, when firms are symmetric, marginal costs cancel out of the critical discount factor and, as a result, the sustainability of collusion is unaffected. In summary, Proposition 2 part (i) reflects Lambertini and Trombetta (2002)'s important finding, extending their insight to the case of differentiated product price competition.[24]

Proposition 2 part (ii) reports that asymmetric sales weights (i.e., $\theta_1 \neq \theta_2$) hinder the sustainability of collusion.[25] This occurs because a difference in sales weights generates an asymmetry in the objectives of managers. To see this, suppose that $\theta_1 > \theta_2$ (i.e., manager 1's compensation is more strongly related to sales than manager 2's compensation). Recall that a positive sales weight causes a reduction in a manager's perceived marginal cost. The difference in perceived marginal cost when $\theta_1 > \theta_2$ is $(1 - \theta_2)\,c - (1 - \theta_1)\,c = (\theta_1 - \theta_2)\,c > 0$ while the difference in perceived marginal cost is 0 when $\theta_1 = \theta_2$. Thus, an asymmetry in

certain models (e.g., Häckner (1994)). Our main qualitative findings are not dependent on the fact that high cost (and/or low product quality) firms have the greatest incentives to defect from collusion in our model. Sales-based compensation effectively reduces asymmetries between firms which is likely to facilitate collusion regardless of which firm is most likely to defect.

[22] Collusion under sales-based compensation and homogeneous firms was first analyzed, in a homogeneous product Cournot framework, by Lambertini and Trombetta (2002).

[23] $\delta^*(0,0) = \begin{cases} \frac{(2-b)^2}{b^2-8b+8} & \text{if } b < -1+\sqrt{3} \\ \frac{(2-b)^2(1-b-b^2)}{4-8b+b^2+3b^3-2b^4} & \text{if } b \geq -1+\sqrt{3} \end{cases}$ when $c_1 = c_2$ and $a = 0$.

[24] This finding also holds under differentiated product quantity competition (Lee and Turner, 2023).

[25] Lambertini and Trombetta (2002) assume managers do not collude when sales weights are asymmetric because, in their homogeneous product Cournot framework, asymmetric sales weights result in one firm producing all output during collusion.

sales weights causes an asymmetry in perceived marginal cost which hinders collusion (Ivaldi et al., 2003; Rothschild, 1999).

4 Sustainability of Collusion

In this section, we analyze the impact of sales-based compensation on the sustainability of collusion when firms are heterogenous.

Condition 1. $X(\theta_1, \theta_2) = |(1 - \theta_2)\, c_2 - (1 - \theta_1)\, c_1 - a| < |c_2 - c_1 - a| = X(0, 0)$

Proposition 3 presents our main conclusion in this setting. The proposition refers to Condition 1.

Proposition 3. $\delta^*(\theta_1, \theta_2) < \delta^*(0, 0)$ *if Condition 1 holds.*

Proposition 3 states that sales-based compensation can facilitate collusion for θ_1 and θ_2 values that reduce the perceived asymmetry $X(\theta_1, \theta_2)$ between managers. When $\theta_1 = \theta_2 = 0$, both managers maximize firm profit. However, due to differences in marginal cost and/or product quality, collusion is difficult to sustain (Rothschild, 1999; Ivaldi et al., 2003; Villar et al., 2012; Vasconcelos, 2005; Häckner, 1994). In the current setting, the assumption of joint profit maximization during collusion results in colluding managers setting prices such that a large share of output is produced by the firm with the lowest quality-adjusted marginal cost. Formally, the majority of output is produced by firm 2 if $c_2 - a < c_1$, and the majority of output is produced by firm 1 if $c_1 < c_2 - a$. As a result, the firm with the higher quality-adjusted marginal cost earns relatively low profits from collusion and, thus, has a relatively strong incentive to deviate and undercut the low cost firm (as demonstrated in Proposition 1). In summary, the asymmetry between the two firms makes collusion difficult to sustain when managers are compensated solely on the basis of profit. However, sales-based compensation can effectively reduce the asymmetry between managers, enhancing the sustainability of collusion. As discussed in Section 2, sales-based compensation results in a perceived marginal cost of $\tilde{c}_i = (1 - \theta_i)\, c_i$ for manager i. When θ_1 and θ_2 satisfy Condition 1, sales-based compensation results in a perceived degree of asymmetry between managers which is less than the true asymmetry. By reducing the perceived degree of asymmetry, sales-based compensation facilitates collusion. Intuitively, when Condition 1 holds, sales-based compensation homogenizes the payoff functions of managers and aligns their objectives, facilitating collusion.

To illustrate the range of managerial compensation structures (i.e., sales weights θ_1 and θ_2) that satisfy Condition 1, we analyze separately the cases of marginal cost and product quality asymmetries. First, we assume firms differ in marginal cost but offer products of equivalent quality (i.e., $a = 0$ and $c_1 \neq c_2$). In this case, Proposition 3 states that sales-based compensation can reduce the critical discount factor

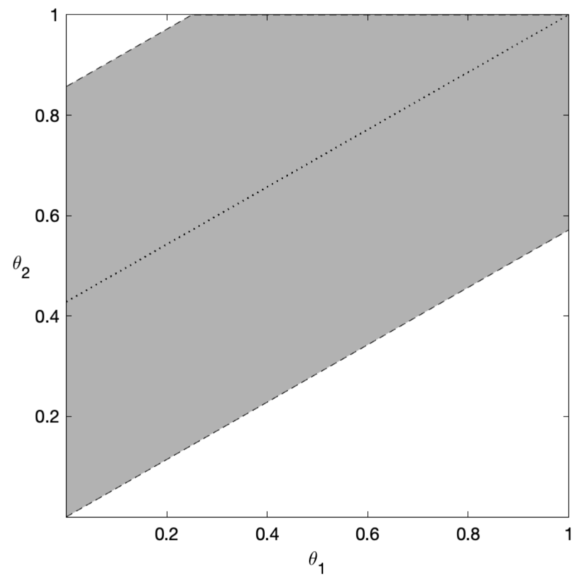

Figure 1: θ_1 and θ_2 values where Condition 1 is satisfied when $c_1 = .1$, $c_2 = .175$, and $a = 0$. The dotted line denotes θ_1 and θ_2 values for which $(1 - \theta_2)\, c_2 = (1 - \theta_1)\, c_1$.

for θ_1 and θ_2 values that homogenize the perceived marginal costs of managers. Formally, Condition 1 requires $|(1 - \theta_2)\, c_2 - (1 - \theta_1)\, c_1| < |c_2 - c_1|$. In this case, Condition 1 holds for any degree of asymmetry if $\theta_1 = \theta_2 = 1$ (i.e., managers are only concerned with maximizing sales and not profits). Condition 1 also holds for any $\theta_1 = \theta_2 > 0$ (i.e., any symmetric manager pay structures that place a nonzero weight on sales).

