On the Optimal Design of TOTEX Regulation

by

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Abstract

We examine the optimal design of TOTEX regulation, which allows a regulated firm to retain during the prevailing regulatory regime the entire cost reduction it achieves. Although TOTEX eliminates input bias, it often provides insufficient incentive to implement new cost-reducing technologies. TOTEX can be particularly likely to deliver inefficiently limited incentive when expected technology implementation costs are high, uncertainty about these costs is limited, the new technology admits moderately large cost reductions, regulatory regimes are short, managerial compensation is not closely linked to the firm's profit, and the firm has operated relatively efficiently historically.

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1 Introduction.

Historically, capacity-constraining increases in the demand for electricity typically have led to expanded network infrastructure investment. Today, grid-enhancing technologies (GETs) provide a less capital-intensive, and often less costly, means to meet increased demand for electricity. GETs "maximize the transmission of electricity across the existing system through a family of technologies that include sensors, power flow control devices, and analytical tools" (U.S. Department of Energy, 2022, p. ii).[1]

Although GETs can enhance operational efficiency and reduce costs, regulated electricity suppliers can be reluctant to adopt them. This is the case because standard rate of return regulation treats capital expenses (CAPEX) and operating expenses (OPEX) asymmetrically. Specifically, CAPEX often commands a reliable (and sometimes relatively generous) return on investment that OPEX does not (Averch and Johnson, 1962). Consequently, regulated electricity suppliers can prefer CAPEX solutions to OPEX solutions even when the latter are more economical.[2]

To encourage regulated electricity suppliers to employ cost-minimizing production technologies, some regulators have replaced standard rate of return regulation with TOTEX regulation.[3] Under TOTEX regulation, the regulator sets the firm's authorized revenue equal to the firm's expected total cost. A fraction of this cost is treated as OPEX (i.e., it is expensed immediately) and the remaining fraction is treated as CAPEX (i.e., it is financed over time). Importantly, these fractions do not change during the prevailing regulatory regime, even if the firm's actual mix of CAPEX and OPEX changes. Consequently, the firm benefits financially if it can reduce its production costs, regardless of the mix of inputs it employs to secure the cost reduction.

Because TOTEX (regulation) effectively awards to the firm the entire cost reduction it achieves for the duration of the prevailing regulatory regime, TOTEX induces the regulated firm to adopt the cost-minimizing mix of inputs.[4] The efficient operation that TOTEX induces is beneficial. However, the induced efficiency does not imply that TOTEX necessarily

[1] Also see Odhiambo (2023) and Siemens Energy (2025), for example.

[2] See Marques et al. (2014, 2022), Ofgem (2017), Smith et al. (2019), the European Union Agency for the Cooperation of Energy Regulators (2021), Oxera (2018, 2021), Ruiz et al. (2023), and Bergaentzlé (2024), for example. Frontier Economics (2017, p. 5) reports that regulated electricity suppliers can "favour capex solutions . . . over opex . . . because capex facilitate[s] growth in the businesses' regulated asset bases (RABs) and a steady return on that capital investment over the assumed regulatory life of those assets."

[3] Ofgem employs TOTEX regulation in the UK electricity sector (Ofgem, 2025). Frontier Economics (2017) reports the implementation of similar policies in Germany, the Netherlands, and Victoria.

[4] See, for example, Jenkins and Perez-Arriaga (2017), Brunekreeft and Rammerstorfer (2021), and von Bebenburg et al. (2023).

implements the ideal sharing of realized cost reductions between the regulated firm and its customers. Indeed, it might seem that awarding the entire realized cost reduction to the firm during the prevailing regulatory regime is unduly generous to the firm. Reserving a portion of the realized cost reduction for consumers during the regime might enhance consumer welfare by reducing expected procurement costs (which are payments by consumers to the firm for providing essential services).

We analyze the division of realized cost reductions between the firm and its customers that minimizes the present discounted value (PDV) of expected procurement costs. We do so in a setting where implementation of a cost-reducing technology is challenging for the firm's manager, and where the regulator is uncertain about the relevant managerial technology implementation costs. The regulator specifies the fraction of realized cost reductions that will be awarded to the firm during the initial regulatory regime. The firm's manager then decides whether to implement the new cost-reducing technology or continue to operate with the technology the firm has employed historically.

We find that TOTEX often provides insufficient incentive to implement the new cost-saving technology. Specifically, if the firm were awarded during the initial regulatory regime more than the cost reduction it achieves, the probability that the new technology is adopted would increase sufficiently to reduce the PDV of expected procurement costs, even after accounting for the firm's more generous compensation.[5]

We also identify conditions under which the extent to which TOTEX provides insufficient incentive for new technology adoption is particularly pronounced. Not surprisingly, this is the case when expected managerial technology implementation costs are high. TOTEX also provides particularly limited incentives for technology adoption when the regulator values future consumer welfare highly. In this case, the regulator optimally establishes greater incentives for technology adoption than does TOTEX to increase the probability that consumers enjoy the full benefit of a substantial cost reduction during future regulatory regimes.

Perhaps more subtly, TOTEX is particularly likely to provide insufficient incentive for new technology adoption when the firm has operated relatively efficiently (e.g., with little over-capitalization) historically. In this case, the incremental cost reduction the firm can secure when it operates under the new technology rather then the original technology is relatively large. Consequently, an increase in the firm's share (s_r) of realized cost reductions increases the incremental profit the firm secures by implementing the new technology rela-

[5] Alternatively, the firm might be awarded beyond the prevailing regulatory regime the full cost reduction it achieves.

tively rapidly, thereby increasing relatively rapidly the probability that the new technology is implemented. When the regulator's s_r instrument is relatively effective in this sense, the instrument is optimally employed more extensively than it is employed under TOTEX.

TOTEX can also provide insufficient incentive for new technology adoption when regulatory regimes are relatively short and/or managerial compensation is not closely linked to the firm's profit.[6] Under these conditions, consumers benefit in the long run when the regulated firm is awarded during the prevailing regulatory regime more than the full cost reduction it achieves. The enhanced award helps to offset the diminished incentives for new technology adoption created by short regulatory regimes and/or manager compensation that does not increase substantially as the firm's profit increases.

To our knowledge, the optimal design of TOTEX-like regulation has received little attention in the literature. von Bebenburg et al. (2023) study a single-period model of TOTEX regulation. The authors prove that TOTEX regulation induces the regulated enterprise to adopt a cost-minimizing mix of inputs.[7] The authors do not analyze the optimal share of realized cost reductions to award to the regulated firm. The Florence School (2023) discusses the potential merits of sharing a portion of realized cost reductions with the regulated firm. However, this study does not analyze the optimal sharing rate, which is the focus of our analysis.[8]

Our analysis proceeds as follows. Section 2 describes our model. Section 3 analyzes the manager's choice of technology and inputs. Section 4 presents our analytic characterization of the regulatory policy that minimizes the PDV of expected procurement costs. Section 5 presents numerical solutions to further characterize the optimal regulatory policy. Section 6 identifies settings in which TOTEX is particularly likely to provide insufficient incentive for new technology adoption. Section 7 reviews our key findings and suggests directions for further research. The Appendix presents the proofs of all formal conclusions in the paper.

2 Model Elements

We consider a setting in which a regulated monopoly supplier operates under an infinite sequence of regulatory regimes. Each regulatory regime consists of $T > 1$ periods (e.g., years). At the start of period 1 (which is the first period in the first regulatory regime), the firm has the opportunity to replace the technology it has employed historically ("technology

[6] Additional conditions under which TOTEX provides insufficient incentive for new technology adoption are identified in Section 6 below.

[7] See Brunekreeft and Rammerstorfer (2021) for related observations.

[8] Brunekreeft (2023) examines how TOTEX regulation affects investment incentives.

α_0") with a new technology ("technology α_1"). The new technology admits lower production costs.

The firm's manager makes the technology implementation decision. The personal cost the manager incurs if he retains the original technology is normalized to 0. If the manager adopts the new technology, he incurs an unmeasured personal (adjustment) cost $K \in \left[\underline{K}, \overline{K}\right]$, where $\overline{K} > \underline{K} > 0$. This cost might reflect in part additional effort the manager must exert to fully understand the new technology and implement it successfully. The manager's cost might also reflect personal disutility he experiences when he is compelled to replace long-time colleagues with new individuals who have the expertise required to operate the new technology.

In deciding which technology to implement, the manager acts to maximize $\delta\,\Pi - K$, where Π denotes the present discounted value (PDV) of the firm's profit, and $\delta \in (0, 1\,]$ is a parameter. Higher values of δ reflect increased congruence between the manager's objective and the firm's objective. Such increased congruence might arise, for example, when a larger portion of the manager's compensation takes the form of options to purchase the firm's stock at a relatively low price.[9]

The manager knows the magnitude of his personal cost of implementing the new technology (K). The regulator does not know K. Her beliefs about K are captured by the distribution function $F(k)$ and corresponding density function $f(k)$, where $k \equiv \frac{K}{\delta}$. The regulator seeks to minimize the PDV of the expected cost of inducing the firm to continue to produce in every period the level of output $(\,\overline{Q}\,)$ the firm has produced historically.

We consider a class of regulatory policies that includes TOTEX as a special case. In essence, the policies award to the firm during the initial regulatory regime a fraction of any cost reduction it achieves during the regime. Formally, during the initial regulatory regime, the regulator sets the firm's revenue in period t (R_t) equal to the difference between the firm's historic revenue $(\,\overline{R}\,)$ and the fraction s_c of the difference between the firm's historic per-period total cost ($\overline{C} \leq \overline{R}$) and C_t, the firm's observed total cost in period t. Formally:

$$R_t \;=\; \overline{R} - s_c\left[\,\overline{C} - C_t\,\right] \quad \text{for } \; t \in \{1, ..., T\}\,. \tag{1}$$

The regulatory policy in (1) reflects TOTEX when $s_c = 0$ because when $s_c = 0$, the firm's revenue does not decline as it reduces its costs below historic levels. More generally, the regulatory policies we consider allow for some immediate sharing with consumers of any cost reduction the firm achieves. The sharing persists throughout the initial regulatory

[9] Higher values of δ can also reflect a higher probability that the manager's tenure at the regulated firm will continue.

regime (i.e., during periods $1, ..., T$). In subsequent regulatory regimes, the regulator sets the firm's revenue equal to its observed total cost (i.e., $R_t = C_t$ for $t \geq T+1$). These policies reflect the standard practice of allowing a regulated enterprise to benefit from achieved efficiencies during the prevailing regulatory regime, but effectively awarding the efficiencies to consumers in subsequent regimes.[10]

The sharing parameter s_c can be viewed as the fraction of any achieved cost reduction that is awarded to consumers during the initial regulatory regime. The regulator faces a fundamental trade-off in setting s_c. By increasing s_c, the regulator secures for consumers during the initial regulatory regime a larger fraction of any achieved cost reduction. However, the corresponding decline in the share of realized cost reductions awarded to the firm can reduce the probability that a cost reduction is achieved by diminishing the manager's incentive to implement the new technology.

$Q(I \,|\, \alpha)$ is the maximum level of output the firm can produce when it operates using technology $\alpha \in \{\alpha_0, \alpha_1\}$ and employs input vector I. $C(I \,|\, \alpha_i)$ is the firm's corresponding per-period total cost of producing output $\overline{Q}$.[11] It is common knowledge that for $i \in \{1, 2\}$:

$$\overline{C} \;\geq\; C_0^* \;>\; C_1^*\,, \text{ where } C_i^* \;\equiv\; \min_I \{\, C(I \,|\, \alpha_i) \text{ subject to } Q(I \,|\, \alpha_i) \geq \overline{Q}\,\}. \qquad (2)$$

The $\overline{C} \geq C_0^*$ inequality in (2) indicates that the firm may not have employed the cost-minimizing input mix historically.[12] This inefficiency might reflect over-capitalization induced by a particularly generous allowed rate of return on capital, for example. When a historic inefficiency prevails, the regulated firm can reduce its operating costs (by adopting the cost-minimizing input mix) even when the firm continues to employ the original technology. Regardless of whether a historic inefficiency prevails, the firm can reduce its operating if it implements the new technology (because $C_1^* < \overline{C}$).

Activity in the model proceed as follows. Before the start of period 1, the regulator specifies the sharing rate (s_c) that will prevail throughout the initial regulatory regime (which lasts for T periods) and the associated compensation policy specified in (1). At the start of period 1, the firm's manager decides which technology to implement and what inputs to

[10] The Florence School (2003, p. 12) observes that sharing a portion of realized cost reductions with the regulated firm "is analogous to leaving any cost saving ... to the [firm] until the end of the regulatory period." In some jurisdictions, regulators award achieved efficiencies to the regulated enterprise for a fixed period of time (e.g., five years), even if the specified time period spans multiple regulatory regimes (e.g., Turner and Sappington, 2025). In our model, any achieved efficiency gains arise at the start of period 1. Therefore, a promise to award efficiency gains to the firm for T periods never entails a commitment that spans multiple regulatory regimes.

[11] For simplicity, we assume that, holding constant the prevailing technology and input vector, the firm's per-period total cost of producing $\overline{Q}$ units of output does not vary over time.

[12] Recall that $\overline{C}$ is the firm's historic per-period total cost of producing output $\overline{Q}$.

employ. Upon observing the firm's associated total production cost in period $t \in \{1, ..., T\}$, the regulator delivers compensation R_t to the firm, as specified in (1). In each period after period T, the regulator eliminates the firm's profit by delivering compensation that reflects the firm's total cost.

3 The Choice of Technology and Inputs

Before characterizing the regulator's choice of s_c, it is helpful to characterize the manager's choice of inputs, given the prevailing technology. Let I_i denote the vector of inputs the manager employs after implementing technology $\alpha_i \in \{\alpha_0, \alpha_1\}$. Because the manager's payoff $(\delta \Pi - K)$ increases as the firm's profit (Π) increases during the initial regulatory regime, the manager chooses I_i to:

$$\text{Maximize} \sum_{t=1}^{T} b^{t-1} [R_t - C(I_i \mid \alpha_i)] \quad \text{subject to } Q(I_i \mid \alpha_i) \geq \overline{Q}, \tag{3}$$

where $b \in (0,1)$ is the manager's inter-period discount factor.[13] (1) implies that the firm's revenue in period $t \in \{1, ..., T\}$ when it operates under technology α_i and employs inputs I_i is:

$$R_t = \overline{R} - s_c \left[\overline{C} - C(I_i \mid \alpha_i) \right] = \overline{R} - s_c \overline{C} + s_c C(I_i \mid \alpha_i). \tag{4}$$

(3) and (4) imply that the manager chooses I_i to:

$$\text{Maximize} \; - \sum_{t=1}^{T} b^{t-1} [1 - s_c] \, C(I_i \mid \alpha_i) \quad \text{subject to } Q(I_i \mid \alpha_i) \geq \overline{Q}. \tag{5}$$

It is apparent from (5) that if consumers are awarded more than the entire cost reduction achieved during the initial regulatory regime (i.e., if $s_c > 1$), the PDV of the firm's profit declines as its costs decline. Therefore, as Lemma 1 reports, the manager will eschew any cost reduction if $s_c > 1$. Instead, the manager will implement the original technology and decline to reduce the firm's cost below $\overline{C}$.[14]

Lemma 1. *If $s_c > 1$, then $C(\cdot) = \overline{C}$ in every period.*

It is also apparent from (5) that whenever $s_c < 1$, the firm's profit is maximized when its costs are minimized. Therefore, to maximize his payoff, the manager employs the inputs that minimize the firm's production costs, given the prevailing technology. Consequently, when $s_c \leq 1$, every regulatory policy in the class of policies under consideration induces

[13] Formally, b is the value the manager derives at the start of the current period from a dollar that he will receive at the beginning of the next period. For expositional ease, we assume the firm and its manager share the same inter-period discount factor.

[14] We assume the regulator can preclude costs above the historic level, $\overline{C}$.

cost-minimizing production by the firm.[15]

Although all regulatory policies under consideration induce an efficient choice of inputs when $s_c \leq 1$, the policies differ in the technology choice they induce. (4) implies that the firm's per-period profit (not counting the manager's implementation cost) during the initial regulatory regime when it operates with technology $\alpha_i \in \{\alpha_0, \alpha_1\}$ is:

$$\overline{R} - s_c \left[\, \overline{C} - C_i^* \right] - C_i^* \;=\; \overline{R} - s_c \, \overline{C} - [\, 1 - s_c \,] \, C_i^* \, . \tag{6}$$

Because the firm's profit is zero after period T, (6) implies that the PDV of the firm's corresponding profit when it operates with technology α_i is:

$$\Pi(\alpha_i) \;\equiv\; b_T \left[\, \overline{R} - s_c \, \overline{C} - (\, 1 - s_c \,) \, C_i^* \right] \quad \text{where } b_T \;\equiv\; \sum_{t=1}^{T} b^{t-1} \;=\; \frac{1 - b^T}{1 - b} \, . \tag{7}$$

(7) implies that the PDV of the incremental profit the firm secures when it operates under technology α_1 rather than technology α_0 is:

$$\Delta_\Pi \;\equiv\; \Pi(\alpha_1) - \Pi(\alpha_0) \;=\; b_T \, [\, 1 - s_c \,] \, \Delta_C^* \quad \text{where } \Delta_C^* \;\equiv\; C_0^* - C_1^* \;>\; 0 \, . \tag{8}$$

(8) implies that the manager prefers to implement technology α_1 rather than technology α_0 if and only if:[16]

$$\delta \, \Pi(\alpha_1) - K \;>\; \delta \, \Pi(\alpha_0) \quad \Leftrightarrow \quad k \;\equiv\; \frac{K}{\delta} < \; \Delta_\Pi \;=\; b_T \, [\, 1 - s_c \,] \, \Delta_C^* \;\equiv\; \widehat{k} \, . \tag{9}$$

(9) implies that the manager implements the new (α_1) technology rather than continue to operate with the original (α_0) technology if and only if his implementation cost is sufficiently small (i.e., $k < \widehat{k}$). (9) further implies that the manager never implements the new technology if the share of realized cost reductions awarded to consumers is sufficiently large, whereas the manager always implements the new technology if this share is sufficiently small.[17] These conclusions are illustrated in Figure 1 and recorded formally in Lemma 2.

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[15] Intuitively, when the manager is effectively awarded a share of realized cost reductions, he chooses inputs to minimize realized costs. This conclusion reflects the findings of von Bebenburg et al. (2023), for example. For analytic ease, we assume that when the manager is indifferent between employing the cost-minimizing input combination and an alternative input combination, he employs the cost-minimizing input combination. This assumption avoids an "open set" problem, wherein the regulator might otherwise seek to set the highest value of s_c (strictly below 1) that induces the manager to employ the cost-minimizing input combination.

[16] As (9) indicates, the manager implements the new technology rather than the old technology if and only if doing so strictly increases the manager's payoff. This "tie-breaking rule" does not affect our key qualitative findings.

[17] In principle, the manager could continue to operate with technology α_0 and set $C_t = \overline{C}$ for all $t \geq 1$. Doing so would generate profit $\overline{R} - \overline{C} \geq 0$ in every period. For expositional ease, we assume $\overline{R} - \overline{C}$ and b are sufficiently small that this strategy never generates the highest PDV of profit for the firm. This assumption simplifies the characterization of the manager's technology adoption decision without affecting our primary conclusions regarding the regulator's optimal choice of s_c.

[**Figure 1 about Here**]

Lemma 2. *The manager implements the new technology for all K realizations if $s_c < \underline{s}_c \equiv 1 - \frac{\overline{k}}{b_T\, \Delta_C^*}$. The manager never implements the new technology if $s_c \geq \overline{s}_c \equiv 1 - \frac{\underline{k}}{b_T\, \Delta_C^*}$.*

4 Characterizing the Optimal Regulatory Policy

To characterize the optimal regulatory policy, observe from (9) that the regulator views $F(\widehat{k})$ to be the *ex ante* probability the manager implements the new technology. The regulator's corresponding expectation of the firm's per-period cost of producing output $\overline{Q}$ is:

$$\widehat{C} \;\equiv\; F(\widehat{k})\, C_1^* + \left[1 - F(\widehat{k}) \right] C_0^* \,. \tag{10}$$

(10) implies that the PDV of the regulator's expected procurement cost when $s_c \leq 1$ is:

$$\begin{aligned} P(s_c) \;&\equiv\; \sum_{t=1}^{\infty} \beta^{t-1}\, \overline{R} - \sum_{t=1}^{T} \beta^{t-1} \left[\overline{C} - \widehat{C} \right] s_c - \sum_{t=T+1}^{\infty} \beta^{t-1} \left[\overline{C} - \widehat{C} \right] \\ &=\; [\, \beta_T + \beta_\infty \,]\, \overline{R} - \beta_T \left[\overline{C} - \widehat{C} \right] s_c - \beta_\infty \left[\overline{C} - \widehat{C} \right] \end{aligned} \tag{11}$$

where β is the regulator's intertemporal discount factor, $\beta_T \equiv \sum_{t=1}^{T} \beta^{t-1} = \frac{1-\beta^T}{1-\beta}$, and $\beta_\infty \equiv \sum_{t=T+1}^{\infty} \beta^{t-1} = \frac{\beta^T}{1-\beta}$.[18] (11) reflects the fact that the regulator secures for consumers the fraction s_c of the firm's achieved cost reduction during the initial regulatory regime, and secures the entire achieved cost reduction thereafter. The regulator's problem, [RP], is to choose s_c to minimize $P(s_c)$.

(9) – (11) imply that s_c^*, the value of s_c at the solution to [RP], typically varies with the properties of $f(k)$. However, some features of s_c^* hold quite generally, as Findings 1 – 3 report.

Finding 1. $s_c^* \in [\, \underline{s}_c, \overline{s}_c) \cup 1\,$.[19]

Finding 1 implies that s_c^* is never less than $\underline{s}_c$. Whenever $s_c < \underline{s}_c$, the firm's share of any realized cost reduction is so large that the manager always incurs K to implement the new technology. In this case, the regulator can increase s_c to secure a larger share of realized cost reductions for consumers while continuing to ensure the manager always implements the new technology. Therefore, $s_c^* \geq \underline{s}_c$.

[18] Lemma 1 implies that if $s_c > 1$, then $P(s_c) = [\, \beta_T + \beta_\infty \,]\, \overline{R}$ (because $C(\cdot) = \overline{C}$ in every period) .

[19] The set of possible values for s_c^* identfied in Finding 1 appear in the bolded (green) portion of the horizontal line in Figure 1.

Finding 1 also implies that if $s_c^* \neq 1$, then s_c^* is less than $\overline{s}_c$. Whenever $s_c \geq \overline{s}_c$, the firm's share of any realized cost reduction is so small that the manager never incurs K to implement the new technology. Therefore, if $s_c \in [\,\overline{s}_c, 1)$, the regulator can increase s_c toward 1, thereby increasing the share of any cost reduction the firm secures (by eliminating historic inefficiencies) that is awarded to consumers. The regulator can do so without increasing the firm's expected cost (because the manager always implements the old technology).

Finding 1 further implies that s_c^* never exceeds 1. If $s_c > 1$, the firm's cost is never reduced below $\overline{C}$, so consumers never derive any benefit from the existing potential to reduce the firm's production cost. (Recall Lemma 1.)

Although s_c^* never exceeds 1, it can be 1. That is, the regulator may optimally award all realized cost reductions to consumers, as Finding 2 reports.

Finding 2. *$s_c^* = 1$ if Δ_C^* is sufficiently small and $\overline{C} > C_0^*$.*

Finding 2 indicates that the regulator does not award any portion of achieved cost reductions to the firm if its historic operations have been inefficient and the incremental cost reduction admitted by the new technology is sufficiently small. The manager never implements the new technology when $s_c^* = 1$. However, the associated increase in expected procurement cost is small when Δ_C^* is small. Consequently, the regulator intentionally foregoes any consumer gains the new technology might provide in order to secure for consumers the entire $(\,\overline{C} - C_0^*)$ gain from eliminating inefficiencies under the original technology. She does so by setting $s_c^* = 1$.

In contrast, if Δ_C^* is sufficiently large relative to $\frac{\overline{K}}{\delta}$, the regulator will award to the firm the smallest share of realized cost reductions that ensures the manager implements the new technology for all K realizations. Finding 3 explains how this share changes as model parameters change in this case. Throughout the ensuing analysis, s_r $(= 1 - s_c)$ denotes the share of realized cost reductions awarded to the regulated firm during the initial regulatory regime. s_r^* $(= 1 - s_c^*)$ denotes this share at the solution to [RP].

Finding 3. *When $s_c^* = \underline{s}_c$ so $s_r^* = 1 - \underline{s}_c$: (i) s_r^* increases as $\overline{K}$ increases; (ii) s_r^* declines as Δ_C^*, b, T, or δ increases; and (iii) s_r^* does not change as $\underline{K}$, $\overline{C}$, or β changes.*

Finding 3 reflects the following considerations. As $\overline{K}$ increases, the regulator must increase s_r to ensure the manager continues to implement the new technology for all K realizations. As Δ_C^*, b, or T increases, the PDV of the profit the firm secures when it operates under the new technology increases, *ceteris paribus*. Consequently, the regulator can reduce the use of her costly s_r instrument while continuing to ensure the manager always implements the new technology.[20] As δ increases, the manager values the firm's profit more highly. Consequently, once again, the regulator can reduce the firm's share of achieved cost reductions while still motivating the manager to implement the new technology for all K realizations.

When s_r is set to ensure the manager implements the new technology for the highest K realization, the same s_r will continue to induce this behavior as $\underline{K}$ changes or $\overline{C}$ changes. Furthermore, a change in β does not affect the PDV of the manager's payoff. Consequently, the level of s_r required to ensure the manager always implements the new technology does not change as $\underline{K}$, $\overline{C}$, or β changes.

When $\frac{\overline{K}}{\delta}$ is intermediate in magnitude relative to Δ_C^*, the regulator optimally induces the manager to implement the new technology for some, but not all, values of K, i.e., $s_c^* \in (\underline{s}_c, \overline{s}_c)$.[21] In this case, a closed-form solution for s_c^* (and thus for s_r^*) can be derived when all K realizations are equally likely, i.e., when Assumption U holds.

Assumption U. $F(k) = \frac{k-\underline{k}}{\overline{k}-\underline{k}}$ for all $k \in \left[\underline{k}, \overline{k}\right]$.

Finding 4. *Suppose Assumption U holds and $s_c^* \in (\underline{s}_c, \overline{s}_c)$. Then:*

$$s_c^* = \frac{1}{2} + \frac{\left[\,\overline{k} - \underline{k}\,\right]\left[\,\overline{C} - C_0^*\,\right]}{2\, b_T \left[\,\Delta_C^*\,\right]^2} - \frac{b_T\, \beta_\infty\, \Delta_C^* + \beta_T\, \underline{k}}{2\, b_T\, \beta_T\, \Delta_C^*}\,. \tag{12}$$

The following corollaries of Finding 4 provide some conclusions about the magnitude of s_r^* $(= 1 - s_c^*)$ and about how s_r^* changes as industry parameters change when k has a uniform distribution.

Corollary 1. *Suppose the conditions in Finding 4 hold and $\overline{C} - C_0^*$ is sufficiently small. Then $s_r^* > 1$ (so $s_c^* < 0$) if $\beta^T \geq \frac{1}{2}$* [22]

Corollary 1 reports that when the historic inefficiency $(\overline{C} - C_0^*)$ is sufficiently small and the regulator values future consumer welfare sufficiently highly, she optimally awards the firm more than the entire achieved cost reduction during the initial regulatory regime.[23] In

[20] The regulator's s_r instrument is "costly" in the sense that consumers receive a smaller portion of any achieved cost reduction as s_r increases.

[21] Lemma 10 in the Appendix identifies sufficient conditions for $s_c^* \in (\underline{s}_c, \overline{s}_c)$.

[22] In jurisdictions where incentive regulation prevails, a regulatory regime typically lasts for approximately four or five years (Sappington and Weisman, 2024). If $T = 5$, then $\beta^T > \frac{1}{2} \Leftrightarrow \beta > \left[\frac{1}{2}\right]^{\frac{1}{5}} \approx 0.87$.

[23] As discussed further in Section 5 below, Law (2014) reports limited empirical evidence of inefficiency due to an Averch-Johnson bias in regulated industries in recent years.

this sense, even though TOTEX awards the entire achieved cost reduction to the firm during the initial regulatory regime, the award may induce the implementation of the new technology with unduly low probability. The regulator optimally increases this implementation probability by awarding the firm more than the entire achieved cost reduction during the initial regulatory regime. Doing so ensures that, in all periods after period T, consumers enjoy with relatively high probability the full cost reduction that arises when the new technology is implemented.

Corollary 2. *Suppose the conditions in Finding 4 hold. Then* $\frac{\partial s_r^*}{\partial \Delta_C^*} < 0$ *if* $\overline{C} = C_0^*$. *Furthermore, if* $\overline{C} > C_0^*$, *then for some* $\Delta_{C2} \geq \Delta_{C1} \geq 0$*: (i)* $s_r^* = 0$ *if* $\Delta_C^* < \Delta_{C1}$*; (ii)* $\frac{\partial s_r^*}{\partial \Delta_C^*} > 0$ *if* $\Delta_C^* \in (\Delta_{C1}, \Delta_{C2})$*; and (iii)* $\frac{\partial s_r^*}{\partial \Delta_C^*} < 0$ *if* $\Delta_C^* > \Delta_{C2}$.

Corollary 2 explains how the optimal sharing rate ($s_r^* = 1 - s_c^*$) varies with the magnitude of the incremental cost reduction admitted by the new technology ($\Delta_C^* \equiv C_0^* - C_1^*$). This variation depends in part on $\overline{C} - C_0^*$, which can be viewed as a measure of the prevailing inefficiency under the original technology. In the absence of any such inefficiency (i.e., when $\overline{C} - C_0^* = 0$), the firm only profits from an increase in s_r if it operates under the new technology. Consequently, the manager's incentive to implement the new technology increases relatively rapidly as s_r increases. In this sense, the regulator's s_r instrument is relatively powerful when $\overline{C} - C_0^* = 0$. In this case, as the gain from implementing the new technology (Δ_C^*) increases, the regulator employs her relatively powerful – but costly – instrument less extensively, i.e., she reduces s_r. Doing so captures a larger share of realized cost reductions for consumers without reducing unduly the probability that the manager implements the new technology.

When $\overline{C} - C_0^* > 0$, the firm's profit increases as s_r increases both when the firm operates under the original technology and when it operates under the new technology. Consequently, the manager's incentive to implement the new technology rather than the original technology increases less rapidly as s_r increases. In this sense, the regulator's s_r instrument becomes less powerful as $\overline{C} - C_0^*$ increases, *ceteris paribus*. When $\overline{C} > C_0^*$ and Δ_C^* is sufficiently small, the regulator optimally foregoes any potential increase in consumer welfare under the new technology in order to capture for consumers the full cost reduction that arises as historic inefficiencies are eliminated under the original technology, i.e., she sets $s_r^* = 0$ (so $s_c^* = 1$). (Recall Finding 2.)

As Δ_C^* increases above the level at which $s_r^* = 0$, the regulator continues to set a relatively low s_r. She does so to avoid awarding the firm substantial profit even when the new technology is not implemented. When s_r is small and $\overline{C} - C_0^*$ is relatively large, the firm's incremental profit from operating under the new technology $(s_r \Delta_C^*)$ increases relatively slowly as Δ_C^* increases. To further enhance the manager's incentive to implement the new technology as Δ_C^* increases (and thereby increase the probability that the larger potential cost reduction is realized), the regulator may increase s_r.[24]

When Δ_C^* is large, this large potential cost reduction itself provides the manager with substantial incentive to implement the new technology. Consequently, as Δ_C^* increases above a threshold (Δ_{C2}) the regulator can secure a larger fraction of realized cost reductions for consumers by reducing s_r (thereby increasing $s_c = 1 - s_r$) without reducing unduly the probability that the manager implements the new technology. If Δ_C^* is sufficiently pronounced that the regulator induces the manager to implement the new technology for all K realizations (i.e., if $s_c^* = \underline{s}_c$, so $s_r^* = 1 - \underline{s}_c$), then the regulator can reduce s_r (and thereby increase s_c) as Δ_C^* increases while continuing to ensure the manager always implements the new technology.[25]

Corollary 3. *Suppose the conditions in Finding 4 hold. Then s_r^* declines as: (i) $\overline{C}$ increases; (ii) β or $\underline{K}$ declines; (iii) $k^e \equiv \frac{1}{2}[\underline{k} + \overline{k}]$ declines, holding $\Delta_k \equiv \overline{k} - \underline{k}$ constant; or (iv) Δ_k increases, holding k^e constant.*

Corollary 3 explains how the optimal sharing rate changes as other industry parameters change. As $\overline{C}$ increases, the cost reduction that arises when the original technology is implemented $(\overline{C} - C_0^*)$ increases. Consequently, the regulator perceives a smaller "loss" when the firm operates under the original technology. This reduced loss leads the regulator to award a larger fraction of realized cost reductions to consumers (i.e., to increase s_c by reducing s_r), even though doing so reduces the probability that the manager implements the new technology.

s_r^* also declines as β, the regulator's inter-period discount factor, declines. The PDV of future (post period T) consumer welfare gains from a cost reduction declines as β declines. This reduced value of future gains leads the regulator to secure more pronounced short-term gains for consumers by reducing s_r (in order to increase s_c), even though doing so reduces the probability that the manager implements the new technology.

[24] Δ_{C1} and Δ_{C2} can be identical when $\overline{C} - C_0^*$ is sufficiently small. Corollary 2 implies that $\frac{\partial s_r^*}{\partial \Delta_C^*} < 0$ in this case whenever $s_r^* > 0$.

[25] Recall that $1 - \underline{s}_c = \frac{\overline{k}}{b_T \Delta_C^*}$, which declines as Δ_C^* increases.

s_r^* declines as $\underline{K}$ declines for two reasons. First, the manager's expected cost of implementing the new technology (k^e) declines as $\underline{K}$ declines. The lower expected cost leads the regulator to reduce s_r because doing so secures for consumers a larger fraction of realized cost reductions without reducing unduly the probability that the manager implements the new technology.

Second, Δ_k, the range of possible k realizations, increases as $\underline{K}$ declines, *ceteris paribus*. The increase in Δ_k (which might be viewed as a measure of the "uncertainty" the regulator faces) renders the manager's technology implementation decision less sensitive to variations in s_r. Formally, because $\frac{d\widehat{k}}{ds_r} = b_T\,\Delta_C^*$ from (9) and because $\frac{dF(\widehat{k})}{d\widehat{k}} = \frac{d}{d\widehat{k}}\left(\frac{\widehat{k}-\underline{k}}{\Delta_k}\right) = \frac{1}{\Delta_k}$ when Assumption U holds:

$$\frac{dF(\widehat{k})}{ds_r} = \frac{dF(\widehat{k})}{d\widehat{k}}\,\frac{d\widehat{k}}{ds_r} = \frac{b_T\,\Delta_C^*}{\Delta_k} \;\Rightarrow\; \frac{d}{d\Delta_k}\left(\frac{dF(\widehat{k})}{ds_r}\right) < 0\,. \qquad (13)$$

(13) indicates that the regulator's s_r instrument becomes less powerful as Δ_k increases (due to a reduction in $\underline{K}$).[26] The regulator optimally employs her instrument less extensively as it becomes less powerful.

The remaining conclusions in Corollary 3 reflect related observations. As k^e, which is proportional to the manager's expected implementation cost, declines (holding Δ_k constant), the regulator reduces s_r because doing so secures a larger fraction of realized cost reductions for consumers without reducing unduly the probability that the manager implements the new technology. As Δ_k, which is proportional to the potential variation in the manager's technology implementation cost, increases (holding k^e constant), the regulator's s_r instrument becomes less powerful. (Recall (13).) The regulator optimally employs her costly s_r instrument less extensively as it becomes less powerful.

Corollary 4. *Suppose the conditions in Finding 4 hold and* $\frac{\overline{C}-C_1^*}{\overline{C}-C_0^*}$ *is sufficiently large. Then* s_r^* *declines as* b, T, *or* δ *increases.*

Corollary 4 considers settings in which the cost reduction admitted by the new technology ($\overline{C} - C_1^*$) is large relative to the cost reduction that can be achieved by reducing historic inefficiencies under the original technology ($\overline{C} - C_0^*$). In such settings, the manager's incentive to implement the new technology increases relatively rapidly as s_r increases. This incentive increases as the manager's discount factor (b), the duration of the initial regulatory regime (T), or the manager's valuation of the firm's profit (δ) increases. In response

[26] The s_r instrument becomes "less powerful" as Δ_k increases in the sense that the larger is Δ_k, the slower is the rate at which the probability the manager implements the new technology rises as s_r increases.

to the enhanced incentive, the regulator reduces her use of the costly s_r instrument because she can do so without diminishing unduly the manager's incentive to implement the new technology.[27]

5 Numerical Solutions

We now employ numerical solutions to further characterize the optimal sharing rate s_c^* (and $s_r^* = 1 - s_c^*$) and to demonstrate that the qualitative conclusions drawn in Finding 4 and its corollaries persist for other distributions of k. We initially do so in a representative *baseline setting*. We subsequently demonstrate that our qualitative conclusions hold it many settings other than the baseline setting.