Condition 1 is also satisfied by more asymmetric compensation structures. For example, suppose that firm 2 is the high cost firm (i.e., $c_2 > c_1$). Additionally, suppose firm 2's manager is compensated solely on the basis of sales (i.e., $\theta_2 = 1$) and firm 1's manager is compensated on the basis of both profits and sales with $\theta_1 = \frac{1}{2}$. Collusion is easier to sustain under this compensation structure than under strictly profit-based compensation structures (i.e., $\theta_1 = \theta_2 = 0$) if $\frac{3}{2} c_1 < c_2$ (i.e., the asymmetry in marginal costs is sufficiently large). Figure 1 depicts θ_1 and θ_2 values that satisfy Condition 1 when $c_1 = .1$ and $c_2 = .175$. The dotted line in Figure 1 denotes θ_1 and θ_2 values where $(1 - \theta_2)\, c_2 = (1 - \theta_1)\, c_1$ and, as a result, $\delta^*(\theta_1, \theta_2)$ equals the critical discount factor when firms are homogenous (as in Section 3) and managerial compensation structures are symmetric.[26]

Next, we assume both firms' marginal cost of production is c, but firm 2's product is of a higher quality than firm 1's product (i.e., $a > 0$ and $c_2 = c_1 = c$). In this case, the perceived asymmetry between the two managers is $X(\theta_1, \theta_2) = |(1 - \theta_2)\, c - (1 - \theta_1)\, c - a| = |(\theta_1 - \theta_2)\, c - a|$. When manager compensation

[26] Throughout the paper, parameter values in figures are chosen to illustrate the boundaries of the relevant condition or set of θ_1 and θ_2 values, not necessarily to demonstrate the size of the corresponding set. For some parameter values, the corresponding condition is satisfied for all $\theta_1 \in [0, 1]$ and/or $\theta_2 \in [0, 1]$.

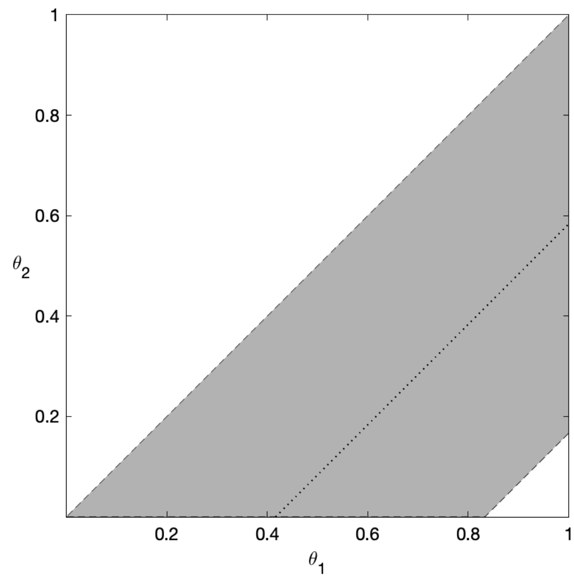

Figure 2: θ_1 and θ_2 values where Condition 1 is satisfied when $c = .3$ and $a = .125$. The dotted line denotes θ_1 and θ_2 values for which $(\theta_1 - \theta_2)\,c = a$.

structures are entirely based on profits ($\theta_1 = \theta_2 = 0$): $X(0,0) = a$. Thus, sales-based compensation reduces the perceived asymmetry, and therefore facilitates collusion, if $|(\theta_1 - \theta_2)\,c - a| < a$ (or $\theta_1 > \theta_2$ and $\frac{(\theta_1-\theta_2)c}{2} < a$). In summary, sales-based compensation facilitates collusion between managers offering products of differing quality when the compensation of the manager producing the lower quality product is tied more strongly to sales levels than that of the manager producing the higher quality product (i.e., $\theta_1 > \theta_2$). Additionally, sales weights must be sufficiently symmetric (i.e., $\theta_1 - \theta_2 < \frac{2a}{c}$) for sales-based compensation to facilitate collusion. This is the case because highly asymmetry compensation structures (e.g., $\theta_1 = 1$ and $\theta_2 = 0$) can increase the perceived asymmetry between managers, hindering collusion. Figure 2 depicts θ_1 and θ_2 values that satisfy Condition 1 when $c = .3$ and $a = .125$. The dotted line in Figure 2 denotes θ_1 and θ_2 values where $(\theta_1 - \theta_2)\,c = a$ and, as a result, $\delta^*(\theta_1, \theta_2)$ equals the critical discount factor when firms are homogenous (as in Section 3) and managerial compensation structures are symmetric.

5 Firm Profit and Consumer Surplus

Section 4 demonstrates that sales-based compensation can facilitate collusion when firms are heterogeneous. In this section, we explore the impact of collusion, between managers compensated on the basis of sales, on firm profit and consumer surplus.