The firm's historic operating cost ($\overline{C}$) is normalized to 100 in the baseline setting. Successful implementation of the new technology admits a 10% cost reduction (so $C_1^* = 90$) in this setting. The U.S. Department of Energy (DOE, 2022) identifies settings where the implementation of grid-enhancing technologies (GETs) generated substantially larger percentage cost reductions. However, the DOE cautions that the settings it describes are settings in which cost savings from GETs are likely to be relatively pronounced.[28] The maximum potential cost reduction under the original technology is taken to be 0.5%, so $C_0^* = 99.5$, in the baseline setting. This relatively modest cost reduction reflects the fact that regulators labor diligently to foster efficient production under prevailing technologies. Furthermore, evidence of a severe Averch-Johnson bias in practice seems limited (e.g., Law, 2014).[29]

To reflect managerial implementation costs that are a small fraction of the cost savings engendered by the new technology, implementation costs in the baseline setting range between $\underline{K} = 0.1$ and $\overline{K} = 1$. The manager is primarily concerned with his personal costs in this setting, so $\delta = 0.01$. Each regulatory regime lasts for five periods (so $T = 5$). The manager and the regulator have the same inter-period discount factor (so $b = \beta$), which is set at 0.95. These parameter values in the baseline setting are recorded in Table 1.

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[27] When $[\,\overline{C} - C_1^*\,] \,/\, [\,\overline{C} - C_0^*\,]$ is relatively small, the manager's incentive to implement the new technology is relatively weak. This incentive increases as b, T, or δ increases. In response to the increased incentive, the regulator may optimally increase her use of the now more effective s_r instrument.

[28] Furthermore, DOE (2022) focuses on specific projects that account for a relatively small portion of the regulated firm's overall operations.

[29] Law (2014, p. 51) observes that "A few studies have ... found evidence suggestive of the AJW [Averch-Johnson-Wellisz] effect. Studies from the same period in different industries found no evidence and more recent papers have found no evidence of the AJW effect. Either there never was a very significant AJW effect and/or regulators read the economics literature, too, and took steps to mitigate the AJW effect."

Parameter	Value
$\overline{C}$	100
C_0^*	99.5
C_1^*	90
$\underline{K}$	0.1
$\overline{K}$	1.0

Parameter	Value
δ	0.01
T	5
b	0.95
β	0.95

Table 1. Parameter Values in the Baseline Setting.

The variable $k \equiv \frac{K}{\delta}$ is assumed to have a uniform density in the baseline setting. However, the ensuing analysis also considers settings in which k has a truncated normal density on $\left[\underline{k}, \overline{k}\right]$.[30] These densities are illustrated in Figure 2.

[**Figure 2 about Here**]

Figure 3 illustrates how $P(s_c)$, the PDV of expected procurement cost, varies with the sharing rate, s_c, in the baseline setting. Figure 3 also illustrates how $P(s_c)$ varies with s_c when parameter values are as specified in Table 1 and $f(k)$ is the truncated normal density density. In both cases, $P(s_c)$ is a U-shaped function of s_c that attains its minimum value when s_c is negative. Thus, as in Corollary 1, the regulator optimally awards to the firm during the initial regulatory regime more than the full cost reduction it achieves. Doing so ensures a relatively high probability that consumers enjoy in all subsequent regulatory regimes the full cost reduction admitted by the new technology.

[**Figure 3 about Here**]

Recall from Corollary 2 that when sufficient historic inefficiency in the firm's operation ($\overline{C} - C_0^* > 0$) prevails and $f(k)$ is the uniform density: (i) $s_r^* = 0$ when Δ_C^* is below a critical value (Δ_{C1}); and (ii) s_r^* initially increases, and eventually declines, as Δ_C^* increases above Δ_{C1}. Figure 4 illustrates this relationship between s_r^* and Δ_C^* in the baseline setting and when $f(k)$ is the truncated normal density.[31] The inverted-U shape of the curves in Figure 4 arises because: (i) when s_r is relatively small, the regulator optimally enhances the

[30] The truncated normal density has mean $\mu = \frac{1}{2}\left[\underline{k} + \overline{k}\right]$ and standard deviation $\sigma = 30$. The corresponding cumulative distribution function (CDF) is $\frac{\Phi(k) - \Phi(\underline{k})}{\Phi(\overline{k}) - \Phi(\underline{k})}$ for $k \in \left[\underline{k}, \overline{k}\right]$, where $\Phi(\cdot)$ is the CDF of a normal distribution with mean μ and standard deviation σ. When $\sigma = 30$ and parameter values are as specified in Table 1, the range of the density, $\overline{k} - \underline{k}$, is three standard deviations. The resulting unimodal density is meaningfully more concentrated around the midrange of the support than is the uniform distribution. Other standard densities with an inverted-U shape, including the parabolic density and the piecewise linear density, give rise to conclusions simlilar to those drawn below.

[31] For both densities, the model parameters that underlie Figure 4 are those specified in Table 1.

manager's increased incentive to implement the new technology as Δ_C^* increases by increasing s_r; and (ii) when s_r is relatively large, the regulator optimally reduces the use of her costly instrument as an increase in Δ_C^* enhances the manager's incentive to implement the new technology.[32]

[**Figure 4 about Here**]

Figures B1 – B7 in the Appendix illustrate how s_r^* varies with other model parameters in the baseline setting and when $f(k)$ is the truncated normal density. These figures thereby illustrate the conclusions drawn in Corollaries 3 and 4, and demonstrate that the conclusions hold more generally.

Tables 2A and 2B characterize industry outcomes as model parameters diverge from the levels specified in Table 1 when $f(k)$ is the uniform density.[33] The first column in the tables identifies the value of the parameter that is changing, while all other parameter values remain at their levels in the baseline setting. The second column specifies the optimal sharing rate, s_r^*. The third column in the tables reports $F(\widehat{k}(s_r^*))$, the probability that the manager implements the new technology when $s_r = s_r^*$. The fourth column presents the ratio of $F(\widehat{k}(1))$, the probability that the manager implements the new technology under TOTEX (i.e., when $s_r = 1$), to $F(\widehat{k}(s_r^*))$.

The fifth column in Tables 2A and 2B presents $P(s_r^*)$, the PDV of expected procurement cost when $s_r = s_r^*$.[34] The last column in the tables provides an indicator of TOTEX's performance, $M \equiv \frac{P(s_r < 0) - P(s_r = 1)}{P(s_r < 0) - P(s_r = s_r^*)}$, where: (i) $P(s_r < 0)$ is the PDV of expected procurement cost when $s_r < 0$, so the manager never implements the new technology (recall Lemma 1); (ii) $P(s_r = 1)$ is the PDV of expected procurement cost under TOTEX, where $s_r = 1$; and (iii) $P(s_r = s_r^*)$ is the minimum PDV of expected procurement cost the regulator can attain.[35] Observe that the numerator of M is the amount by which the implementation of TOTEX reduces the PDV of expected procurement cost below the PDV of procurement cost when the historic cost ($\overline{C}$) always prevails (i.e., below $P(s_r < 0) = \frac{\overline{C}}{1-\beta}$). Further observe that the denominator of M is the difference between $\frac{\overline{C}}{1-\beta}$ and the minimum PDV of expected procurement cost the regulator can attain. Therefore, M can be viewed as the fraction of

[32]In essence, when $\overline{C} - C_0^* > 0$ is sufficiently pronounced that s_r is a relatively weak instrument, s_r and Δ_C^* optimally act as complements in enhancing the manager's incentive to implement the new technology when s_r is small, whereas they optimally act as substitutes when s_r is large.

[33]Tables B1 and B2 in the Appendix provide the characterization when $f(k)$ is the truncated normal density.

[34]For expositional ease, in Tables 2A and 2B and in the ensuing analysis, the PDV of expected procurement cost, $P(\cdot)$, is expressed as a function of the share of achieved cost reductions awarded to the firm (rather than to consumers) during the initial regulatory regime.

[35]Formally, $P(s_r = s_r^*)$ is the PDV of expected procurement cost at the solution to [RP].

the maximum attainable reduction in the PDV of expected procurement cost that TOTEX secures.

Parameter	s_r^*	$F(\widehat{k}(s_r^*))$	$\frac{F(\widehat{k}(1))}{F(\widehat{k}(s_r^*))}$	$P(s_r^*)$	M
$\overline{C} = 105$	1.720	0.711	0.515	$1,950.4$	0.929
$\overline{C} = 100$	2.271	0.974	0.376	$1,905.2$	0.650
$\overline{C} = 99.5$	2.326	1.000	0.366	$1,900.0$	0.599
$C_0^* = 99.5$	2.271	0.974	0.376	$1,905.2$	0.650
$C_0^* = 95.0$	1.775	0.737	0.497	$1,856.4$	0.914
$C_1^* = 95$	2.210	0.389	0.296	$1,977.5$	0.700
$C_1^* = 90$	2.271	0.974	0.376	$1,905.2$	0.650
$C_1^* = 80$	1.113	1.000	0.869	$1,702.6$	0.908
$\underline{K} = 0.05$	2.210	0.947	0.422	$1,905.0$	0.700
$\underline{K} = 0.1$	2.271	0.974	0.376	$1,905.2$	0.650
$\underline{K} = 0.2$	2.326	1.000	0.287	$1,950.3$	0.527
$\overline{K} = 0.5$	1.163	1.000	0.825	$1,852.6$	0.875
$\overline{K} = 1.0$	2.271	0.974	0.376	$1,905.2$	0.650
$\overline{K} = 2.0$	2.210	0.447	0.389	$1,952.5$	0.700

Table 2A. Outcomes as Cost Parameters Vary from their Baseline Values.

Parameter	s_r^*	$F(\widehat{k}(s_r^*))$	$\frac{F(\widehat{k}(1))}{F(\widehat{k}(s_r^*))}$	$P(s_r^*)$	M
$\delta = 0.005$	2.333	0.446	0.287	$1,955.3$	0.593
$\delta = 0.01$	2.271	0.974	0.376	$1,905.2$	0.650
$\delta = 0.02$	1.163	1.000	0.844	$1,852.6$	0.895
$b = 0.90$	2.278	0.874	0.367	$1,914.7$	0.644
$b = 0.95$	2.271	0.974	0.376	$1,905.2$	0.650
$b = 0.98$	2.191	1.000	0.396	$1,899.1$	0.654
$\beta = 0.90$	1.282	0.501	0.731	975.0	0.941
$\beta = 0.95$	2.271	0.974	0.376	$1,905.2$	0.650
$\beta = 0.98$	2.327	1.000	0.366	$4,611.8$	0.463
$T = 3$	3.603	0.974	0.195	$1,905.2$	0.417
$T = 5$	2.271	0.974	0.376	$1,905.2$	0.650
$T = 7$	1.703	0.974	0.540	$1,905.2$	0.809

Table 2B. Outcomes as Other Parameters Vary from their Baseline Values.

Tables 2A and 2B indicate that in the baseline setting and for substantial variation in this setting, TOTEX is unduly stringent in the sense that it awards too small a share of realized cost reductions to the firm (i.e., $s_r^* > 1$). This stringency reduces the probability that the new technology is implemented ($F(\widehat{k}(1))$) below its optimal level ($F(\widehat{k}(s_r^*))$). This reduction in implementation probability is substantial in the baseline setting, as $F(\widehat{k}(1))$ is less than 40% of $F(\widehat{k}(s_r^*))$.[36] The sub-optimal implementation probability in the baseline setting causes TOTEX to secure less than two-thirds of the reduction in the PDV of expected procurement cost that the optimal regulatory policy secures (i.e., $M = 0.650$ in the baseline setting).

Tables 2A and 2B indicate that M, a measure of TOTEX's efficacy in reducing procurement costs relative to the optimal (s_r^*) policy, generally declines as s_r^* increases further above 1. In this sense, TOTEX generally performs more poorly as the share of cost reductions that is optimally awarded to the firm during the initial regulatory regime (s_r^*) increases further above 1. Industry conditions under which s_r^* tend to be relatively high, so TOTEX performs poorly in the sense that M is relatively low, are discussed in Section 6.

Before proceeding to Section 6, we note that Tables 2A and 2B do not imply that s_r^* always exceeds 1.[37,38] Settings do exist in which $s_r^* < 1$. However, these settings seem unlikely to prevail in practice. To illustrate, consider the setting of primary interest where managerial technology implementation costs (K) are such that the regulator optimally induces the manager to implement the new technology for some, but not all, K realizations.[39] In this setting, consider separately for each model parameter the feasible values of the parameter for which $s_r^* < 1$ when all other parameters are as specified in Table 1 and $f(k)$ is the uniform density.[40] It can be shown that there are no such feasible values of $\overline{C}$, C_0^*, C_1^*, $\underline{K}$, $\overline{K}$, b,

[36] As Tables 2A and 2B report, $\frac{F(\widehat{k}(1))}{F(\widehat{k}(s_r^*))} = 0.376$ in the baseline setting.

[37] We also note that Tables 2A and 2B illustrate the extent to which s_r^* declines in the baseline setting as $\overline{C}$ increases, or as β or $\underline{K}$ declines. (Recall Corollary 3.) Furthermore, Table 2B implies that $\frac{\overline{C} - C_1^*}{\overline{C} - C_0^*}$ is sufficiently large in the baseline setting to ensure that s_r^* declines as b, T, or δ increases. (Recall Corollary 4.)

[38] Two other elements of Tables 2A and 2B warrant brief mention. First, Table 2A illustrates that s_r^* does not vary monotonically with $\overline{K}$. The non-monotonicity arises in part because $k^e \equiv \frac{1}{2}[\underline{k} + \overline{k}]$ and $\Delta_k \equiv \overline{k} - \underline{k}$ both increase as $\overline{K}$ increases. The increase in k^e serves to increase s_r^* whereas the increase in $\overline{K}$ serves to reduce s_r^*, *ceteris paribus*. (Recall Corollary 3.) Second, Table 2B implies that $\widehat{k}(s_r^*)$ does not vary as T changes. It can be shown that this conclusion reflects the assumption that $b = \beta$ in the baseline setting.

[39] The Appendix considers settings in which the regulator induces the manager to implement the new technology for all realizations of $K \in [\underline{K}, \overline{K}]$.

[40] Feasible values for $\overline{C}$, C_0^*, and C_1^* are, respectively, $\overline{C} \geq C_0^*$, $C_0^* \in (C_1^*, \overline{C}]$, and $C_1^* \in (0, C_0^*)$. Feasible values for $\underline{K}$ and $\overline{K}$ are, respectively, $\underline{K} \in (0, \overline{K})$ and $\overline{K} > \underline{K}$. Feasible values for the other model

or δ. The relevant feasible values of T and β are $T > 15$ and $\beta \in (0, 0.856)$.[41] We are not aware of any incentive regulation regime that has lasted more than 15 years.[42] Furthermore, annual interest rates typically are well below 14% in developed countries,[43] which suggests that values of β below 0.856 are unlikely to prevail in practice.

6 Settings Where TOTEX Performs Poorly

The analysis in Section 5 indicates that TOTEX tends to perform more poorly (in the sense that M declines) as s_r^* increases further above 1. Section 4 identifies conditions that promote higher values of s_r^*, and thus generally promote lower values of M when $s_r^* > 1$. Specifically, the analysis in Section 4 identifies five factors that systematically increase s_r^* in the setting of primary interest where the manager is optimally induced to implement the new technology for some, but not all, realizations of $K \in [\underline{K}, \overline{K}]$.[44]

First, s_r^* increases as expected managerial implementation costs (k^e) increase.[45] In practice, k^e might be relatively high when, for example, the firm's managers have little experience implementing new technologies in the regulated industry or in any other industries where they have worked in the past. k^e might also be relatively high when the regulated firm in question is an industry leader in the sense that no other firms in the regulated industry have yet attempted to implement the new technology. As k^e increases, the probability that the manager implements the new technology declines, *ceteris paribus*. The regulator optimally increases s_r to avoid an unduly large reduction in this probability.

Second, s_r^* increases as regulatory uncertainty about managerial technology implementation costs declines in the sense that $\overline{k} - \underline{k}$ declines, holding k^e constant.[46] Such regulatory uncertainty can decline, for example, when the regulated firm in question is an industry

parameters are $\delta > 0$, $T > 1$, $b \in (0, 1)$, and $\beta \in (0, 1)$.

[41] If T is very large (e.g., $T > 15$), the firm secures a share (s_r^*) of realized cost reductions for such an extended period of time that the manager might implement the new technology for nearly all K realizations even when $s_r^* < 1$. If β is sufficiently small (e.g., $\beta < 0.856$), the regulator may place such a low valuation on consumer welfare after period T that she optimally secures a portion of realized cost reductions for consumers during the initial regulatory regime by setting $s_r^* < 1$, despite the associated reduction in the probability that the manager implements the new technology.

[42] Ofgem's RIIO incentive regulation plan for UK electricity distribution companies lasted for eight years, from 2015 to 2023 (Ofgem, 2017). Incentive regulation plans typically last for only four or five years (Sappington and Weisman, 2024).

[43] The U.S. prime rate has not exceeded 14% in more than forty years (FedPrimeRate.com, 2025).

[44] Finding 3 identifies parameter changes that increase s_r^* when the manager is induced to implement the new technology for all realizations of $K \in [\underline{K}, \overline{K}]$.

[45] Recall Corollary 3 and see Figure B1 in the Appendix.

[46] Recall Corollary 3 and see Figure B2 in the Appendix.

laggard in the sense that regulated firms in other jurisdictions have already gained considerable experience implementing the new technology. As such regulatory uncertainty declines, the probability that the manager implements the new technology becomes more responsive to changes in s_r. (Recall (13).) The regulator optimally employs her instrument more extensively (i.e., s_r increases) as the instrument becomes systematically more effective in this sense.[47]

Third, s_r^* increases as the regulator's valuation of future consumer welfare increases relative to her valuation of present consumer welfare (i.e., when β increases).[48] This change in relative valuation might stem from reduced pressure to secure immediate rate relief for consumers in the regulated industry, for example.[49] The reduced pressure might prevail, for example, when the economy is robust, so wages and incomes are high, and unemployment is low. Pressure for immediate rate relief can also decline when consumer prices in the regulated industry have been stable (or declining) in recent years.[50] As this pressure declines, the regulator optimally increases s_r to enhance the probability that the new technology is implemented, thereby increasing the probability that consumers enjoy substantial gains in future regulatory regimes.

Fourth, s_r^* increases as the historic inefficiency of the firm's operation declines, perhaps because of concerted regulatory effort to limit over-capitalization, for instance.[51] As $\overline{C} - C_0^*$ declines, the firm secures a smaller increment in profit (by eliminating historic inefficiencies) if it continues to employ the original technology. Consequently, an increase in s_r raises the probability that the manager implements the new technology relatively rapidly. The regulator optimally increases s_r as the instrument becomes more effective in this sense. Doing so increases the probability that the new technology is implemented sufficiently to ensure that the PDV of expected procurement cost declines.

Fifth, s_r^* can increase as the incremental cost saving admitted by the new technology ($\Delta_C^* \equiv C_0^* - C_1^*$) increases when sufficient historic inefficiency ($\overline{C} - C_0^* > 0$) prevails and

[47] s_r^* does not necessarily increase as other measures of regulatory uncertainty decline. To illustrate, s_r^* declines as the variance of the truncated normal density declines when model parameters are as specified in Table 1. In general, the impact of reduced "uncertainty" on s_r^* varies with the value of s_r^* and the manner in which reduced uncertainty affects the density $f(k)$ in the neighborhood of $\widehat{k}(s_r^*)$.

[48] Recall Corollary 3 and see Figure B3 in the Appendix.

[49] β may also increase as the regulator's tenure increases, which can cause her to value long-term consumer gains relatively highly.

[50] β can increase in non-election years, when incumbent politicians may perceive less pressure to secure immediate voter approval.

[51] Recall Corollary 3 and see Figure B4 in the Appendix.

Δ_C^* is not too pronounced.[52] As Δ_C^* increases but remains relatively small, the manager's incremental gain from implementing the new technology increases more rapidly as s_r increases. The regulator optimally employs s_r more extensively as the instrument becomes more powerful in this sense.[53] However, once Δ_C^* becomes sufficiently pronounced, the regulator can employ her costly instrument less extensively (i.e., she can reduce s_r) as Δ_C^* increases further without reducing unduly the probability that the manager implements the new technology.

We have also identified three factors that promote an increase in s_r^* when $\frac{\overline{C} - C_1^*}{\overline{C} - C_0^*}$ is sufficiently pronounced, so the regulator's s_r instrument is relatively effective at inducing the manager to implement the new technology. First, s_r^* increases in this setting as managerial compensation becomes less closely linked to the firm's realized profit.[54] More limited linkage can arise, for example, when managers in the regulated firm secure a smaller fraction of their compensation in the form of stock options. This reduced linkage renders the manager's technology implementation decision less responsive to variation in the firm's profit. To help offset the manager's reduced incentive to implement the new technology, the regulator optimally increases s_r to avoid an undue reduction in the probability that the new technology is implemented.

Second, s_r^* increases in this setting as the length of a regulatory regime (T) declines.[55] As T declines, the increase in the firm's profit engendered by an increase in s_r becomes less enduring. The reduced time period during which the manager can benefit from an achieved cost reduction diminishes the manager's incentive to implement the new technology. To help offset this diminished incentive, the regulator optimally increases s_r to avoid an undue reduction in the probability that the new technology is implemented.

Third, s_r^* increases in this setting as the manager's valuation of future returns (b) declines.[56] In practice, b might decline when managers become less likely to be employed by the regulated firm for the full duration of the initial regulatory regime.[57] As b declines, the

[52] Recall Corollary 2 and Figure 4.

[53] When the regulator's s_r instrument is relatively powerful because $\overline{C} = C_0^*$, the regulator optimally employs s_r less extensively as Δ_C^* increases. (Recall Corollary 2.)

[54] Formally, $\frac{ds_r^*}{d\delta} < 0$ when $\frac{\overline{C} - C_1^*}{\overline{C} - C_0^*}$ is sufficiently pronounced, as Corollary 4 reports. Also see Figure B5 in the Appendix.

[55] Recall Corollary 4 and see Figure B6 in the Appendix. Sappington and Weisman (2010) report that the length of regulatory regimes often varies over time and across regulatory settings.

[56] Recall Corollary 4 and see Figure B7 in the Appendix.

[57] A manager may be less likely to remain with the regulated firm for an extended time period if, for example, the manager is relatively old or the firm lacks effective policies to identify, promote, and retain promising young managers.

manager effectively values less highly the profit the firm secures during periods $2, ..., T$, which reduces his incentive to implement the new technology. When the regulator's s_r instrument is relatively effective (because $\frac{\overline{C}-C_1^*}{\overline{C}-C_0^*}$ is large), the regulator optimally increases her use of the instrument as the manager's incentive to implement the new technology declines (due to the reduction in b). The increase in s_r avoids an excessive reduction in the probability that the manager implements the new technology.

In summary, TOTEX often provides less incentive to implement the new technology than does the optimal (s_r^*) regulatory policy. The associated reduction in the extent to which TOTEX reduces the PDV of expected procurement costs can be especially pronounced when expected managerial technology implementation costs are large, regulatory uncertainty about these costs is limited, the regulator values future consumer welfare relatively highly, or the regulated firm's historic inefficiency is limited. TOTEX also tends to perform more poorly in this sense as the incremental cost reduction admitted by the new technology increases in a range of moderate such cost reductions when historic inefficiency is relatively pronounced. Furthermore, TOTEX tends to provide unduly limited incentive for new technology adoption when the adoption admits a substantial cost reduction (in the sense that $\frac{\overline{C}-C_1^*}{\overline{C}-C_0^*}$ is large) and managerial compensation is not closely linked to the firm's realized profit, regulatory regimes are of relatively limited duration, or the firm's managers value short-term profit highly relative to long-term profit.

7 Conclusions

We have analyzed the optimal sharing of realized cost reductions between a regulated firm and its customers in a setting where the regulator has limited knowledge of the difficulties the firm's managers face in implementing a new cost-reducing technology. We found that TOTEX, which effectively awards to the firm during the initial regulatory regime the full cost reduction it achieves, often provides insufficient incentive to implement the new technology. Enhanced incentive would increase the probability that the new technology is implemented sufficiently to reduce the PDV of expected procurement costs, even after accounting for the firm's more generous compensation.

We also identified conditions under which TOTEX is particularly likely to provide insufficient incentive for new technology adoption. As might be expected, these conditions include relatively high expected technology implementation costs and relatively pronounced concern with future consumer welfare. Perhaps more subtly, these conditions also include relatively limited uncertainty about managerial technology adoption costs, relatively limited historic inefficiency (e.g., over-capitalization) in the firm's operations, and moderately large (but not especially large) potential incremental cost reductions under the new technology when historic inefficiency prevails.

These findings suggest the potential merits of enhancing incentives for new technology adoption in regulated industries. The enhanced incentives might be provided by extending the period during which the regulated firm is awarded the full benefit of realized cost reductions, for example. The optimal length of this extension generally will vary across settings, reflecting the factors we have identified (including, for instance, whether the firm in question is an industry leader or an industry laggard in new technology adoption).

We have analyzed a streamlined setting in which the only friction the regulator faces is limited knowledge of managerial technology implementation costs. In practice, regulators typically have limited information about other industry conditions, including the full set of potential cost-reducing technologies and the cost reductions these technology admit, for example. These additional frictions would likely change the details of our analysis. However, the key trade-offs we have analyzed seem likely to persist in the presence of these additional frictions. These trade-offs also seem likely to persist in the presence of nonlinear sharing rules, and when production costs and consumer demand vary over time.[58]

[58] These trade-offs also seem likely to persist in settings where managerial effort can affect the extent to which the full potential of the prevailing technology is achieved.

Appendix

Part A of this Appendix provides the proofs of the formal conclusions in the text. Part B presents additional characterization of the numerical solutions.

A. Proofs of the Formal Conclusions in the Text.

Proof of Lemma 2. $s_c \;\geq\; \overline{s}_c \;\Leftrightarrow\; s_c \;\geq\; 1 - \dfrac{\underline{k}}{b_T\,\Delta_C^*} \;\Leftrightarrow\; \dfrac{\underline{k}}{b_T\,\Delta_C^*} \;\geq\; 1 - s_c$

$$\Leftrightarrow\; \underline{k} \;\geq\; [\,1 - s_c\,]\, b_T\,\Delta_C^* \;=\; \widehat{k}\,;$$

$$s_c \;<\; \underline{s}_c \;\Leftrightarrow\; s_c \;<\; 1 - \frac{\overline{k}}{b_T\,\Delta_C^*} \;\Leftrightarrow\; \frac{\overline{k}}{b_T\,\Delta_C^*} \;<\; 1 - s_c$$

$$\Leftrightarrow\; \overline{k} \;<\; [\,1 - s_c\,]\, b_T\,\Delta_C^* \;=\; \widehat{k}\,. \tag{14}$$

(9) implies that the manager implements the new technology if and only if $k < \widehat{k}$. Therefore, (14) implies that the manager: (i) never implements the new technology if $s_c \geq \overline{s}_c$; and (ii) implements the new technology for all K realizations if $s_c < \underline{s}_c$. ■

Proof of Finding 1. The proof consists of Step A, Step B, and Step C.

Step A. Prove that $s_c^* \leq 1$.

If $s_c > 1$, then $C_t(\cdot) = \overline{C}$ for all $t = 1, ..., T$. (Lemma 1.) Therefore, (11) implies:

$$P(s_c) \;=\; [\,\beta_T + \beta_\infty\,]\;\overline{R} \quad \text{when } \; s_c \;>\; 1\,. \tag{15}$$

Case 1. $\overline{C} \;>\; C_0^*$.

The manager never implements the new technology when $s_c = 1$. $\widehat{C} = C_0^*$ in this case, so (11) implies:

$$P(1) \;=\; [\,\beta_T + \beta_\infty\,]\;\overline{R} - [\,\beta_T + \beta_\infty\,]\,[\,\overline{C} - C_0^*\,]\,. \tag{16}$$

(15) and (16) imply that $P(1) < P(s_c > 1)$, so $s_c^* \leq 1$, when $\overline{C} \;>\; C_0^*$.

Case 2. $\overline{C} \;=\; C_0^*$.

First suppose $\widehat{k} \;\leq\; \underline{k}$, so $F(\widehat{k}\,) = 0$. Then (10) implies that $\widehat{C} \;=\; C_0^* \;=\; \overline{C}$. Consequently, (11) and (15) imply that the PDV of expected procurement cost is $[\,\beta_T + \beta_\infty\,]\;\overline{R} \;=\; P(s_c > 1)$. Therefore, the regulator cannot reduce the PDV of expected procurement cost strictly below $P(1)$ by increasing s_c above 1.

Now suppose $\widehat{k} \;>\; \underline{k}$, so $F(\widehat{k}\,) > 0$. Then $\widehat{C} \;<\; \overline{C}$ when $s_c \;=\; 0$. Consequently, (11) implies that when $\overline{C} \;=\; C_0^*$:

$$P(0) \;=\; [\,\beta_T + \beta_\infty\,]\;\overline{R} - \beta_\infty \left[\,\overline{C} - \widehat{C}(0)\,\right] \;<\; [\,\beta_T + \beta_\infty\,]\;\overline{R} \;=\; P(1)\,. \tag{17}$$

The last equality in (17) reflects (16) because $\overline{C} = C_0^*$ in this case. (15) and (17) imply that $s_c^* \leq 1$ when $\overline{C} = C_0^*$.

Step B. Prove that $s_c^* < \overline{s}_c$ if $s_c^* \neq 1$.

The manager never implements the new technology if $s_c \geq \overline{s}_c$ (Lemma 2). Therefore, (11) implies that when $s_c \geq \overline{s}_c$:

$$P(s_c) = [\beta_T + \beta_\infty]\,\overline{R} - \beta_T\, s_c\,\left[\,\overline{C} - C_0^*\right] - \beta_\infty\left[\,\overline{C} - C_0^*\right]$$

$$\Rightarrow \quad P'(s_c) = -\beta_T\left[\,\overline{C} - C_0^*\right] \overset{s}{=} -\left[\,\overline{C} - C_0^*\right]. \tag{18}$$

First suppose $\overline{C} > C_0^*$. (18) implies that the regulator can strictly reduce the PDV of expected procurement cost in this case by increasing s_c when $s_c \geq \overline{s}_c$. Therefore, $s_c^* < \overline{s}_c$ in this case.

Now suppose $\overline{C} = C_0^*$. (18) implies that $P(s_c) = [\beta_T + \beta_\infty]\,\overline{R}$ for all $s_c \in [\,\overline{s}_c, 1\,]$ when $\overline{C} = C_0^*$. We will show that the regulator can reduce $P(s_c)$ below $[\beta_T + \beta_\infty]\,\overline{R}$ by reducing s_c below $\overline{s}_c$ in this case. To do so, first observe from (9) that:

$$\frac{d\widehat{k}}{ds_c} = -\,b_T\,\Delta_C^*. \tag{19}$$

(10) and (19) imply:

$$\frac{\partial\widehat{C}}{\partial s_c} = F'(\widehat{k})\,[\,C_1^* - C_0^*\,]\,\frac{\partial\widehat{k}}{\partial s_c} = -\,F'(\widehat{k})\,\Delta_C^*\,[-\,b_T\,\Delta_C^*\,] = b_T\,F'(\widehat{k})\,[\,\Delta_C^*\,]^2. \tag{20}$$

(11) and (20) imply:

$$P'(s_c) = -\beta_T\left[\,\overline{C} - \widehat{C}\,\right] + [\,\beta_T\,s_c + \beta_\infty\,]\,\frac{\partial\widehat{C}}{\partial s_c}$$

$$= -\beta_T\left[\,\overline{C} - \widehat{C}\,\right] + [\,\beta_T\,s_c + \beta_\infty\,]\,b_T\,F'(\widehat{k})\,[\,\Delta_C^*\,]^2. \tag{21}$$

(21) and Lemma 2 imply that when $\overline{C} = C_0^*$:[59]

$$P'(\overline{s}_c^-) = -\beta_T\left[\,\overline{C} - C_0^*\right] + [\,\beta_T\,\overline{s}_c + \beta_\infty\,]\,b_T\,F'(\widehat{k})\,[\,\Delta_C^*\,]^2$$

$$= [\,\beta_T\,\overline{s}_c + \beta_\infty\,]\,b_T\,F'(\widehat{k})\,[\,\Delta_C^*\,]^2 > 0. \tag{22}$$

(22) implies that the regulator can reduce $P(s_c)$ below $[\beta_T + \beta_\infty]\,\overline{R}$ by reducing s_c below $\overline{s}_c$. Therefore, $s_c^* < \overline{s}_c$ in this case.

Step C. Prove that $s_c^* \geq \underline{s}_c$.

If $s_c < \underline{s}_c$, the manager implements the new technology for all K realizations (Lemma 2). Therefore, (11) implies that the PDV of expected procurement cost is:

$$P(s_c) = [\beta_T + \beta_\infty]\,\overline{R} - \beta_T\,s_c\left[\,\overline{C} - C_1^*\right] - \beta_\infty\left[\,\overline{C} - C_1^*\right]$$

[59] $P'(\overline{s}_c^-)$ denotes the left-hand derivative of $P(s_c)$, evaluated at $s_c = \overline{s}_c$.

$$\Rightarrow \quad P'(s_c) \; = \; - \beta_T \left[\, \overline{C} - C_1^* \right] \; < \; 0 \, . \tag{23}$$

(23) implies that the regulator can strictly reduce the PDV of expected procurement cost by increasing s_c toward $\underline{s}_c$. Therefore, $s_c^* \geq \underline{s}_c$. ■

Proof of Finding 2. Finding 1 implies that Finding 2 holds if:

$$P(1) \; < \; \min_{s_c \, \in \, [\, \underline{s}_c , \, \overline{s}_c \,]} P(s_c) \tag{24}$$

when $\overline{C} > C_0^*$ and Δ_C^* is sufficiently small. (11) and (16) imply that when $\overline{C} > C_0^*$:

$$P(1) \; = \; [\beta_T + \beta_\infty \,] \, \overline{R} - [\beta_T + \beta_\infty \,] \left[\, \overline{C} - C_0^* \right] \quad \text{and}$$

$$P(s_c) \; = \; [\beta_T + \beta_\infty \,] \, \overline{R} - [\beta_T \; s_c + \beta_\infty \,] \left[\, \overline{C} - \widehat{C}(s_c) \right] . \tag{25}$$

Suppose $s_c < 1$ at the solution to [RP]. Then (25) implies:

$$\begin{aligned} P(s_c) \; &= \; [\beta_T + \beta_\infty \,] \, \overline{R} - [\beta_T \; s_c + \beta_\infty \,] \left[\, \overline{C} - \widehat{C}(s_c) \right] \\ &\leq \; [\beta_T + \beta_\infty \,] \, \overline{R} - [\beta_T + \beta_\infty \,] \left[\, \overline{C} - C_0^* \right] \; = \; P(1) \, . \end{aligned} \tag{26}$$

The inequality in (26) reflects the assumption that s_c is the solution to [RP]. (26) implies:

$$[\beta_T + \beta_\infty \,] \left[\, \overline{C} - C_0^* \right] \; \leq \; [\beta_T \; s_c + \beta_\infty \,] \left[\, \overline{C} - \widehat{C}(s_c) \right] \; \Rightarrow \; \beta_T \; s_c + \beta_\infty \; \geq \; 0 \, . \tag{27}$$

(25) and (27) imply:

$$\begin{aligned} \min_{s_c \, \in \, [\, \underline{s}_c , \, \overline{s}_c \,]} P(s_c) \; &= \; \min_{s_c \, \in \, [\, \underline{s}_c , \, \overline{s}_c \,]} \left\{ [\beta_T + \beta_\infty \,] \, \overline{R} - [\beta_T \; s_c + \beta_\infty \,] \left[\, \overline{C} - \widehat{C}(s_c) \right] \right\} \\ &\geq \; \min_{s_c \, \in \, [\, \underline{s}_c , \, \overline{s}_c \,]} \left\{ [\beta_T + \beta_\infty \,] \, \overline{R} - [\beta_T \; s_c + \beta_\infty \,] \left[\, \overline{C} - C_1^* \right] \right\} \\ &\geq \; [\beta_T + \beta_\infty \,] \, \overline{R} - [\beta_T \; \overline{s}_c + \beta_\infty \,] \left[\, \overline{C} - C_1^* \right] \\ &= \; [\beta_T + \beta_\infty \,] \, \overline{R} - [\beta_T \; \overline{s}_c + \beta_\infty \,] \left[\, \overline{C} - C_0^* \right] - [\beta_T \; \overline{s}_c + \beta_\infty \,] \, \Delta_C^* \, . \end{aligned} \tag{28}$$