First, we explore the impact of sales-based compensation on firm profit. Specifically, we compare firm profit under sales-based compensation with firm profit under strictly profit-based compensation. Let $\pi_i^C(\theta_1, \theta_2)$ denote firm i's profit when manager 1's sales weight is θ_1, manager 2's sales weight is θ_2, and managers collude when setting prices (i.e., managers set prices which maximize their joint pay). Analogously, let $\pi_i^N(\theta_1, \theta_2)$ denote firm i's profit when managers compete when setting prices. Let

$$\pi_i(\theta_1, \theta_2) = \begin{cases} \pi_i^C(\theta_1, \theta_2) & \text{if } \delta \geq \delta^*(\theta_1, \theta_2) \\ \pi_i^N(\theta_1, \theta_2) & \text{if } \delta < \delta^*(\theta_1, \theta_2) \end{cases} \tag{8}$$

denote firm i's profit, assuming that managers will collude if $\delta \geq \delta^*(\theta_1, \theta_2)$ and will engage in Nash competition otherwise.

Proposition 3 shows that sales-based compensation can facilitate collusion when firms are asymmetric. All else equal, manager collusion increases firm profits (we refer to this effect as the collusive effect of sales-based compensation). However, sales-based compensation can also reduce firm profit. This is the case because sales-based compensation distorts the pricing decisions of managers. A manager compensated on the basis of sales wishes to reduce their price below the profit maximizing level in order to increase sales (we refer to this effect as the price-distorting effect of sales-based compensation).[27] When managers reduce prices below the profit maximizing level, firm profit is reduced.

Thus, there are two effects which determine the impact of sales-based compensation on firm profit: the collusive effect and the price-distorting effect. These two effects point in opposite directions. In this section, we establish when the collusive effect can dominate and, as a result, sales-based compensation can increase firm profit.

Condition 2. *a)* $\theta_1 < \min\left\{\frac{1-c_1}{c_1}, \frac{(1-\tilde{c}_2+a)-b(1-c_1)}{bc_1} + 4\frac{\left((2-b^2)(1-c_2+a)-b(1-c_1)\right)^2}{b(4-b^2)^2(2c_2-\tilde{c}_2-a-1)c_1}\right\}$ *and*
$\theta_2 < \min\left\{\frac{1-c_2+a}{c_2}, \frac{(1-\tilde{c}_1)-b(1-c_2+a)}{bc_2} + 4\frac{\left((2-b^2)(1-c_1)-b(1-c_2+a)\right)^2}{b(4-b^2)^2(2c_1-\tilde{c}_1-1)c_2}\right\}$
b) $\delta^*(\theta_1, \theta_2) \leq \delta < \delta^*(0, 0) < 1$

Proposition 4 compares firm profit under sales-based compensation with firm profit under profit-based compensation (i.e., $\theta_1 = \theta_2 = 0$).

Proposition 4. $\pi_1(\theta_1, \theta_2) > \pi_1(0, 0)$ *and* $\pi_2(\theta_1, \theta_2) > \pi_2(0, 0)$ *if Condition 2 holds.*

Proposition 4 establishes that sales-based compensation may increase firm profit if sales-based compensation induces manager collusion. Condition 2 has two parts. Condition 2a determines the set of θ_1 and θ_2 values where the collusive effect of sales-based compensation (which raises prices and owner prof-

[27] Sales-based compensation reduces a manager's perceived marginal cost below the actual level of marginal cost (i.e., $(1-\theta_i)c_i < c_i$). This causes managers to price more aggressively (i.e., set lower prices) than under profit-based compensation.

its) dominates the price distorting effect (which reduces prices and owner profits). Thus, Condition 2a ensures that, if collusion occurs under sales-based compensation but not under profit-based compensation, owner profits under sales-based compensation will exceed owner profits under profit-based compensation (i.e., $\pi_1^C(\theta_1, \theta_2) > \pi_1^N(0,0)$ and $\pi_2^C(\theta_1, \theta_2) > \pi_2^N(0,0)$). The price-distorting effect is larger when managerial compensation is more strongly linked to sales (i.e., larger sales weights) because managers, seeking to maximize their compensation, have a relatively strong incentive to reduce price in order to increase sales. Contrarily, if managerial compensation structures are not tied too strongly to sales levels (i.e., θ_1 and θ_2 are sufficiently small) and are instead, partially, dependent on firm profits, the magnitude of the price distorting effect is reduced and managers set higher prices during collusion, enhancing owners' profits. Therefore, Condition 2a is satisfied for sufficiently small values of θ_1 and θ_2.

Condition 2b ensures that managers will collude under sales-based compensation but not under profit-based compensation.[28] Condition 2b stipulates that managers' discount factor δ must exceed the critical discount factor under sales-based compensation (ensuring collusion is sustainable when managers are compensated on the basis of sales), but not exceed the critical discount factor under solely profit-based compensation (ensuring collusion is not sustainable if managers are compensated solely on the basis of profit). Recall that $\delta^*(\theta_1, \theta_2)$ is increasing in managers' perceived asymmetry $X(\theta_1, \theta_2)$. Thus, Condition 2b is most likely to hold for sales weights θ_1 and θ_2 that substantially reduce the perceived asymmetry between managers (see Figure 1 and 2). Altogether, Proposition 4 implies that, when Condition 2 holds, managers will collude under sales-based compensation but not under solely profit-based compensation, and such collusion will raise the profits of owners (as the collusive effect will dominate the price-distorting effect). Figure 3 depicts θ_1 and θ_2 values that satisfy Condition 2 when $c_1 = .1$, $c_2 = .15$, $b = .2$, $a = 0$, and $\delta = .56$.

Next, we analyze the impact of sales-based compensation on consumer surplus. Let $CS^C(\theta_1, \theta_2)$ denote consumer surplus when manager 1's sales weight is θ_1, manager 2's sales weight is θ_2, and managers collude when setting prices. Analogously, let $CS^N(\theta_1, \theta_2)$ denote consumer surplus when managers compete. Let

$$CS(\theta_1, \theta_2) = \begin{cases} CS^C(\theta_1, \theta_2) & \text{if } \delta \geq \delta^*(\theta_1, \theta_2) \\ CS^N(\theta_1, \theta_2) & \text{if } \delta < \delta^*(\theta_1, \theta_2) \end{cases} \tag{9}$$

denote consumer surplus, assuming that managers will collude if $\delta \geq \delta^*(\theta_1, \theta_2)$ and will engage in Nash competition otherwise.