The first inequality in (28) holds because $\widehat{C}(s_c) \geq C_1^*$ and $\beta_T \; s_c + \beta_\infty \geq 0$ (from (27)). The second inequality in (28) holds because $s_c \leq \overline{s}_c$ when $s_c \in [\, \underline{s}_c, \, \overline{s}_c \,]$. (25) and (28) imply that (24) holds if:

$$\begin{aligned} & [\beta_T + \beta_\infty \,] \, \overline{R} - [\beta_T \; \overline{s}_c + \beta_\infty \,] \left[\, \overline{C} - C_0^* \right] - [\beta_T \; \overline{s}_c + \beta_\infty \,] \, \Delta_C^* \\ & \qquad > \; [\beta_T + \beta_\infty \,] \, \overline{R} - [\beta_T + \beta_\infty \,] \left[\, \overline{C} - C_0^* \right] \\ \Leftrightarrow \; & [\beta_T + \beta_\infty \,] \left[\, \overline{C} - C_0^* \right] \; > \; [\beta_T \; \overline{s}_c + \beta_\infty \,] \left[\, \overline{C} - C_0^* \right] + [\beta_T \; \overline{s}_c + \beta_\infty \,] \, \Delta_C^* \\ \Leftrightarrow \; & \beta_T \, [\, 1 - \overline{s}_c \,] \left[\, \overline{C} - C_0^* \right] \; > \; [\beta_T \; \overline{s}_c + \beta_\infty \,] \, \Delta_C^* \end{aligned}$$

$$\Leftrightarrow \quad \beta_T \left[\frac{\underline{k}}{b_T \, \Delta_C^*} \right] [\, \overline{C} - C_0^* \,] \;\; > \;\; \left[\beta_T \left(1 - \frac{\underline{k}}{b_T \, \Delta_C^*} \right) + \beta_\infty \right] \Delta_C^*$$

$$\Leftrightarrow \quad \frac{\underline{k}}{\Delta_C^*} [\, \overline{C} - C_0^* \,] \left[\frac{\beta_T}{b_T} \right] \;\; > \;\; \left[\beta_T + \beta_\infty - \frac{\underline{k}}{\Delta_C^*} \left(\frac{\beta_T}{b_T} \right) \right] \Delta_C^*$$

$$\Leftrightarrow \quad \frac{\underline{k}}{\Delta_C^*} [\, \overline{C} - C_0^* \,] \;\; > \;\; \frac{b_T}{\beta_T} [\, \beta_T + \beta_\infty \,] \, \Delta_C^* - \underline{k}$$

$$\Leftrightarrow \quad \frac{\underline{k}}{\Delta_C^*} [\, \overline{C} - C_0^* \,] \;\; > \;\; \frac{b_T \, \Delta_C^*}{\beta_T \, [\, 1 - \beta \,]} - \underline{k} \, .$$

The last inequality here holds when $\overline{C} > C_0^*$ and Δ_C^* is sufficiently small. ■

Proof of Finding 3. It is apparent that $\underline{s}_c = 1 - \frac{\overline{k}}{b_T \, \Delta_C^*}$ does not change as $\underline{K}$, $\overline{C}$, or β changes. It is also apparent that: (i) $\underline{s}_c$ increases as Δ_C^* increases; and (ii) $\underline{s}_c$ declines as $\overline{K}$ increases or δ declines (because $\overline{k} = \frac{\overline{K}}{\delta}$). Furthermore:

$$\frac{\partial \underline{s}_c}{\partial T} \;\; = \;\; \frac{\partial}{\partial T} \left(1 - \frac{\overline{k}}{b_T \, \Delta_C^*} \right) \;\; = \;\; \left[\frac{\overline{k}}{(b_T)^2 \, \Delta_C^*} \right] \frac{\partial b_T}{\partial T} \;\; > \;\; 0 \, .$$

The inequality here holds because, from (7):

$$\frac{\partial b_T}{\partial T} \;\; = \;\; \frac{\partial}{\partial T} \left(\frac{1 - b^T}{1 - b} \right) \;\; \stackrel{s}{=} \;\; - \frac{\partial b^T}{\partial T} \;\; = \;\; - \, b^T \ln b \;\; > \;\; 0 \, .$$

The inequality here reflects the fact that $\ln b < 0$ because $b \in (0, 1)$.

(7) also implies:

$$\frac{\partial}{\partial b} \left(1 - \frac{\overline{k}}{b_T \, \Delta_C^*} \right) \;\; \stackrel{s}{=} \;\; - \left[- \frac{1}{(b_T)^2} \right] \frac{\partial b_T}{\partial b} \;\; \stackrel{s}{=} \;\; \frac{\partial b_T}{\partial b}$$

$$= \;\; \frac{\partial}{\partial b} \left(\sum_{t=1}^{T} b^{t-1} \right) \;\; = \;\; \sum_{t=1}^{T} [\, t - 1 \,] \, b^{t-2} \;\; > \;\; 0 \, . \;\; ■ \qquad (29)$$

The following assumption and definition are employed in Lemmas 3 and 4, which are helpful in proving Finding 4.

Assumption L. $|\, F''(k) \,|$ is sufficiently close to 0 for all $k \in [\, \underline{k}, \overline{k} \,]$.

Definitions. $$\widetilde{s}_c \;\; \equiv \;\; \frac{\overline{C} - \widehat{C}}{b_T \; F'(\widehat{k}) \, [\, \Delta_C^* \,]^2} - \frac{\beta_\infty}{\beta_T} \, . \qquad \widehat{s}_c \;\; = \;\; \underset{s_c \in [\, \underline{s}_c, \, \overline{s}_c \,]}{\arg\min} \; P(s_c) \, . \qquad (30)$$

Lemma 3. *Suppose Assumption L holds. Then $P(s_c)$ is a strictly convex function of s_c for $s_c \in [\,\underline{s}_c,\, \overline{s}_c\,]$. Furthermore, if $\widetilde{s}_c \in (\underline{s}_c,\, \overline{s}_c\,)$, then: (i) $P'(s_c) < 0$ for $s_c \in [\,\underline{s}_c,\, \widetilde{s}_c\,)$; and (ii) $P'(s_c) > 0$ for $s_c \in (\,\widetilde{s}_c,\, \overline{s}_c\,]$.*

Proof. (19) – (21) imply:

$$\begin{aligned} P''(s_c) &= \beta_T \frac{\partial \widehat{C}}{\partial s_c} + \beta_T \, b_T \; F'(\widehat{k}) \, [\, \Delta_C^* \,]^2 + [\, \beta_T \; s_c + \beta_\infty \,] \, b_T \; F''(\widehat{k}) \, [\, \Delta_C^* \,]^2 \; \frac{\partial \widehat{k}}{\partial s_c} \\ &= \beta_T \, b_T \; F'(\widehat{k}) \, [\, \Delta_C^* \,]^2 + \beta_T \, b_T \; F'(\widehat{k}) \, [\, \Delta_C^* \,]^2 + [\, \beta_T \; s_c + \beta_\infty \,] \, b_T \; F''(\widehat{k}) \, [\, \Delta_C^* \,]^2 \; \frac{\partial \widehat{k}}{\partial s_c} \\ &= 2 \, \beta_T \, b_T \; F'(\widehat{k}) \, [\, \Delta_C^* \,]^2 + [\, \beta_T \; s_c + \beta_\infty \,] \, b_T \; F''(\widehat{k}) \, [\, \Delta_C^* \,]^2 \, [\, - \, b_T \; \Delta_C^* \,] \\ &= 2 \, \beta_T \, b_T \; F'(\widehat{k}) \, [\, \Delta_C^* \,]^2 - [\, \beta_T \; s_c + \beta_\infty \,] \, [\, \Delta_C^* \,]^3 \, [\, b_T \,]^2 \, F''(\widehat{k}) \; > \; 0 \, . \end{aligned} \tag{31}$$

The inequality in (31) reflects Assumption L.

(21) implies:

$$\begin{aligned} P'(s_c) \; = \; 0 \quad &\Leftrightarrow \quad [\, \beta_T \; s_c + \beta_\infty \,] \, b_T \; F'(\widehat{k}) \, [\, \Delta_C^* \,]^2 \; = \; \beta_T \left[\, \overline{C} - \widehat{C} \, \right] \\ &\Leftrightarrow \quad \beta_T \; s_c + \beta_\infty \; = \; \frac{\beta_T \left[\, \overline{C} - \widehat{C} \, \right]}{b_T \; F'(\widehat{k}) \, [\, \Delta_C^* \,]^2} \\ &\Leftrightarrow \quad s_c \; = \; \frac{\overline{C} - \widehat{C}}{b_T \; F'(\widehat{k}) \, [\, \Delta_C^* \,]^2} - \frac{\beta_\infty}{\beta_T} \; \equiv \; \widetilde{s}_c \, . \end{aligned} \tag{32}$$

The strict convexity of $P(s_c)$ established in (31) ensures that if $\widetilde{s}_c \in (\underline{s}_c,\, \overline{s}_c)$, then $P'(s_c) < 0$ for $s_c \in [\,\underline{s}_c,\, \widetilde{s}_c\,)$, and $P'(s_c) > 0$ for $s_c \in (\,\widetilde{s}_c,\, \overline{s}_c\,]$. ■

Lemma 4. *Suppose Assumption L holds. Then:*

$$\widehat{s}_c \; = \; \begin{cases} \underline{s}_c & \textit{if} \;\; \widetilde{s}_c \, \leq \, \underline{s}_c \\ \widetilde{s}_c & \textit{if} \;\; \widetilde{s}_c \, \in \, (\, \underline{s}_c,\, \overline{s}_c \,) \\ \overline{s}_c & \textit{if} \;\; \widetilde{s}_c \, \geq \, \overline{s}_c \, . \end{cases}$$

Proof. Lemma 3 establishes that $P(s_c)$ is increasing in s_c for all $s_c \in [\,\underline{s}_c,\, \overline{s}_c\,]$ if $\widetilde{s}_c \leq \underline{s}_c$. Therefore, $\underline{s}_c = \underset{s_c \in [\,\underline{s}_c, \overline{s}_c\,]}{\arg\min} \; P(s_c)$ in this case.

Lemma 3 also establishes that $\widetilde{s}_c = \underset{s_c \in [\,\underline{s}_c, \overline{s}_c\,]}{\arg\min} \; P(s_c)$ if $\widetilde{s}_c \in (\underline{s}_c,\, \overline{s}_c)$.

In addition, Lemma 3 establishes that $P(s_c)$ is decreasing in s_c for all $s_c \in [\,\underline{s}_c,\, \overline{s}_c\,]$ if $\overline{s}_c \leq \widetilde{s}_c$. Therefore, $\overline{s}_c = \underset{s_c \in [\,\underline{s}_c, \overline{s}_c\,]}{\arg\min} \; P(s_c)$ in this case. ■

Proof of Finding 4. (9) implies that when Assumption U holds:

$$F(\,\widehat{k}\,) \;=\; \frac{\widehat{k} \,-\, \underline{k}}{\overline{k} \,-\, \underline{k}} \;=\; \frac{[\,1 - s_c\,]\, b_T \;\Delta_C^* - \underline{k}}{\overline{k} - \underline{k}}\,. \tag{33}$$

(10) and (33) imply that when Assumption U holds:

$$\begin{aligned} \overline{C} - \widehat{C} \;&=\; \overline{C} - C_0^* + F(\,\widehat{k}\,)\,[\,C_0^* - C_1^*\,] \;=\; \overline{C} - C_0^* + F(\,\widehat{k}\,)\;\Delta_C^* \\ &=\; \frac{1}{\overline{k} - \underline{k}}\;\Big\{\,\big[\,\overline{k} - \underline{k}\,\big]\;\big[\,\overline{C} - C_0^*\,\big] + [\,(\,1 - s_c\,)\,b_T\;\Delta_C^* - \underline{k}\,]\,\Delta_C^*\,\Big\}\,. \end{aligned} \tag{34}$$

$F'(\,\widehat{k}\,) \;=\; \frac{1}{\overline{k} - \underline{k}}$ when Assumption U holds. Therefore, (34) implies that when Assumption U holds:

$$\frac{\overline{C} - \widehat{C}}{F'(\,\widehat{k}\,)} \;=\; \big[\,\overline{k} - \underline{k}\,\big]\;\big[\,\overline{C} - C_0^*\,\big] + [\,(\,1 - s_c\,)\,b_T\;\Delta_C^* - \underline{k}\,]\,\Delta_C^*\,. \tag{35}$$

(32), (35), and Lemma 4 imply that if Assumption U holds and $s^* \in [\,\underline{s}_c, \overline{s}_c\,]$, then:

$$\begin{aligned} s_c^* \;&=\; \frac{\overline{C} - \widehat{C}}{b_T\;F'(\,\widehat{k}\,)\,[\,\Delta_C^*\,]^2} - \frac{\beta_\infty}{\beta_T} \\ &=\; \frac{\big[\,\overline{k} - \underline{k}\,\big]\;\big[\,\overline{C} - C_0^*\,\big] + [\,(\,1 - s_c^*\,)\,b_T\;\Delta_C^* - \underline{k}\,]\,\Delta_C^*}{b_T\;[\,\Delta_C^*\,]^2} - \frac{\beta_\infty}{\beta_T} \\ &=\; \frac{\beta_T\,\big[\,\overline{k} - \underline{k}\,\big]\;\big[\,\overline{C} - C_0^*\,\big] + \beta_T\,[\,(\,1 - s_c^*\,)\,b_T\;\Delta_C^* - \underline{k}\,]\,\Delta_C^* - b_T\;\beta_\infty\;[\,\Delta_C^*\,]^2}{b_T\,\beta_T\;[\,\Delta_C^*\,]^2} \end{aligned}$$

$$\begin{aligned} \Rightarrow\quad b_T\,\beta_T\;[\,\Delta_C^*\,]^2\;s_c^* \;&=\; \beta_T\,\big[\,\overline{k} - \underline{k}\,\big]\;\big[\,\overline{C} - C_0^*\,\big] \\ &\qquad +\; \beta_T\,[\,(\,1 - s_c^*\,)\,b_T\;\Delta_C^* - \underline{k}\,]\,\Delta_C^* - b_T\;\beta_\infty\;[\,\Delta_C^*\,]^2 \end{aligned}$$

$$\Rightarrow\quad 2\,b_T\,\beta_T\;[\,\Delta_C^*\,]^2\;s_c^* \;=\; \beta_T\,\big[\,\overline{k} - \underline{k}\,\big]\;\big[\,\overline{C} - C_0^*\,\big] + b_T\,\beta_T\,[\,\Delta_C^*\,]^2 - \beta_T\,\underline{k}\;\Delta_C^* - b_T\;\beta_\infty\,[\,\Delta_C^*\,]^2$$

$$\Rightarrow\quad s_c^* \;=\; \frac{1}{2} + \frac{\big[\,\overline{k} - \underline{k}\,\big]\;\big[\,\overline{C} - C_0^*\,\big]}{2\,b_T\,[\,\Delta_C^*\,]^2} - \frac{b_T\;\beta_\infty\;\Delta_C^* + \beta_T\;\underline{k}}{2\,b_T\,\beta_T\,\Delta_C^*}\,. \quad \blacksquare$$

Proof of Corollary 1. (12) implies that under the specified conditions:

$$\begin{aligned} s_r^* \;>\; 1 \quad\Leftrightarrow\quad s_c^* \;<\; 0 \quad\Leftrightarrow\quad & \frac{1}{2} + \frac{\big[\,\overline{k} - \underline{k}\,\big]\;\big[\,\overline{C} - C_0^*\,\big]}{2\,b_T\,[\,\Delta_C^*\,]^2} - \frac{b_T\;\beta_\infty\;\Delta_C^* + \beta_T\;\underline{k}}{2\,b_T\,\beta_T\,\Delta_C^*} \;<\; 0 \\ \Leftrightarrow\quad & \frac{b_T\;\beta_\infty\;\Delta_C^* + \beta_T\;\underline{k}}{2\,b_T\,\beta_T\,\Delta_C^*} \;>\; \frac{1}{2} + \frac{\beta_T\,\big[\,\overline{k} - \underline{k}\,\big]\;\big[\,\overline{C} - C_0^*\,\big]}{2\,b_T\,[\,\Delta_C^*\,]^2} \end{aligned}$$

$$\Leftrightarrow \quad b_T\, \beta_\infty\, \Delta_C^* + \beta_T\, \underline{k} \; > \; b_T\, \beta_T\, \Delta_C^* + \frac{[\, \overline{k} - \underline{k} \,]\, [\, \overline{C} - C_0^* \,]}{b_T\, \Delta_C^*}$$

$$\Leftrightarrow \quad \beta_T\, \underline{k} \; > \; b_T\, [\, \beta_T - \beta_\infty \,]\, \Delta_C^* + \frac{[\, \overline{k} - \underline{k} \,]\, [\, \overline{C} - C_0^* \,]}{b_T\, \Delta_C^*}$$

$$\Leftrightarrow \quad \beta_T\, \underline{k} \; > \; b_T \left[\frac{1 - \beta^T - \beta^T}{1 - \beta} \right] \Delta_C^* + \frac{[\, \overline{k} - \underline{k} \,]\, [\, \overline{C} - C_0^* \,]}{b_T\, \Delta_C^*}$$

$$\Leftrightarrow \quad \underline{k} \; > \; \frac{b_T}{\beta_T} \left[\frac{1 - 2\, \beta^T}{1 - \beta} \right] \Delta_C^* + \frac{[\, \overline{k} - \underline{k} \,]\, [\, \overline{C} - C_0^* \,]}{b_T\, \Delta_C^*} \,. \tag{36}$$

Because $\underline{k} > 0$, the last inequality in (36) holds if $\beta^T \geq \frac{1}{2}$ and $\overline{C} - C_0^*$ is sufficiently small. ■

Lemmas 5 – 10 help to prove Corollary 2**.**

Lemma 5. *$s_c^* \in [\, \underline{s}_c, \overline{s}_c \,)$ if $\overline{C} = C_0^*$.*

Proof. $s_c^* \geq \underline{s}_c$ from Finding 1. Finding 1 implies that if $s_c^* > \overline{s}_c$, then $s_c^* = 1$, so $P(s_c) = [\, \beta_T + \beta_\infty \,]\, \overline{R}$ when $\overline{C} = C_0^*$, from (16). (22) implies the regulator can reduce $P(s_c)$ below $[\, \beta_T + \beta_\infty \,]\, \overline{R}$ when $\overline{C} = C_0^*$. Therefore, $s_c^* \leq \overline{s}_c$ in this case. Consequently, Finding 1 implies that $s_c^* < \overline{s}_c$. ■

Lemma 6. *If $s_c^* < 1$ when $\Delta_C^* = \Delta_0 > 0$, then $s_c^* < 1$ when $\Delta_C^* > \Delta_0$.*

Proof. Let $s_c^*(\Delta)$ denote the value of s_c at the solution to [RP] when $\Delta_C^* = \Delta$. Similarly, let $\widehat{C}(s_c; \Delta)$ denote the firm's expected cost when sharing rate s_c is imposed and $\Delta_C^* = \Delta$. Also let $P(s_c; \Delta)$ denote the PDV of expected procurement cost when sharing rate s_c is imposed and $\Delta_C^* = \Delta$. Define $\widehat{k}(s_c; \Delta) = b_T\, [\, 1 - s_c \,]\, \Delta$ analogously.