[28] The inequality $\delta^*(0,0) < 1$ in Condition 2b holds when the asymmetry between firms (i.e., $c_2 - c_1 - a$) is moderate (see Lee and Turner (2023) Subsection A.4 for a proof). When the efficiency advantage of one firm (relative to its rival) is extreme, the more efficient firm is a monopolist and serves all demand (and, thus, earns relatively large profits) in the Nash equilibrium under profit-based compensation. Thus, the owner of the more efficient firm is unlikely to benefit from sales-based compensation and managerial collusion.

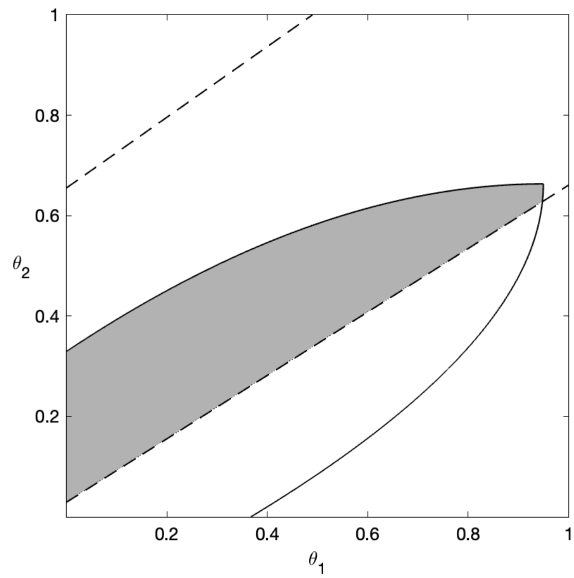

Figure 3: θ_1 and θ_2 values where Condition 2 is satisfied when $c_1 = .1$, $c_2 = .15$, $b = .2$, $a = 0$, and $\delta = .56$. The solid lines delineate θ_1 and θ_2 values satisfying Condition 2a. The dashed lines delineate θ_1 and θ_2 values satisfying Condition 2b.

In a static setting without the possibility of collusion, sales-based compensation results in a higher level of consumer surplus than profit-based compensation. This is the case because managers reduce their prices below the profit-maximizing level in order to increase sales and, as a result, their compensation (the price-distorting effect). However, this result does not necessarily hold in a dynamic setting where managers can collude. As shown in Proposition 3, sales-based compensation can reduce the critical discount factor and make collusion feasible in markets where collusion would not occur if managers were compensated only on the basis of profit. Collusion results in elevated prices (the collusive effect) and, all else equal, reduces consumer welfare.

Thus, sales-based compensation has two effects (i.e., the collusive effect and price-distorting effect) on consumer surplus which point in opposite directions. Next, we show that the collusive effect can dominate and, as a result, sales-based compensation may result in diminished consumer welfare.

Condition 3. *a)* $\theta_1 < \frac{1}{c_1}\left(\frac{2b(1-c_2+a)+b^2(1-c_1)}{4-b^2}\right)$ *and* $\theta_2 < \frac{1}{c_2}\left(\frac{2b(1-c_1)+b^2(1-c_2+a)}{4-b^2}\right)$

b) $\delta^*(\theta_1, \theta_2) \leq \delta < \delta^*(0,0) < 1$

Proposition 5 compares consumer surplus under sales-based compensation with consumer surplus under profit-based compensation.

Proposition 5. $CS(\theta_1, \theta_2) < CS(0,0)$ *if Condition 3 holds.*

Proposition 5 reports that sales-based compensation may reduce consumer surplus. Condition 3 has two parts. Condition 3a ensures that, if managers collude under sales-based compensation but not under profit-based compensation, then consumer surplus under profit-based compensation exceeds the level of consumer surplus under sales-based compensation (i.e., $CS^C(\theta_1, \theta_2) < CS^N(0,0)$). Condition 3a is satisfied when the collusive effect of sales-based compensation (which reduces consumer surplus) dominates the price-distorting effect, which occurs for sufficiently small values of θ_1 and θ_2. When sales weights are sufficiently small, managers do not face strong incentives to reduce price in order to increase sales (i.e., the price-distorting effect is weak). As a result, the collusive effect dominates and prices under sales-based compensation (when managers collude) exceed prices under profit-based compensation (when managers compete), which results in diminished consumer surplus.

Condition 3b (which mirrors Condition 2b) stipulates that managers' discount factor δ must exceed the critical discount factor under sales-based compensation (ensuring collusion is sustainable when managers are compensated on the basis of sales), but not the critical discount factor under solely profit-based compensation (ensuring collusion is not sustainable if managers are compensated solely on the basis of profit).[29] Altogether, Proposition 5 implies that, when Condition 3 holds, managers will collude under sales-based compensation but not under solely profit-based compensation, and such collusion will result in reduced consumer surplus. Figure 3 depicts θ_1 and θ_2 values that satisfy Condition 3 when $c_1 = .1$, $c_2 = .175$, $a = 0$, $b = .2$, and $\delta = .58$.

[29]The inequality $\delta^*(0,0) < 1$ in Condition 3b holds when the asymmetry between firms (i.e., $c_2 - c_1 - a$) is moderate (see Lee and Turner (2023) Subsection A.4 for a proof). When the efficiency advantage of one firm (relative to its rival) is extreme, the more efficient firm is a monopolist and serves all demand in the Nash equilibrium under profit-based compensation. The product variety offered by the less efficient firm is not purchased, and consumers pay relatively high prices for the other product variety (which results in a relatively low level of consumer surplus). In this case, sales-based compensation (which, all else equal, lowers prices due to the price-distorting effect) and collusion (which, to be sustainable, must involve positive production by both firms) is less likely to harm consumers.