(11) implies that when $s_c^*(\Delta_C^*) < 1$:

$$P(s_c^*(\Delta_C^*); \Delta_C^*) \; = \; [\, \beta_T + \beta_\infty \,]\, \overline{R} \\ - [\, \beta_T\; s_c^*(\Delta_C^*) + \beta_\infty \,] \left[\overline{C} - \widehat{C}(s_c^*(\Delta_C^*); \Delta_C^*) \right] \; \leq \; P(1)\,. \tag{37}$$

Suppose $s_c^*(\Delta) = 1$ for some $\Delta > \Delta_C^*$. (11) implies that because $s_c^*(\Delta) = 1$:

$$P(1) \; = \; P(s_c^*(\Delta); \Delta) \; = \; [\, \beta_T + \beta_\infty \,]\, \overline{R} - [\, \beta_T\; s_c^*(\Delta) + \beta_\infty \,] \left[\overline{C} - \widehat{C}(s_c^*(\Delta); \Delta) \right]$$

$$\leq \; [\, \beta_T + \beta_\infty \,]\, \overline{R} - [\, \beta_T\; s_c^*(\Delta_C^*) + \beta_\infty \,] \left[\overline{C} - \widehat{C}(s_c^*(\Delta_C^*); \Delta) \right]$$

$$< \; [\, \beta_T + \beta_\infty \,]\, \overline{R} - [\, \beta_T\; s_c^*(\Delta_C^*) + \beta_\infty \,] \left[\overline{C} - \widehat{C}(s_c^*(\Delta_C^*); \Delta_C^*) \right] \; = \; P(s_c^*(\Delta_C^*); \Delta_C^*)\,. \tag{38}$$

The first inequality in (38) holds because $s_c^*(\Delta)$ constitutes the solution to [RP] when $C_0^* - C_1^* = \Delta$. The last inequality in (38) holds because (10) implies:

$$\begin{aligned}\widehat{C}(s_c^*(\Delta_C^*);\Delta) \;&=\; C_0^* - F(\widehat{k}(s_c^*(\Delta_C^*),\Delta))\,\Delta \;\leq\; C_0^* - F(\widehat{k}(s_c^*(\Delta_C^*),\Delta_C^*))\,\Delta \\ &<\; C_0^* - F(\widehat{k}(s_c^*(\Delta_C^*),\Delta_C^*))\,\Delta_C^* \;=\; \widehat{C}(s_c^*(\Delta_C^*);\Delta_C^*)\,. \end{aligned} \tag{39}$$

The first inequality in (39) holds because (9) implies:

$$\widehat{k}(s_c^*(\Delta_C^*),\Delta_C^*) \;=\; b_T\;\Delta_C^*\,[\,1 - s_c^*(\Delta_C^*)\,] \;<\; b_T\;\Delta\,[\,1 - s_c^*(\Delta_C^*)\,] \;=\; \widehat{k}(s_c^*(\Delta_C^*),\Delta)\,. \tag{40}$$

The inequality in (40) holds because $s_c^*(\Delta_C^*) < 1$ and $\Delta > \Delta_C^*$. The second inequality in (39) holds because $\Delta_C^* < \Delta$ and $F(\widehat{k}(s_c^*(\Delta_C^*),\Delta_C^*)) > 0$. This last inequality here holds because $\widehat{k}(s_c^*(\Delta_C^*),\Delta_C^*) = b_T\;\Delta_C^*\,[\,1 - s_c^*(\Delta_C^*)\,] > \underline{k}$, from Finding 1 (which establishes that $s_c < \overline{s}_c$ at the solution to [RP] if $s_c \neq 1$ at this solution).[60]

(38) implies that (37) does not hold. Therefore, $s_c^*(\Delta) < 1$ for all $\Delta > \Delta_C^*$. ■

Lemma 7. *There exists a $\Delta_C^* > 0$ for which $s_c^* < 1$.*

Proof. Because $\overline{s}_c < 1$, Lemma 5 implies that $s_c^* < 1$ for all $\Delta_C^* > 0$ if $\overline{C} = C_0^*$. The remainder of the proof considers the case where $\overline{C} > C_0^*$.

Define $s_{c0} \equiv 1 - \frac{\underline{k}+\overline{k}}{2\,b_T\,\Delta_C^*}$ and suppose $s_c^* = 1$ for all Δ_C^*. (10) and (11) imply that because $s_c = 1$ for all Δ_C^*:

$$\begin{aligned} P(1) \;\leq\; P(s_{c0}) \quad\Leftrightarrow\quad & [\,\beta_T + \beta_\infty\,]\,\overline{R} - [\,\beta_T + \beta_\infty\,]\,[\,\overline{C} - C_0^*\,] \\ & \leq\; [\,\beta_T + \beta_\infty\,]\,\overline{R} - [\,\beta_T\;s_{c0} + \beta_\infty\,]\left[\,\overline{C} - \widehat{C}(s_{c0})\,\right] \\ \Leftrightarrow\quad & [\,\beta_T\;s_{c0} + \beta_\infty\,]\left[\,\overline{C} - \widehat{C}(s_{c0})\,\right] \;\leq\; [\,\beta_T + \beta_\infty\,]\,[\,\overline{C} - C_0^*\,] \\ \Leftrightarrow\quad & [\,\beta_T\;s_{c0} + \beta_\infty\,]\left[\,\overline{C} - C_0^* + F(\widehat{k}(s_{c0}))\,\Delta_C^*\,\right] \;\leq\; [\,\beta_T + \beta_\infty\,]\,[\,\overline{C} - C_0^*\,] \\ \Leftrightarrow\quad & [\,\beta_T\;s_{c0} + \beta_\infty\,]\,F(\widehat{k}(s_{c0}))\;\Delta_C^* \;\leq\; [\,1 - s_{c0}\,]\,\beta_T\,[\,\overline{C} - C_0^*\,]\,. \end{aligned} \tag{41}$$

(9) implies:

$$\begin{aligned}\widehat{k}(s_{c0}) \;&=\; b_T\,[\,1 - s_{c0}\,]\,\Delta_C^* \;=\; b_T\left[1 - \left(1 - \frac{\underline{k}+\overline{k}}{2\,b_T\,\Delta_C^*}\right)\right]\Delta_C^* \\ &=\; b_T\left[\frac{\underline{k}+\overline{k}}{2\,b_T\,\Delta_C^*}\right]\Delta_C^* \;=\; \frac{\underline{k}+\overline{k}}{2}\,. \end{aligned} \tag{42}$$

(42) implies that $F(\widehat{k}(s_{c0})) = F(\frac{\underline{k}+\overline{k}}{2}) > 0$ for all $\Delta_C^* > 0$. Therefore, the inequality in (41) holds if and only if:

[60] Observe that $s_c^* < \overline{s}_c \;\Leftrightarrow\; s_c^* < 1 - \frac{\underline{k}}{b_T\;\Delta_C^*} \;\Leftrightarrow\; \frac{\underline{k}}{b_T\;\Delta_C^*} < 1 - s_c^* \;\Leftrightarrow\; \underline{k} < b_T\,[\,1 - s_c^*\,]\,\Delta_C^*$.

$$[\,\beta_T\; s_{c0} + \beta_\infty\,]\; F(\frac{\underline{k} + \overline{k}}{2})\; \Delta_C^* \;\leq\; [\,1 - s_{c0}\,]\,\beta_T\,[\,\overline{C} - C_0^*\,]\,. \tag{43}$$

$s_{c0} \rightarrow 1$ as $\Delta_C^* \rightarrow \infty$. Therefore, as $\Delta_C^* \rightarrow \infty$, the right hand side of the inequality in (43) approaches 0 whereas the left hand side of the inequality becomes infinitely large. Therefore, the inequality in (41) does not hold, which implies it is not the case that $s_c^* = 1$ for all Δ_C^*. ■

Lemma 8. *Suppose $\overline{C} > C_0^*$. Then there exists a $\Delta > 0$ such that: (i) $s_c^* = 1$ if $\Delta_C^* < \Delta$; whereas (ii) $s_c^* < 1$ if $\Delta_C^* > \Delta$.*

Proof. Finding 2 establishes that there exist values of $\Delta > 0$ such that $s_c^* = 1$ if $\Delta_C^* < \Delta$. Let $\overline{\Delta}$ be the largest of these Δ. Lemmas 6 and 7 establish that such a $\overline{\Delta}$ exists and is finite. Lemma 6 implies that $s_c^* < 1$ if $\Delta_C^* > \overline{\Delta}$. ■

Lemma 9. *Suppose Assumption U holds. Then $s_c^* = \underline{s}_c$ if:*

$$\overline{k} \;<\; \min\left\{ \frac{b_T\,[\,\Delta_C^*\,]^2}{\beta_T\,[\,1-\beta\,]\,[\,\overline{C} - C_1^*\,]}\,,\; \frac{\underline{k}\,[\,\overline{C} - C_1^*\,] + \left[\,\frac{1}{1-b}\,\right] \left[\frac{1-b^T}{1-\beta^T}\right] [\,\Delta_C^*\,]^2}{\overline{C} - C_1^* + \Delta_C^*} \right\}. \tag{44}$$

Proof. $s_c^* \neq 1$ under the specified conditions. This is the case because (11) implies:

$$\begin{aligned}
P(1) \;>\; P(\underline{s}_c) \;\;&\Leftrightarrow\;\; [\,\beta_T + \beta_\infty\,]\;\overline{R} - \beta_T\,[\,\overline{C} - C_0^*\,] - \beta_\infty\,[\,\overline{C} - C_0^*\,] \\
&\qquad > \;[\,\beta_T + \beta_\infty\,]\;\overline{R} - \beta_T\;\underline{s}_c\,[\,\overline{C} - C_1^*\,] - \beta_\infty\,[\,\overline{C} - C_1^*\,]
\end{aligned}$$

$$\Leftrightarrow\;\; \beta_T\,[\,\overline{C} - C_0^*\,] + \beta_\infty\,[\,\overline{C} - C_0^*\,] \;<\; \beta_T\;\underline{s}_c\,[\,\overline{C} - C_1^*\,] + \beta_\infty\,[\,\overline{C} - C_1^*\,]$$

$$\Leftrightarrow\;\; \beta_T\,[\,\overline{C} - C_0^*\,] - \beta_\infty\,[\,C_0^* - C_1^*\,] \;<\; \underline{s}_c\,\beta_T\,[\,\overline{C} - C_1^*\,]$$

$$\Leftrightarrow\;\; \underline{s}_c \;>\; \frac{\beta_T\,[\,\overline{C} - C_0^*\,] - \beta_\infty\,[\,C_0^* - C_1^*\,]}{\beta_T\,[\,\overline{C} - C_1^*\,]}$$

$$\Leftrightarrow\;\; \underline{s}_c \;>\; \frac{\beta_T\,[\,\overline{C} - C_1^*\,] - \beta_T\,[\,C_0^* - C_1^*\,] - \beta_\infty\,[\,C_0^* - C_1^*\,]}{\beta_T\,[\,\overline{C} - C_1^*\,]}$$

$$\Leftrightarrow\;\; \underline{s}_c \;>\; 1 - \frac{\beta_T\,[\,C_0^* - C_1^*\,] + \beta_\infty\,[\,C_0^* - C_1^*\,]}{\beta_T\,[\,\overline{C} - C_1^*\,]} \;=\; 1 - \frac{[\,\beta_T + \beta_\infty\,]\,\Delta_C^*}{\beta_T\,[\,\overline{C} - C_1^*\,]}$$

$$\Leftrightarrow\;\; 1 - \frac{\overline{k}}{b_T\;\Delta_C^*} \;>\; 1 - \frac{[\,\beta_T + \beta_\infty\,]\,\Delta_C^*}{\beta_T\,[\,\overline{C} - C_1^*\,]} \;\;\Leftrightarrow\;\; \frac{\overline{k}}{b_T\;\Delta_C^*} \;<\; \frac{[\,\beta_T + \beta_\infty\,]\,\Delta_C^*}{\beta_T\,[\,\overline{C} - C_1^*\,]}$$

$$\Leftrightarrow\;\; \overline{k} \;<\; \frac{b_T\,[\,\beta_T + \beta_\infty\,]\,[\,\Delta_C^*\,]^2}{\beta_T\,[\,\overline{C} - C_1^*\,]} \;\;\Leftrightarrow\;\; \overline{k} \;<\; \frac{b_T\,[\,\Delta_C^*\,]^2}{\beta_T\,[\,1-\beta\,]\,[\,\overline{C} - C_1^*\,]}\,. \tag{45}$$

Because $s_c^* \neq 1$, Lemma 4 implies that $s_c^* = \underline{s}_c$ if:

$$\frac{1}{2} + \frac{\beta_T \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - b_T \, \beta_\infty \left[\, \Delta_C^* \,\right]^2 - \beta_T \, \underline{k} \, \Delta_C^*}{2 \, b_T \, \beta_T \left[\, \Delta_C^* \,\right]^2} \;\leq\; 1 - \frac{\overline{k}}{b_T \, \Delta_C^*}$$

$$\Leftrightarrow \quad \frac{\beta_T \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - b_T \, \beta_\infty \left[\, \Delta_C^* \,\right]^2 - \beta_T \, \underline{k} \, \Delta_C^*}{2 \, b_T \, \beta_T \left[\, \Delta_C^* \,\right]^2} + \frac{\overline{k}}{b_T \, \Delta_C^*} \;\leq\; \frac{1}{2}$$

$$\Leftrightarrow \quad \beta_T \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - b_T \, \beta_\infty \left[\, \Delta_C^* \,\right]^2 - \beta_T \, \underline{k} \, \Delta_C^* + 2 \, \beta_T \, \Delta_C^* \, \overline{k} \;\leq\; b_T \, \beta_T \left[\, \Delta_C^* \,\right]^2$$

$$\Leftrightarrow \quad \beta_T \left\{ \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - \underline{k} \, \Delta_C^* + 2 \, \Delta_C^* \, \overline{k} \,\right\} \;\leq\; b_T \left[\, \beta_T + \beta_\infty \,\right] \left[\, \Delta_C^* \,\right]^2$$

$$\Leftrightarrow \quad \frac{1 - \beta^T}{1 - \beta} \left\{ \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - \underline{k} \, \Delta_C^* + 2 \, \Delta_C^* \, \overline{k} \,\right\} \;\leq\; \frac{1 - b^T}{1 - b} \left[\frac{1}{1 - \beta} \right] \left[\, \Delta_C^* \,\right]^2$$

$$\Leftrightarrow \quad \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - \underline{k} \, \Delta_C^* + 2 \, \Delta_C^* \, \overline{k} \;\leq\; \left[\frac{1}{1 - b} \right] \left[\frac{1 - b^T}{1 - \beta^T} \right] \left[\, \Delta_C^* \,\right]^2 \tag{46}$$

$$\Leftrightarrow \quad \overline{k} \left[\, \overline{C} - C_0^* + 2 \, \Delta_C^* \,\right] \;\leq\; \underline{k} \left[\, \overline{C} - C_0^* + C_0^* - C_1^* \,\right] + \left[\frac{1}{1 - b} \right] \left[\frac{1 - b^T}{1 - \beta^T} \right] \left[\, \Delta_C^* \,\right]^2$$

$$\Leftrightarrow \quad \overline{k} \left[\, \overline{C} - C_1^* + \Delta_C^* \,\right] \;\leq\; \underline{k} \left[\, \overline{C} - C_1^* \,\right] + \left[\frac{1}{1 - b} \right] \left[\frac{1 - b^T}{1 - \beta^T} \right] \left[\, \Delta_C^* \,\right]^2$$

$$\Leftrightarrow \quad \overline{k} \;\leq\; \frac{\underline{k} \left[\, \overline{C} - C_1^* \,\right] + \left[\frac{1}{1 - b} \right] \left[\frac{1 - b^T}{1 - \beta^T} \right] \left[\, \Delta_C^* \,\right]^2}{\overline{C} - C_1^* + \Delta_C^*} \,. \quad \blacksquare$$

Lemma 10. *Suppose Assumption U holds and* $s_c^* \neq 1$.[61] *Then:*

$s_c^* \in (\underline{s}_c, \overline{s}_c) \;\Leftrightarrow\; \Delta_C^* \in (\underline{\Delta}_C^*, \overline{\Delta}_C^*)$, *where, for* $z \equiv \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right]$

$$\underline{\Delta}_C^* \;\equiv\; \frac{1 - \beta}{2 \, b_T} \left[\underline{k} \, \beta_T + \sqrt{ \left[\, \underline{k} \, \beta_T \,\right]^2 + \frac{4 \, b_T \, \beta_T \, z}{1 - \beta} } \;\right] \quad \textit{and}$$

$$\overline{\Delta}_C^* \;\equiv\; \frac{1 - \beta^T}{2 \, b_T} \left[2 \, \overline{k} - \underline{k} + \sqrt{ \left[\, 2 \, \overline{k} - \underline{k} \,\right]^2 + \frac{4 \, z \, b_T}{1 - \beta^T} } \;\right] . \tag{47}$$

Proof. (46) implies:

$$\widetilde{s}_c \;>\; \underline{s}_c \quad \Leftrightarrow \quad \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - \underline{k} \, \Delta_C^* + 2 \, \Delta_C^* \, \overline{k} \;>\; \left[\frac{1}{1 - b} \right] \left[\frac{1 - b^T}{1 - \beta^T} \right] \left[\, \Delta_C^* \,\right]^2$$

[61] It can be shown that $s_c^* \neq 1$ if $\frac{\beta_T \, \widehat{s}_c + \beta_\infty}{\beta_T \left[\, 1 - \widehat{s}_c \,\right]} \left[\, C_0^* - \widehat{C}(\, \widehat{s}_c \,) \,\right] > \overline{C} - C_0^*$.

$$\Leftrightarrow \quad \left[\frac{b_T}{1 - \beta^T} \right] [\, \Delta_C^* \,]^2 - \left[\, 2\,\overline{k} - \underline{k} \,\right] \Delta_C^* - z \;<\; 0\,. \tag{48}$$

The roots of the quadratic equation associated with (48) are:

$$\Delta_C^* \;=\; \frac{1 - \beta^T}{2\, b_T} \left[2\,\overline{k} - \underline{k} \pm \sqrt{\left[\, 2\,\overline{k} - \underline{k} \,\right]^2 + \frac{4\, z\, b_T}{1 - \beta^T}} \;\right]. \tag{49}$$

(48) and (49) imply that because $\Delta_C^* > 0$:

$$s_c^* \;>\; \underline{s}_c \quad \Leftrightarrow \quad \Delta_C^* \;\in\; (\, 0\, ,\, \overline{\Delta}_C^* \,)\,.$$

Lemma 4 implies:

$$\widetilde{s}_c \;<\; \overline{s}_c \quad \Leftrightarrow \quad \frac{1}{2} + \frac{\beta_T \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - b_T\, \beta_\infty\, [\, \Delta_C^* \,]^2 - \beta_T\, \underline{k}\, \Delta_C^*}{2\, b_T\, \beta_T\, [\, \Delta_C^* \,]^2} \;<\; 1 - \frac{\underline{k}}{b_T\, \Delta_C^*}$$

$$\Leftrightarrow \quad \frac{\beta_T \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - b_T\, \beta_\infty\, [\, \Delta_C^* \,]^2 - \beta_T\, \underline{k}\, \Delta_C^*}{2\, b_T\, \beta_T\, [\, \Delta_C^* \,]^2} + \frac{\underline{k}}{b_T\, \Delta_C^*} \;<\; \frac{1}{2}$$

$$\Leftrightarrow \quad \beta_T \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - b_T\, \beta_\infty\, [\, \Delta_C^* \,]^2 - \beta_T\, \underline{k}\, \Delta_C^* + 2\, \beta_T\, \Delta_C^*\, \underline{k} \;<\; b_T\, \beta_T\, [\, \Delta_C^* \,]^2$$

$$\Leftrightarrow \quad \beta_T \left\{ \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] + \underline{k}\, \Delta_C^* \right\} \;<\; b_T\, [\, \beta_T + \beta_\infty \,]\, [\, \Delta_C^* \,]^2$$

$$\Leftrightarrow \quad \beta_T\, [\, z + \underline{k}\, \Delta_C^* \,] \;<\; \frac{b_T}{1 - \beta}\, [\, \Delta_C^* \,]^2 \quad \Leftrightarrow \quad \frac{b_T}{1 - \beta}\, [\, \Delta_C^* \,]^2 - \beta_T\, \underline{k}\, \Delta_C^* - \beta_T\, z \;>\; 0\,. \tag{50}$$

The roots of the quadratic equation associated with (50) are:

$$\Delta_C^* \;=\; \frac{1 - \beta}{2\, b_T} \left[\underline{k}\, \beta_T \pm \sqrt{[\, \underline{k}\, \beta_T \,]^2 + \frac{4\, b_T\, \beta_T\, z}{1 - \beta}} \;\right]. \tag{51}$$

(50) and (51) imply that because $\Delta_C^* > 0$:

$$s_c^* \;<\; \overline{s}_c \quad \Leftrightarrow \quad \Delta_C^* \;>\; \underline{\Delta}_C^*\,. \quad \blacksquare$$

Proof of Corollary 2. (12) implies that when $\overline{C} = C_0^*$:

$$s_c^* \;=\; \frac{1}{2} - \frac{\beta_\infty}{2\, \beta_T} - \frac{\underline{k}}{2\, b_T\, \Delta_C^*} \quad \Rightarrow \quad \frac{\partial s_c^*}{\partial \Delta_C^*} \;>\; 0 \quad \Rightarrow \quad \frac{\partial s_r^*}{\partial \Delta_C^*} \;<\; 0\,.$$

Now suppose $\overline{C} > C_0^*$. Lemma 8 implies that in this case, there exists a Δ_{C1} such that: (1) $s_c^* = 1$ when $\Delta_C^* < \Delta_{C1}$; and (2) $s_c^* < 1$ when $\Delta_C^* \geq \Delta_{C1}$. Finding 1 and Lemma 3 imply that $s_c^* \in \{\, \underline{s}_c, \widetilde{s}_c \,\}$ when $\Delta_C^* \geq \Delta_{C1}$, where:

$$\widetilde{s}_c \;=\; \frac{1}{2} + \frac{\beta_T\, z - b_T\, \beta_\infty\, [\, \Delta_C^* \,]^2 - \beta_T\, \underline{k}\, \Delta_C^*}{2\, b_T\, \beta_T\, [\, \Delta_C^* \,]^2}\,. \tag{52}$$

(52) implies that, holding $\overline{C} - C_0^*$ constant:

$$\frac{\partial \widetilde{s}_c}{\partial \Delta_C^*} \; = \; - \frac{z}{b_T \, [\, \Delta_C^* \,]^3} + \frac{\underline{k}}{2 \, b_T \, [\, \Delta_C^* \,]^2} \; \stackrel{s}{=} \; \frac{\underline{k}}{2} - \frac{z}{\Delta_C^*}$$

$$\gtreqless \; 0 \quad \Leftrightarrow \quad \underline{k} \; \gtreqless \; \frac{2\,z}{\Delta_C^*} \quad \Leftrightarrow \quad \Delta_C^* \; \gtreqless \; \frac{2\,z}{\underline{k}} \, . \tag{53}$$

(53) implies: (1) $\frac{\partial \widetilde{s}_c}{\partial \Delta_C^*} > 0$ if $\Delta_C^* > \frac{2\,z}{\underline{k}}$; and (2) $\frac{\partial \widetilde{s}_c}{\partial \Delta_C^*} < 0$ if $\Delta_C^* < \frac{2\,z}{\underline{k}}$.

Because $\underline{s}_c$ is increasing in Δ_C^*, conclusions (ii) and (iii) in the Lemma hold if:

$$s_c^* \; = \; \underline{s}_c \quad \Leftrightarrow \quad \Delta_C^* \; \geq \; \Delta \;\; \text{for some} \;\; \Delta \; \geq \; 0 \, . \tag{54}$$

(48) implies that $\widetilde{s}_c \leq \underline{s}_c$ (so $s_c^* = \underline{s}_c$, from Lemma 4) if and only if:

$$\left[\frac{b_T}{1 - \beta^T} \right] [\, \Delta_C^* \,]^2 - \left[\, 2\,\overline{k} - \underline{k} \,\right] \Delta_C^* - z \; \geq \; 0 \, . \tag{55}$$

(49) implies that the positive root of the quadratic equation associated with (55) is $\overline{\Delta}_C^*$, as defined in (47). Consequently, because $\Delta_C^* > 0$, (55) implies that $s_c^* = \underline{s}_c$ if $\Delta_C^* \geq \overline{\Delta}_C^*$ and $\Delta_C^* \geq \Delta_{C1}$. Therefore, (54) holds with $\Delta = \max\{\, \overline{\Delta}_C^*, \, \Delta_{C1} \,\}$. ■

Proof of Corollary 3. (12) implies that under the specified conditions:

$$s_c^* \; = \; \frac{1}{2} + \frac{\left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right]}{2\,b_T\,[\, \Delta_C^* \,]^2} - \frac{b_T \, \beta_\infty \, \Delta_C^* + \beta_T \, \underline{k}}{2\,b_T\,\beta_T\,\Delta_C^*} \, . \tag{56}$$

(56) implies:

$$\frac{ds_c^*}{d\underline{k}} \; = \; - \frac{\overline{C} - C_0^*}{2\,b_T\,[\, \Delta_C^* \,]^2} - \frac{\beta_T}{2\,b_T\,\beta_T\,\Delta_C^*} \;\; < \;\; 0 \quad \text{and} \quad \frac{ds_c^*}{d\overline{C}} \; = \; \frac{\overline{k} - \underline{k}}{2\,b_T\,[\, \Delta_C^* \,]^2} \;\; > \;\; 0 \, . \tag{57}$$

(56) implies that under the specified conditions:

$$s_c^* \; = \; \frac{1}{2} + \frac{\beta_T \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - b_T \, \beta_\infty \, [\Delta_C^*]^2 - \beta_T \, \Delta_C^* \, \underline{k}}{2\,b_T\,\beta_T\,[\, \Delta_C^* \,]^2} \, . \tag{58}$$

(58) implies:

$$\frac{\partial s_c^*}{\partial \beta} \; \stackrel{s}{=} \; \beta_T \left\{ \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] \frac{\partial \beta_T}{\partial \beta} - \underline{k} \, \Delta_C^* \, \frac{\partial \beta_T}{\partial \beta} - b_T \, [\, \Delta_C^* \,]^2 \, \frac{\partial \beta_\infty}{\partial \beta} \right\}$$

$$- \; \left\{ \beta_T \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - b_T \, \beta_\infty \, [\, \Delta_C^* \,]^2 - \beta_T \, \underline{k} \, \Delta_C^* \,\right\} \frac{\partial \beta_T}{\partial \beta}$$

$$= \; \frac{\partial \beta_T}{\partial \beta} \{\, \beta_T \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] - \beta_T \, \underline{k} \, \Delta_C^*$$

$$- \;\; \beta_T \left[\, \overline{k} - \underline{k} \,\right] \left[\, \overline{C} - C_0^* \,\right] + b_T \, \beta_\infty \, [\, \Delta_C^* \,]^2 + \beta_T \, \underline{k} \, \Delta_C^* \,\}$$

$$- \; \frac{\partial \beta_\infty}{\partial \beta} \, b_T \, \beta_T \, [\, \Delta_C^* \,]^2$$

$$= \frac{\partial \beta_T}{\partial \beta} b_T \, \beta_\infty [\, \Delta_C^* \,]^2 - \frac{\partial \beta_\infty}{\partial \beta} b_T \, \beta_T [\, \Delta_C^* \,]^2 \overset{s}{=} \frac{\partial \beta_T}{\partial \beta} \beta_\infty - \frac{\partial \beta_\infty}{\partial \beta} \beta_T \, . \tag{59}$$

Observe that:

$$\frac{\partial \beta_T}{\partial \beta} = \frac{\partial}{\partial \beta} \left(\frac{1 - \beta^T}{1 - \beta} \right) = \frac{- [\, 1 - \beta \,] \, T \, \beta^{T-1} + 1 - \beta^T}{[\, 1 - \beta \,]^2} = \frac{1 - \beta^T}{[\, 1 - \beta \,]^2} - \frac{T \, \beta^{T-1}}{1 - \beta}$$

$$= \left[\frac{1}{1 - \beta} \right] \frac{1 - \beta^T}{1 - \beta} - \frac{T}{\beta} \left[\frac{\beta^T}{1 - \beta} \right] = \frac{\beta_T}{1 - \beta} - \frac{T \, \beta_\infty}{\beta} \, ; \text{ and}$$

$$\frac{\partial \beta_\infty}{\partial \beta} = \frac{\partial}{\partial \beta} \left(\frac{\beta^T}{1 - \beta} \right) = \frac{[\, 1 - \beta \,] \, T \, \beta^{T-1} + \beta^T}{[\, 1 - \beta \,]^2} = \frac{T \, \beta^{T-1}}{1 - \beta} + \frac{\beta^T}{[\, 1 - \beta \,]^2}$$

$$= \frac{T}{\beta} \left[\frac{\beta^T}{1 - \beta} \right] + \left[\frac{1}{1 - \beta} \right] \frac{\beta^T}{1 - \beta} = \left[\frac{T}{\beta} + \frac{1}{1 - \beta} \right] \beta_\infty \, . \tag{60}$$

(59) and (60) imply:

$$\frac{\partial s_c^*}{\partial \beta} \overset{s}{=} \beta_\infty \left[\frac{\beta_T}{1 - \beta} - \frac{T \, \beta_\infty}{\beta} \right] - \beta_T \, \beta_\infty \left[\frac{T}{\beta} + \frac{1}{1 - \beta} \right]$$

$$\overset{s}{=} \frac{\beta_T}{1 - \beta} - \frac{T \, \beta_\infty}{\beta} - \beta_T \left[\frac{T}{\beta} + \frac{1}{1 - \beta} \right] = - \frac{T \, \beta_\infty}{\beta} - \beta_T \left[\frac{T}{\beta} \right] < 0 \, .$$

Because $k^e \equiv \frac{1}{2} \left[\underline{k} + \overline{k} \right]$ and $\Delta_k \equiv \overline{k} - \underline{k}$:

$$k^e - \frac{\Delta_k}{2} = \frac{1}{2} \left[\underline{k} + \overline{k} \right] - \frac{1}{2} \left[\overline{k} - \underline{k} \right] = \underline{k} \, . \tag{61}$$

(56) and (61) imply that under the specified conditions:

$$s_c^* = \frac{1}{2} + \frac{\Delta_k \left[\, \overline{C} - C_0^* \, \right]}{2 \, b_T \, [\, \Delta_C^* \,]^2} - \frac{b_T \, \beta_\infty \, \Delta_C^* + \beta_T \left[\, k^e - \frac{\Delta_k}{2} \, \right]}{2 \, b_T \, \beta_T \, \Delta_C^*}$$

$$\Rightarrow \frac{\partial s_c^*}{\partial k^e} = - \frac{1}{2 \, b_T \, \Delta_C^*} < 0 \text{ and } \frac{\partial s_c^*}{\partial \Delta_k} = \frac{\overline{C} - C_0^*}{2 \, b_T \, [\, \Delta_C^* \,]^2} + \frac{1}{4 \, b_T \, \Delta_C^*} > 0 \, . \; \blacksquare$$

Proof of Corollary 4. We first prove the conclusion regarding δ. When the conditions in Finding 4 hold, s_c^* is as specified in (56) and (58). (56) implies:

$$\frac{d s_c^*}{d \overline{k}} = \frac{\overline{C} - C_0^*}{2 \, b_T \, [\, \Delta_C^* \,]^2} \, . \tag{62}$$

Because $k = \frac{K}{\delta}$:

$$\frac{\partial s_c^*}{\partial \delta} = \frac{\partial s_c^*}{\partial \overline{k}} \frac{\partial \overline{k}}{\partial \delta} + \frac{\partial s_c^*}{\partial \underline{k}} \frac{\partial \underline{k}}{\partial \delta} = - \frac{1}{\delta^2} \left[\frac{\partial s_c^*}{\partial \overline{k}} \, \overline{K} + \frac{\partial s_c^*}{\partial \underline{k}} \, \underline{K} \right] . \tag{63}$$

(57), (62), and (63) imply:

$$\frac{\partial s_c^*}{\partial \delta} \overset{s}{=} -\overline{K}\,[\,\overline{C}-C_0^*\,]+\underline{K}\,[\,\overline{C}-C_0^*+\Delta_C^*\,] \;=\; \underline{K}\,[\,\overline{C}-C_1^*\,]-\overline{K}\,[\,\overline{C}-C_0^*\,]$$

$$\lesseqgtr 0 \;\Leftrightarrow\; \overline{K}\,[\,\overline{C}-C_0^*\,] \gtreqless \underline{K}\,[\,\overline{C}-C_1^*\,] \;\Leftrightarrow\; \frac{\overline{C}-C_0^*}{\overline{C}-C_1^*} \gtreqless \frac{\underline{K}}{\overline{K}}. \tag{64}$$

We now prove the conclusion regarding b. (58) implies:

$$\frac{\partial s_c^*}{\partial b} \overset{s}{=} -b_T\,\beta_\infty\,[\,\Delta_C^*\,]^2\,\frac{\partial b_T}{\partial b}$$

$$-\;\frac{\partial b_T}{\partial b}\left\{\beta_T\,[\,\overline{k}-\underline{k}\,]\,[\,\overline{C}-C_0^*\,]-b_T\,\beta_\infty\,[\,\Delta_C^*\,]^2-\beta_T\,\underline{k}\,\Delta_C^*\right\}$$

$$= -\frac{\partial b_T}{\partial b}\left\{b_T\,\beta_\infty\,[\,\Delta_C^*\,]^2+\beta_T\,[\,\overline{k}-\underline{k}\,]\,[\,\overline{C}-C_0^*\,]-b_T\,\beta_\infty\,[\,\Delta_C^*\,]^2-\beta_T\,\underline{k}\,\Delta_C^*\right\}$$

$$= -\beta_T\,\frac{\partial b_T}{\partial b}\left\{[\,\overline{k}-\underline{k}\,]\,[\,\overline{C}-C_0^*\,]-\underline{k}\,\Delta_C^*\right\}$$

$$= -\beta_T\,\frac{\partial b_T}{\partial b}\left\{\overline{k}\,[\,\overline{C}-C_0^*\,]-\underline{k}\,[\,\overline{C}-C_0^*+C_0^*-C_1^*\,]\right\}$$

$$= \beta_T\,\frac{\partial b_T}{\partial b}\left\{\underline{k}\,[\,\overline{C}-C_1^*\,]-\overline{k}\,[\,\overline{C}-C_0^*\,]\right\}. \tag{65}$$

(7) implies:

$$\frac{\partial b_T}{\partial b} = \frac{\partial}{\partial b}\left(\sum_{t=1}^{T} b^{t-1}\right) = \sum_{t=1}^{T}[\,t-1\,]\,b^{t-2} > 0. \tag{66}$$

(65) and (66) imply that $\frac{\partial s_c^*}{\partial b} > 0$ if $\frac{\overline{C}-C_1^*}{\overline{C}-C_0^*} > \frac{\overline{k}}{\underline{k}}$.