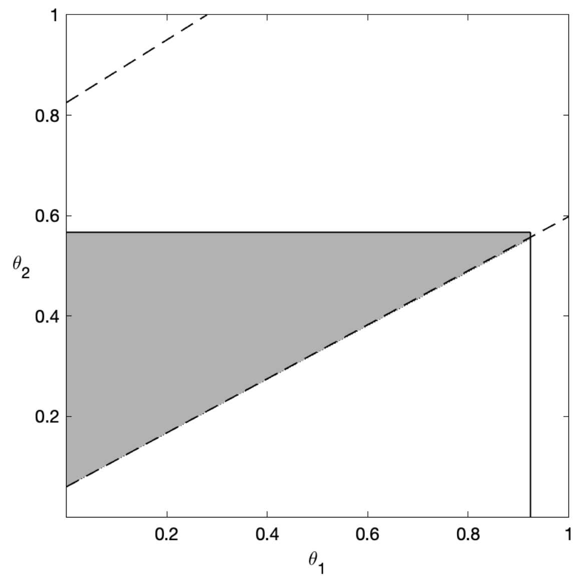

Figure 4: θ_1 and θ_2 values where Condition 3 is satisfied when $c_1 = .1$, $c_2 = .175$, $a = 0$, $b = .2$ and $\delta = .58$. The solid lines delineate θ_1 and θ_2 values satisfying Condition 3a. The dashed lines delineate θ_1 and θ_2 values satisfying Condition 3b.

6 Strategic Delegation

The results of Section 4 and Section 5 demonstrate that sales-based compensation can facilitate collusion between managers and, as a result, increase owners' profits. This suggests that owners may wish to strategically set sales weights in managers' wage contracts in order to induce managers to collude. In this section, we explore this possibility by considering a model of strategic delegation (Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987) wherein θ_1 and θ_2 are determined endogenously by owners. The timing of the game proceeds as follows. In the initial stage (prior to price setting by managers), each owner determines their respective manager's compensation structure. Formally, owners simultaneously and publicly choose sales weights θ_1 and θ_2. After sales weights are determined in the initial stage, the game proceeds according to the model of Section 2. Specifically, managers engage in an infinitely repeated pricing game where managerial compensation contracts are as chosen by owners in the initial stage. Owner i seeks to maximize the discounted present value of firm i's profit while manager i seeks to maximize the discounted present value of their pay.[30]

As in Section 2, collusion is sustained through grim trigger strategies. Specifically, if any manager does not set the price that maximizes their joint payoff in any period, then both managers revert to Nash equilibrium

[30]We consider two alternative delegation games, for comparison, in Subsection A.7 of Lee and Turner (2023). In Subsection A.7.2 of Lee and Turner (2023), we analyze a two stage game wherein owners select sales weights in stage 1 and managers set prices competitively in stage 2. In Subsection A.7.3 of Lee and Turner (2023), we consider a two stage game wherein owners select sales weights in stage 1 and managers set prices collusively in stage 2 (regardless of the sales weights chosen in stage 1).

play for all subsequent periods. We maintain this assumption when modeling deviations by owners: If any owner does not set the agreed upon sales weight, then both managers engage in static Nash equilibrium play in perpetuity. Thus, a deviation by either owner in the initial stage is punished by an infinite repetition of Nash equilibrium play between managers.

The model of this section involves a number of assumptions. First, following prior literature (Dockner and Löffler, 2015; Mujumdar and Pal, 2007; Matsumura and Matsushima, 2012), we assume that owners' choices of θ_1 and θ_2 cannot be changed or renegotiated in subsequent periods (e.g., after the defection of a manager). This assumption is likely to be appropriate when managerial compensation contracts are lengthy relative to the duration of a period in the model (which is determined by the frequency of price adjustments in an industry). Additionally, this assumption is likely to be appropriate when the adjustment of compensation contracts is costly.[31] Second, we assume both managers revert to Nash equilibrium play for all subsequent periods (i.e., grim trigger strategies) if either manager does not set the collusive price or if any owner does not select the agreed upon sales weights in the initial stage. An alternative assumption would allow managers to collude, if sustainable, after deviations by owners in the initial stage. We leave the exploration of this possibility to future research.[32]

Let $M_i^C(\theta_1, \theta_2)$ denote manager i's per-period payoff during collusion when sales weights are θ_1 and θ_2. $\pi_i^C(\theta_1, \theta_2)$ denotes firm i's per-period profit during collusion when sales weights are θ_1 and θ_2. Let $M_i^D(\theta_1, \theta_2)$ denote manager i's payoff when defecting. Let $M_i^N(\theta_1, \theta_2)$ and $\pi_i^N(\theta_1, \theta_2)$ denote manager i's payoff and owner i's profit, respectively, in the static Nash equilibrium. We explore the possibility of owners strategically setting managerial compensation structures in order to induce or facilitate collusion between managers. For this to occur, two conditions must be satisfied. First, the sales weights chosen in the initial stage, denoted θ_1^* and θ_2^*, must be such that no owner wishes to deviate by compensating their manager according to an alternative pay structure. Formally, $\pi_1^C(\theta_1^*, \theta_2^*) \geq \pi_1^N(\theta_1, \theta_2^*)$ for all $\theta_1 \in [0, 1]$, and $\pi_2^C(\theta_1^*, \theta_2^*) \geq \pi_2^N(\theta_1^*, \theta_2)$ for all $\theta_2 \in [0, 1]$, must hold. These inequalities are, hereafter, referred to as the owner incentive compatibility constraints. Second, collusion between managers must be sustainable when sales weights are θ_1^* and θ_2^* (i.e., manager collusion must be sustainable in the (θ_1^*, θ_2^*) sub-game), which occurs if

$$\frac{M_i^C(\theta_1^*, \theta_2^*)}{1-\delta} \geq M_i^D(\theta_1^*, \theta_2^*) + \delta \frac{M_i^N(\theta_1^*, \theta_2^*)}{1-\delta}$$

for $i = 1, 2$, or, as shown in Section 2, $\delta \geq \delta^*(\theta_1^*, \theta_2^*)$. A pair of sales weights (θ_1^*, θ_2^*) is *collusion-compatible*

[31] Recall that sales weights may reflect intrinsic managerial preferences to maximize sales (or profit). In this case, adjusting θ_i may require terminating the contract of a manager and hiring a new manager with differing preferences.

[32] Allowing managers to collude after owner deviations introduces additional complications. For example, owners may have an incentive to marginally adjust their sales weights in order to increase firm profit while maintaining the sustainability of collusion between managers.

if θ_1^* and θ_2^* satisfy the incentive compatibility constraints of owners and collusion between managers compensated according to θ_1^* and θ_2^* is sustainable.

Condition 4. *a)* $\theta_1^* \leq \frac{b(1-\tilde{c}_2+a)+\sqrt{(2c_1-b(1-\tilde{c}_2+a))^2+4K_1}}{2c_1}$ *where*
$K_1 = (1-2c_1)(1-b(1-\tilde{c}_2+a)) - \frac{4\left(\left(2-b^2\right)(1-c_1)-b(1-\tilde{c}_2+a)\right)^2}{(4-b^2)^2}$, *and*
$\theta_2^* \leq \frac{b(1-\tilde{c}_1)+\sqrt{(2c_2-b(1-\tilde{c}_1))^2+4K_2}}{2c_2}$ *where*
$K_2 = (1-2c_2+a)(1+a-b(1-\tilde{c}_1)) - \frac{4\left(\left(2-b^2\right)(1-c_2+a)-b(1-\tilde{c}_1)\right)^2}{(4-b^2)^2}$

b) $\delta \geq \delta^*(\theta_1^*, \theta_2^*)$

Proposition 6 presents our main result in this section. The proposition refers to Condition 4.

Proposition 6. (θ_1^*, θ_2^*) *is collusion-compatible if Condition 4 holds.*

Condition 4 characterizes the set of collusion-compatible managerial compensation structures (i.e., sales weights for which collusion between managers is sustainable and no owner wishes to defect in the initial stage). For Condition 4 to hold, the managerial compensation structures selected by owners in the initial stage (i.e., the sales weights θ_1^* and θ_2^*) must satisfy two restrictions (Condition 4a and Condition 4b). First, Condition 4a requires θ_1^* and θ_2^* to be sufficiently small. Put differently, manager pay must not be tied too strongly to sales levels. Recall, from Section 5, that sales-based compensation induces managers to set prices below profit-maximizing levels (referred to as the price-distorting effect of sales-based compensation) in order to increase sales/pay, reducing owner profits during collusion. The magnitude of the price-distorting effect is large when managerial compensation is primarily based on sales (i.e., θ_i is close to 1). Due to this effect, owners do not gain significantly from collusion between managers compensated primarily on the basis of sales. As a result, owners have an incentive to deviate in the initial stage, despite such a deviation disrupting collusion between managers. Thus, the sales weights θ_1^* and θ_2^* must be sufficiently small that both owners earn sufficient profits during collusion and have no incentive to defect by selecting an alternative managerial compensation structure (i.e., the owner incentive compatibility constraints are satisfied).

Second, Condition 4b ensures that the compensation structures selected in the initial stage are compatible with collusion between managers. Put differently, the sales weights chosen in the initial stage must result in a sufficiently small critical discount factor $\delta^*(\theta_1, \theta_2)$ that collusion between managers, with a common discount factor δ, is sustainable. As shown in Proposition 3, managerial compensation structures that reduce the perceived asymmetry $X(\theta_1, \theta_2)$ between managers facilitate collusion (i.e., reduce the critical discount factor).[33] Thus, Condition 4b is more likely to be satisfied when θ_1^* and θ_2^* substantially reduce the

[33] Note that, when firms are homogenous, sales-based compensation either hinders or does not impact the sustainability of collusion between managers (see Proposition 2). Additionally, sales-based compensation tends to reduce firm profit due to the price distorting effect. Thus, owners of homogenous firms are unlikely to employ sales-based compensation as a means of inducing collusion between managers.

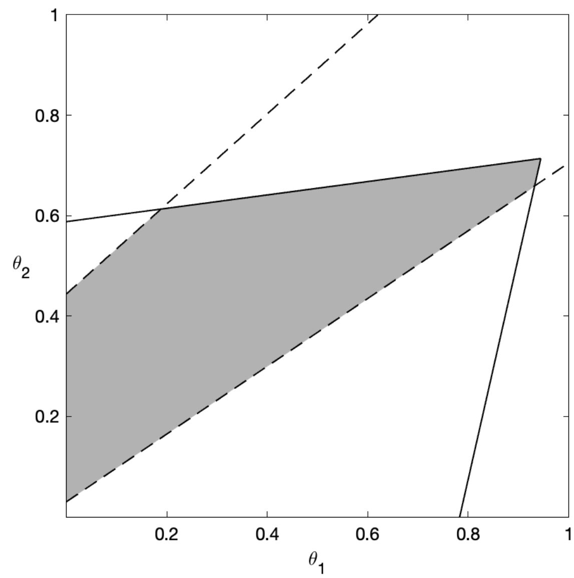

Figure 5: θ_1 and θ_2 values where Condition 4 is satisfied when $c_1 = .35$, $c_2 = .45$, $b = .375$, $a = 0$, and $\delta = .7$ The solid lines delineate θ_1 and θ_2 values satisfying Condition 4a. The dashed lines delineate θ_1 and θ_2 values satisfying Condition 4b.

perceived asymmetry between managers. Contrarily, managerial compensation structures that exacerbate asymmetries between managers (e.g., $\theta_1^* = 1$ and $\theta_2^* = 0$ when $c_2 - a > c_1$) hinder collusion, result in relatively large critical discount factors, and are therefore unlikely to satisfy Condition 4. Figure 5 depicts the set of (θ_1^*, θ_2^*) values that satisfy Condition 4 when $c_1 = .07$, $c_2 = .1$, $b = .06$, $a = 0$, and $\delta = .53$.

The results of this section do not imply that sales-based compensation has arisen solely as a mechanism for inducing collusion between managers. In many cases, sales-based compensation may have arisen due to other considerations.[34] The results of this section suggest the possibility that, in some cases, sales-based compensation may be employed as a means of relaxing competition between managers and facilitating collusion.

7 Conclusion

We have analyzed the possibility of collusion between managers, compensated partly or entirely on the basis of sales, of heterogeneous firms. We have shown that, contrary to when firms are homogeneous, sales-based compensation can facilitate collusion when firms differ in marginal cost or product quality. This result occurs

[34]For example, sales-based compensation may have arisen due to its simplicity, the need to incentivize managers to control costs, or the unavailability of an accurate measure of economic profit. Additionally, as discussed in Section 2, managers may place an intrinsic value on sales volume (e.g., social prestige or long-term career benefits associated with employment at high sales firms), regardless of actual compensation or wage contracts set by owners or higher level executives. While it may be more difficult for owners to influence managerial incentives in these cases, owners may indirectly determine θ_1 and θ_2 by screening and attracting managers with an intrinsic tendency to maximize sales (or profits).

under both price and quantity competition. Intuitively, sales-based compensation can reduce asymmetries between rival managers and align their objectives, facilitating collusion.

As a result, we find that the welfare and profit implications of sales-based compensation in a dynamic setting can differ markedly from those of a static setting. In a static one-period setting, managerial compensation structures based on sales enhance consumer welfare and reduce firm profit, because managers have an incentive to reduce their price in order to increase their compensation. We find that this result does not always arise in a dynamic setting with heterogeneous firms. This is the case because sales-based compensation may facilitate manager collusion, harming consumers and increasing firm profit. Finally, we study a model of strategic delegation wherein owners can endogenously select managerial compensation structures prior to manager competition (or collusion). We find that owners can strategically set sales weights in order to induce collusion between managers and enhance their profits.

The results of Section 5 and 6 suggest that managerial compensation structures that are based on sales but are not tied too strongly to sales levels are of the greatest concern to antitrust authorities. This is the case as managerial compensation structures of this kind have the potential to enhance owner profit (Proposition 4), and reduce consumer surplus (Proposition 5). Additionally, as shown in Proposition 6, these pay structures could be selected strategically by rival owners in order to facilitate collusion between managers and increase owner profit.

We have considered a simplified duopoly setting with constant marginal costs, differentiated product competition, grim trigger strategies, and monopoly pricing during collusion. Our results seem likely to generalize to alternative demand systems, cost structures, collusive pricing strategies, and punishment schemes because our results are driven by the more general fact that collusion is easier to sustain when firms are more symmetric. Sales-based compensation can effectively increase the degree of symmetry between firms/managers and, as a result, facilitate collusion. Thus, our main qualitative findings may also apply to other settings where asymmetries hinder the sustainability of collusion.

Further exploration of the welfare effects and potential antitrust implications of sales-based compensation (particularly in dynamic settings with heterogeneous firms) is an important task for future research. For example, future studies could explore the role of sales-based compensation in predatory pricing (Iacobucci, 2006). Compensating the manager of a low-cost firm on the basis of sales may act as a credible commitment to aggressive or below-cost pricing. This commitment could drive a less efficient rival out of the market and generate a monopoly position.

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A Proofs

Let $\delta_i^*(\tilde{c}_1, x, a)$ denote the critical discount factor of manager i when the perceived marginal cost of manager 1 is $\tilde{c}_1$, the asymmetry in perceived marginal cost is $x \equiv \tilde{c}_2 - \tilde{c}_1$ and the asymmetry in product quality is a. The critical discount factor $\delta^*(\tilde{c}_1, x, a)$ is defined analogously. Details and algebraic computations behind proofs are presented in the online appendix (Lee and Turner, 2023).

Proof of Proposition 1. See Subsection A.5.1 in Lee and Turner (2023). □

Lemma 1. *Suppose $x - a \neq 0$. Then, $\delta^*(\tilde{c}_1, x, a)$ is increasing in $\tilde{c}_1$.*

Proof. Note that $\delta^*(\tilde{c}_1, x, a) < 1$ (which holds by assumption, see Subsection A.4 of Lee and Turner (2023)) implies $|x - a| < (1 - b)(1 - \tilde{c}_1)$ (see Lemma A.1 of Lee and Turner (2023)) and, thus, it is sufficient to examine $\tilde{c}_1$ values which satisfy this condition. It can be shown that $\delta_2^*(\tilde{c}_1, x, a)$ is increasing in $\tilde{c}_1$ for $0 < x - a < (1 - b)(1 - \tilde{c}_1)$ and $\delta_1^*(\tilde{c}_1, x, a)$ is increasing in $\tilde{c}_1$ for $-(1 - b)(1 - \tilde{c}_1) < x - a < 0$ (see Subsection A.5.2 of Lee and Turner (2023)). Since $\delta^*(\tilde{c}_1, x, a) = \delta_1^*(\tilde{c}_1, x, a)$ if $x - a \leq 0$ and $\delta^*(\tilde{c}_1, x, a) = \delta_2^*(\tilde{c}_1, x, a)$ if $x - a \geq 0$ (by Proposition 1), $\delta^*(\tilde{c}_1, x, a)$ is increasing in $\tilde{c}_1$ for any $x - a \neq 0$. □

Lemma 2. *Suppose $x - a \neq 0$. Then, $\delta^*(\tilde{c}_1, x, a)$ is increasing in x for $x > a$ and $\delta^*(\tilde{c}_1, x, a)$ is decreasing in x for $x < a$.*

Proof. Note that $\delta^*(\tilde{c}_1, x, a) < 1$ (which holds by assumption, see Subsection A.4 of Lee and Turner (2023)) implies $|x - a| < (1 - b)(1 - \tilde{c}_1)$ (see Lemma A.1 of Lee and Turner (2023)), and thus, it is sufficient to examine x values that satisfy this condition. It can be shown that $\delta_2^*(\tilde{c}_1, x, a)$ is increasing in x for $0 < x - a < (1 - b)(1 - \tilde{c}_1)$ and $\delta_1^*(\tilde{c}_1, x, a)$ is decreasing in x for $-(1 - b)(1 - \tilde{c}_1) < x - a < 0$ (see Subsection A.5.3 of Lee and Turner (2023)). Since $\delta^*(\tilde{c}_1, x, a) = \delta_1^*(\tilde{c}_1, x, a)$ if $x - a \leq 0$ and $\delta^*(\tilde{c}_1, x, a) = \delta_2^*(\tilde{c}_1, x, a)$ if $x - a \geq 0$ (by Proposition 1), $\delta^*(\tilde{c}_1, x, a)$ is increasing in x when $x > a$ and decreasing in x when $x < a$. □

Proof of Proposition 2. Part i) When $\tilde{c}_2 = \tilde{c}_1$, perceived marginal costs are symmetric and cancel out of the critical discount factor. Part ii) When $\tilde{c}_2 \neq \tilde{c}_1$, there is an asymmetry in perceived marginal cost which increases the critical discount factor by Lemma 2. See Subsection A.5.4 of Lee and Turner (2023) for additional details. □

Proof of Proposition 3. Note that $\delta^*(A, B, C) = \delta^*(A, B - C, 0)$ (see Subsection A.3 of Lee and Turner (2023)). Thus, $\delta^*(\tilde{c}_1, x, a) = \delta^*(\tilde{c}_1, x - a, 0)$. Let $z \equiv c_2 - c_1 - a$. When $\theta_1 = \theta_2 = 0$, the critical discount factor is $\delta^*(c_1, c_2 - c_1, a) = \delta^*(c_1, z, 0)$. We wish to show $\delta^*(\tilde{c}_1, x - a, 0) < \delta^*(c_1, z, 0)$ when Condition 1 holds. Suppose Condition 1 holds (i.e., $|x - a| < |z|$). There are four cases to consider.

Case 1 $z > 0$ and $x - a \geq 0$: The result follows from the observation that $\delta^*(c_1, z, 0) \geq \delta^*(\tilde{c}_1, z, 0) > \delta^*(\tilde{c}_1, x - a, 0)$ where the first inequality follows from Lemma 1 (and $\tilde{c}_1 \leq c_1$) and the second inequality follows from Lemma 2 (and $0 \leq x - a < z$).

Case 2 $z > 0$ and $x - a < 0$: Note that $\delta^*(A, B, 0) = \delta^*(A + B, -B, 0)$ for any A and B by symmetry. Thus, $\delta^*(c_1, z, 0) = \delta^*(c_1 + z, -z, 0)$ holds. The result follows from

$$\delta^*(c_1, z, 0) = \delta^*(c_1 + z, -z, 0) > \delta^*(c_1 + z, x - a, 0) > \delta^*(\tilde{c}_1, x - a, 0)$$

where the first inequality follows from $-z < x - a < 0$ (by $|x - a| < |z|$) and Lemma 2. The second inequality follows from $c_1 + z > \tilde{c}_1$ and Lemma 1.

Case 3 $z < 0$ and $x - a > 0$: The result follows from

$$\delta^*(c_1, z, 0) = \delta^*(c_1 + z, -z, 0) > \delta^*(c_1 + z, x - a, 0) > \delta^*(\tilde{c}_1, x - a, 0)$$

where the first inequality follows from $0 < x - a < -z$ (by $|x - a| < |z|$) and Lemma 2. The second inequality follows from Lemma 1 and $c_1 + z > \tilde{c}_1$ which holds because

$$\tilde{c}_1 = \tilde{c}_2 - x < \tilde{c}_2 - a \leq c_2 - a = c_1 + c_2 - c_1 - a = c_1 + z$$

where the first inequality follows from $x > a$, and the second inequality follows from $(1 - \theta_2)c_2 \leq c_2$.

Case 4 $z < 0$ and $x - a \leq 0$: The result follows from $\delta^*(c_1, z, 0) \geq \delta^*(\tilde{c}_1, z, 0) > \delta^*(\tilde{c}_1, x - a, 0)$ where the first inequality follows from Lemma 1 (and $\tilde{c}_1 \leq c_1$) and the second inequality follows from Lemma 2 (and $z < x - a \leq 0$). □

Proof of Proposition 4. Condition 2b ensures that collusion is sustainable under sales-based compensation and unsustainable under strictly profit-based compensation. Thus, $\pi_i(\theta_1, \theta_2) = \pi_i^C(\theta_1, \theta_2)$ and $\pi_i(0, 0) = \pi_i^N(0, 0)$ for $i = 1, 2$. Condition 2a and $\delta^*(0, 0) < 1$ imply that $\pi_1^C(\theta_1, \theta_2) > \pi_1^N(0, 0)$ and $\pi_2^C(\theta_1, \theta_2) > \pi_2^N(0, 0)$. See Subsection A.5.5 of Lee and Turner (2023) for details. □

Proof of Proposition 5. Condition 3b ensures that collusion is sustainable under sales-based compensation and unsustainable under strictly profit-based compensation. Thus, $CS(0, 0) = CS^N(0, 0)$ and $CS(\theta_1, \theta_2) = CS^C(\theta_1, \theta_2)$. Condition 3a and $\delta^*(0, 0) < 1$ ensure that $p_1^N(0, 0) < p_1^C(\theta_1, \theta_2)$ and $p_2^N(0, 0) < p_2^C(\theta_1, \theta_2)$ (i.e., prices under sales-based compensation and collusion exceed prices under profit-based compensation and Nash competition) which together imply $CS^C(\theta_1, \theta_2) < CS^N(0, 0)$. See Subsection A.5.6 of Lee and Turner (2023) for details. □

Lemma 3. $\pi_i^N(\theta_1, \theta_2)$ *is non-increasing in* θ_i *for* $i = 1, 2$.

Proof. See Subsection A.5.7 of Lee and Turner (2023). □

Proof of Proposition 6. The sustainability of collusion between managers follows directly from Condition 4b. Lemma 3 implies that no owner wishes to defect in the initial stage if $\pi_1^C(\theta_1^*, \theta_2^*) \geq \pi_1^N(0, \theta_2^*)$ and $\pi_2^C(\theta_1^*, \theta_2^*) \geq \pi_2^N(\theta_1^*, 0)$. Solving $\pi_1^C(\theta_1^*, \theta_2^*) \geq \pi_1^N(0, \theta_2^*)$ and $\pi_2^C(\theta_1^*, \theta_2^*) \geq \pi_2^N(\theta_1^*, 0)$ for θ_1^* and θ_2^* yields Condition 4a (see Subsection A.5.8 of Lee and Turner (2023) for details). □