Finally, we prove the conclusion regarding T. Observe that:

$$\frac{\partial \beta_T}{\partial T} = \frac{\partial}{\partial T}\left(\frac{1-\beta^T}{1-\beta}\right) = -\frac{\beta^T\,\ln\beta}{1-\beta}, \quad \frac{\partial \beta_\infty}{\partial T} = \frac{\partial}{\partial T}\left(\frac{\beta^T}{1-\beta}\right) = \frac{\beta^T\,\ln\beta}{1-\beta},$$

$$\text{and}\quad \frac{\partial b_T}{\partial T} = \frac{\partial}{\partial T}\left(\frac{1-b^T}{1-b}\right) = -\frac{b^T\,\ln b}{1-b}. \tag{67}$$

$$\text{Define:}\quad Z \equiv \beta_T\,[\,\overline{k}-\underline{k}\,]\,[\,\overline{C}-C_0^*\,]-b_T\,\beta_\infty\,[\,\Delta_C^*\,]^2-\beta_T\,\underline{k}\,\Delta_C^*. \tag{68}$$

(58), (67), and (68) imply:

$$\frac{\partial s_c^*}{\partial T} \overset{s}{=} b_T\,\beta_T\left\{[\,\overline{k}-\underline{k}\,]\,[\,\overline{C}-C_0^*\,]\,\frac{\partial \beta_T}{\partial T}-\beta_\infty\,[\,\Delta_C^*\,]^2\,\frac{\partial b_T}{\partial T}-b_T\,[\,\Delta_C^*\,]^2\,\frac{\partial \beta_\infty}{\partial T}\right.$$

$$\left.-\,\underline{k}\,\Delta_C^*\,\frac{\partial \beta_T}{\partial T}\right\} - Z\left[b_T\,\frac{\partial \beta_T}{\partial T}+\beta_T\,\frac{\partial b_T}{\partial T}\right]$$

$$
\begin{aligned}
&= b_T \, \frac{\partial \beta_T}{\partial T} \left\{ \beta_T \left[\, \overline{k} - \underline{k} \, \right] \left[\, \overline{C} - C_0^* \, \right] - \beta_T \, \underline{k} \, \Delta_C^* - Z \, \right\} \\
&\qquad - \beta_T \, \frac{\partial b_T}{\partial T} \left\{ b_T \, \beta_\infty \left[\Delta_C^* \right]^2 + Z \right\} - \beta_T \left[b_T \right]^2 \left[\Delta_C^* \right]^2 \, \frac{\partial \beta_\infty}{\partial T} \\
&= b_T \, \frac{\partial \beta_T}{\partial T} \left\{ b_T \, \beta_\infty \left[\Delta_C^* \right]^2 \right\} - \beta_T \left[b_T \right]^2 \left[\Delta_C^* \right]^2 \, \frac{\partial \beta_\infty}{\partial T} \\
&\qquad - \beta_T \, \frac{\partial b_T}{\partial T} \left\{ \beta_T \left[\, \overline{k} - \underline{k} \, \right] \left[\, \overline{C} - C_0^* \, \right] - \beta_T \, \underline{k} \, \Delta_C^* \right\} \\
&= \left[b_T \right]^2 \beta_\infty \left[\Delta_C^* \right]^2 \, \frac{\partial \beta_T}{\partial T} - \beta_T \left[b_T \right]^2 \left[\Delta_C^* \right]^2 \, \frac{\partial \beta_\infty}{\partial T} \\
&\qquad - \left[\beta_T \right]^2 \, \frac{\partial b_T}{\partial T} \left\{ \left[\, \overline{k} - \underline{k} \, \right] \left[\, \overline{C} - C_0^* \, \right] - \underline{k} \, \Delta_C^* \right\} \\
&= \left[b_T \right]^2 \left[\Delta_C^* \right]^2 \left[\beta_\infty \, \frac{\partial \beta_T}{\partial T} - \beta_T \, \frac{\partial \beta_\infty}{\partial T} \right] \\
&\qquad - \left[\beta_T \right]^2 \, \frac{\partial b_T}{\partial T} \left\{ \, \overline{k} \left[\, \overline{C} - C_0^* \, \right] - \underline{k} \left[\, \overline{C} - C_0^* + C_0^* - C_1^* \, \right] \right\} \\
&= \left[b_T \right]^2 \left[\Delta_C^* \right]^2 \left[\beta_\infty \left(- \frac{\beta^T}{1 - \beta} \right) \ln \beta \, - \beta_T \left(\frac{\beta^T}{1 - \beta} \right) \ln \beta \right] \\
&\qquad + \left[\beta_T \right]^2 \left[\frac{b^T}{1 - b} \right] \ln \beta \, \left\{ \, \overline{k} \left[\, \overline{C} - C_0^* \, \right] - \underline{k} \left[\, \overline{C} - C_1^* \, \right] \right\} \\
&= \left[- \ln \beta \right] \left[b_T \right]^2 \left[\Delta_C^* \right]^2 \left[\frac{\beta^T}{1 - \beta} \right] \left[\beta_\infty + \beta_T \right] \\
&\qquad - \left[\beta_T \right]^2 \left[\frac{b^T}{1 - b} \right] \ln \beta \, \left\{ \, \underline{k} \left[\, \overline{C} - C_1^* \, \right] - \overline{k} \left[\, \overline{C} - C_0^* \, \right] \right\} \\
&\overset{s}{=} \frac{\beta^T \left[b_T \right]^2 \left[\Delta_C^* \right]^2}{\left[1 - \beta \right]^2} + \frac{b^T \left[\beta_T \right]^2}{1 - b} \left\{ \, \underline{k} \left[\, \overline{C} - C_1^* \, \right] - \overline{k} \left[\, \overline{C} - C_0^* \, \right] \right\} . \qquad (69)
\end{aligned}
$$

The last " $\overset{s}{=}$ " in (69) reflects the fact that $\ln \beta < 0$ because $\beta \in (0, 1)$. (69) implies that $\frac{\partial s_c^*}{\partial T} > 0$ if $\frac{\overline{C} - C_1^*}{\overline{C} - C_0^*} > \frac{\overline{k}}{\underline{k}}$. ■

B. <u>Additional Characterization of the Numerical Solutions</u>.

Three additional characterizations of the numerical solutions follow. First, Figures B1 – B7 illustrate how s_r^* changes as model parameters change. Second, settings in which $s_r^* < 1$ and the manager is induced to always implement the new technology are considered. Third, Tables B1 and B2 explain how outcomes change as industry parameters change when $f(k)$ is the truncated normal density.

Figures B1 – B7 illustrate how s_r^* changes as model parameters vary from their levels in the baseline setting. In each figure, the parameter that varies from its value in the baseline setting is identified on the horizontal axis. All other parameter values are held constant at the values specified in Table 1. In each figure, the relevant variation in s_r^* is depicted by: (i) the black line when $f(k)$ is the uniform density; and (ii) the red line when $f(k)$ is the truncated normal density. Dotted lines appear in regions where $s_r^* = 1 - \underline{s}_c$, so the manager is induced to implement the new technology for all realizations of $K \in \left[\,\underline{K}, \overline{K}\,\right]$.

[**Figures B1 – B7 Here**]

Now consider settings in which the regulator optimally induces the manager to implement the new technology for all realizations of $K \in \left[\,\underline{K}, \overline{K}\,\right]$ when $f(k)$ is the uniform density. In such settings, consider separately for each model parameter the feasible values of the parameter for which $s_r^* < 1$ (and $s_r^* \neq 0$) when all other parameters are as specified in Table 1. It can be shown that there are no such feasible values of $\overline{C}$, C_0^*, $\underline{K}$, b, β, or T. The relevant feasible values of the other model parameters are $C_1^* \in (0, 77.4)$, $\overline{K} \in (0.1, 0.427)$, and $\delta \in (0.0233, 1\,]$.

The identified values of C_1^* correspond to settings in which the new technology admits at least a 23.6% reduction in the firm's total cost below its historic level $(\overline{C}\,)$. Such substantial cost reductions are conceivable, but would seem to arise with limited frequency in practice.

The identified values of $\overline{K}$ are those that are sufficiently close to $\underline{K}$. When managerial technology implementation costs are always sufficiently low in this sense, the regulator may: (i) optimally induce the manager to always incur these costs; and (ii) do so by awarding the firm during the initial regulatory regime less than the entire cost reduction it achieves.

The identified values of δ are those for which the manager's payoff $(\delta\,\Pi - K\,)$ increases with the firm's profit at a rate above 0.0233. Average CEO compensation in large U.S. utilities was approximately \$9.8 million in 2024 (Sturgis, 2025). Average net income for large U.S. utilities was approximately \$884.47 million in 2024 (CSIMarket.com, 2025). Stock options constitute approximately 70% of executive compensation in S&P 500 companies (Batish, 2024). These statistics suggest that $\widehat{\delta} = 0.00776$ $(\approx 0.70\left[\,\frac{9.8}{884.47}\,\right])$ may be a reasonable estimate of the rate at which utility CEO compensation increases as utility profit increases. $\widehat{\delta}$ is less than one-third of 0.0233.

Finally, Tables B1 and B2 replicate the information in Tables 2A and 2B for the setting where $f(k)$ is the truncated normal density with standard deviation $\sigma = 30$. Tables B1 and

B2 indicate that the primary qualitative conclusions drawn from Tables 2A and 2B generally persist when $f(k)$ is the truncated normal density.

Parameter	s_r^*	$F(\widehat{k}(s_r^*))$	$\frac{F(\widehat{k}(1))}{F(\widehat{k}(s_r^*))}$	$P(s_r^*)$	M
$\overline{C} = 105$	1.765	0.796	0.402	1,943.0	0.842
$\overline{C} = 100$	1.992	0.900	0.356	1,900.6	0.552
$\overline{C} = 99.5$	2.016	0.909	0.352	1,896.1	0.501
$C_0^* = 99.5$	1.992	0.900	0.356	1,900.6	0.552
$C_0^* = 95.0$	1.787	0.807	0.397	1,849.0	0.824
$C_1^* = 95$	2.580	0.462	0.143	1,978.5	0.575
$C_1^* = 90$	1.992	0.900	0.356	1,900.6	0.552
$C_1^* = 80$	1.133	1.000	0.922	1,702.6	0.962
$\underline{K} = 0.05$	1.938	0.892	0.404	1,899.2	0.601
$\underline{K} = 0.1$	1.992	0.900	0.356	1,900.6	0.552
$\underline{K} = 0.2$	2.101	0.921	0.257	1,903.0	0.439
$\overline{K} = 0.5$	1.163	1.000	0.838	1,852.6	0.869
$\overline{K} = 1.0$	1.992	0.900	0.356	1,900.6	0.552
$\overline{K} = 2.0$	2.841	0.716	0.027	1,947.8	0.201

Table B1. Outcomes as Cost Parameters Vary from their Baseline Values when $f(k)$ is the Truncated Normal Density.

Parameter	s_r^*	$F(\widehat{k}(s_r^*))$	$\frac{F(\widehat{k}(1))}{F(\widehat{k}(s_r^*))}$	$P(s_r^*)$	M
$\delta = 0.005$	2.921	0.699	0.016	1,951.6	0.195
$\delta = 0.01$	1.992	0.900	0.356	1,900.6	0.552
$\delta = 0.02$	1.163	1.000	0.860	1,852.6	0.911
$b = 0.90$	2.110	0.865	0.305	1,908.8	0.511
$b = 0.95$	1.992	0.900	0.356	1,900.6	0.552
$b = 0.98$	1.920	0.917	0.391	1,895.8	0.580
$\beta = 0.90$	1.431	0.599	0.534	974.4	0.816
$\beta = 0.95$	1.992	0.900	0.356	1,900.6	0.552
$\beta = 0.98$	2.327	1.000	0.320	4,611.8	0.412
$T = 3$	3.159	0.900	0.140	1,900.6	0.293
$T = 5$	1.992	0.900	0.356	1,900.6	0.552
$T = 7$	1.494	0.900	0.596	1,900.6	0.785
$\sigma = 10$	1.687	0.960	0.120	1,881.0	0.207
$\sigma = 30$	1.992	0.900	0.356	1,900.6	0.552
$\sigma = 50$	2.113	0.916	0.382	1,903.9	0.615

Table B2. Outcomes as Other Parameters Vary from their Baseline Values when $f(k)$ is the Truncated Normal Density.

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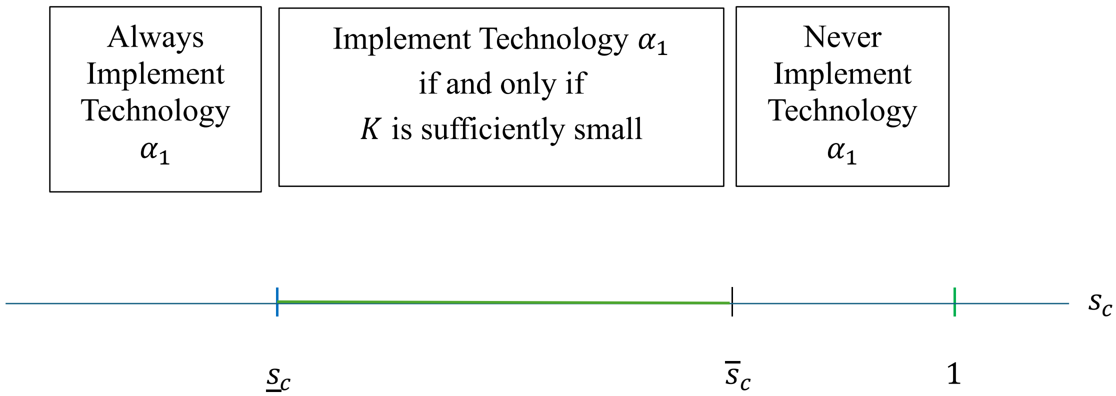

Figure 1. The Technology Implementation Decision.

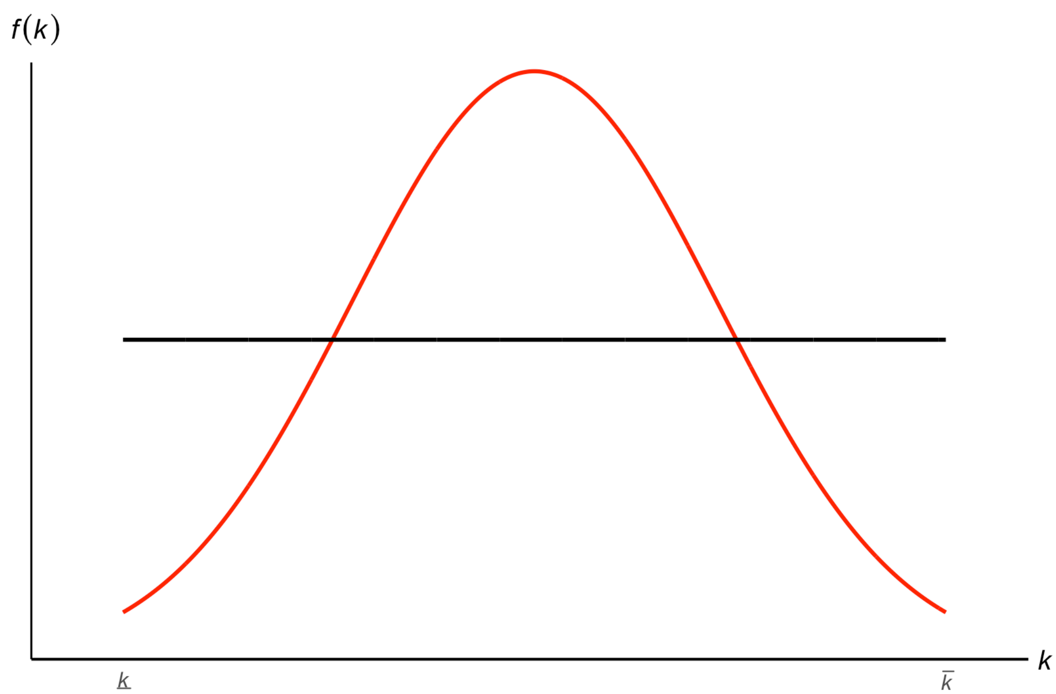

Figure 2. The Uniform (black) and Truncated Normal (red) Densities.

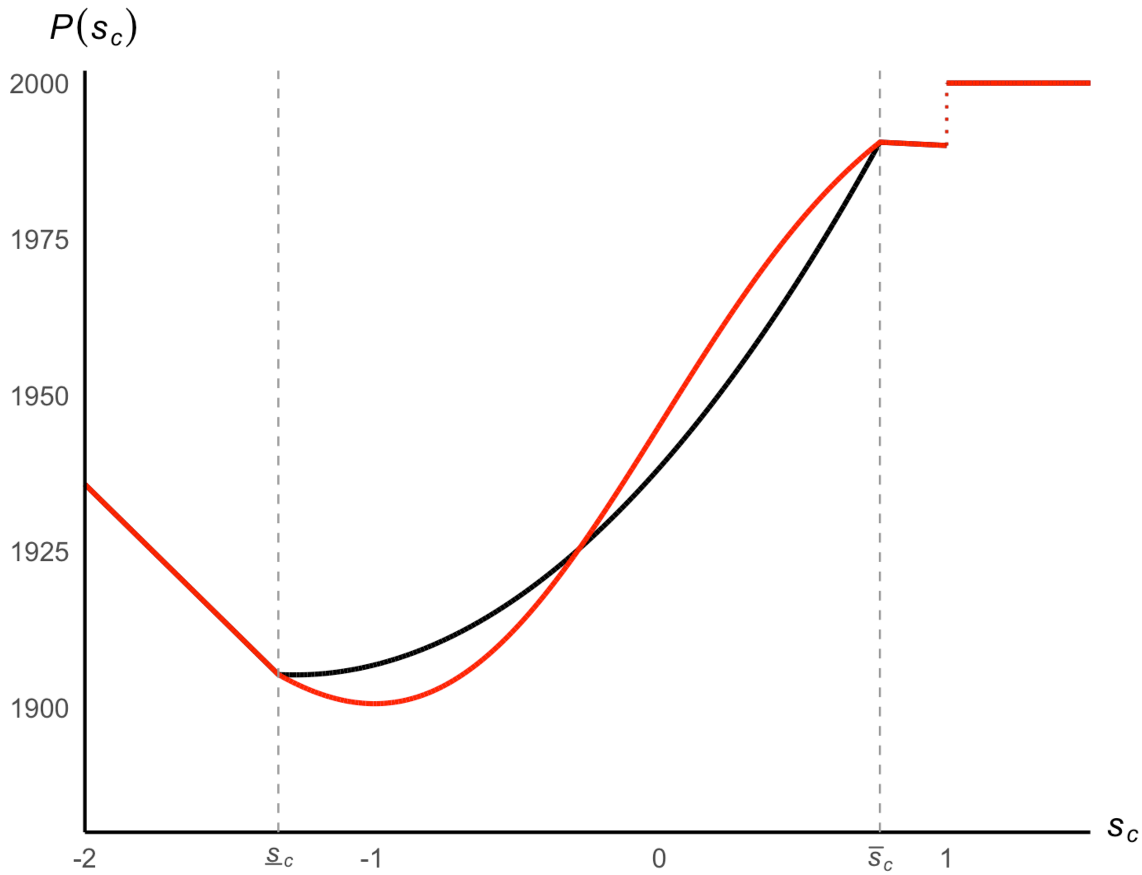

Figure 3. **The PDV of Expected Procurement Cost, $P(s_c)$, for the Uniform (black) and Truncated Normal (red) Densities.**

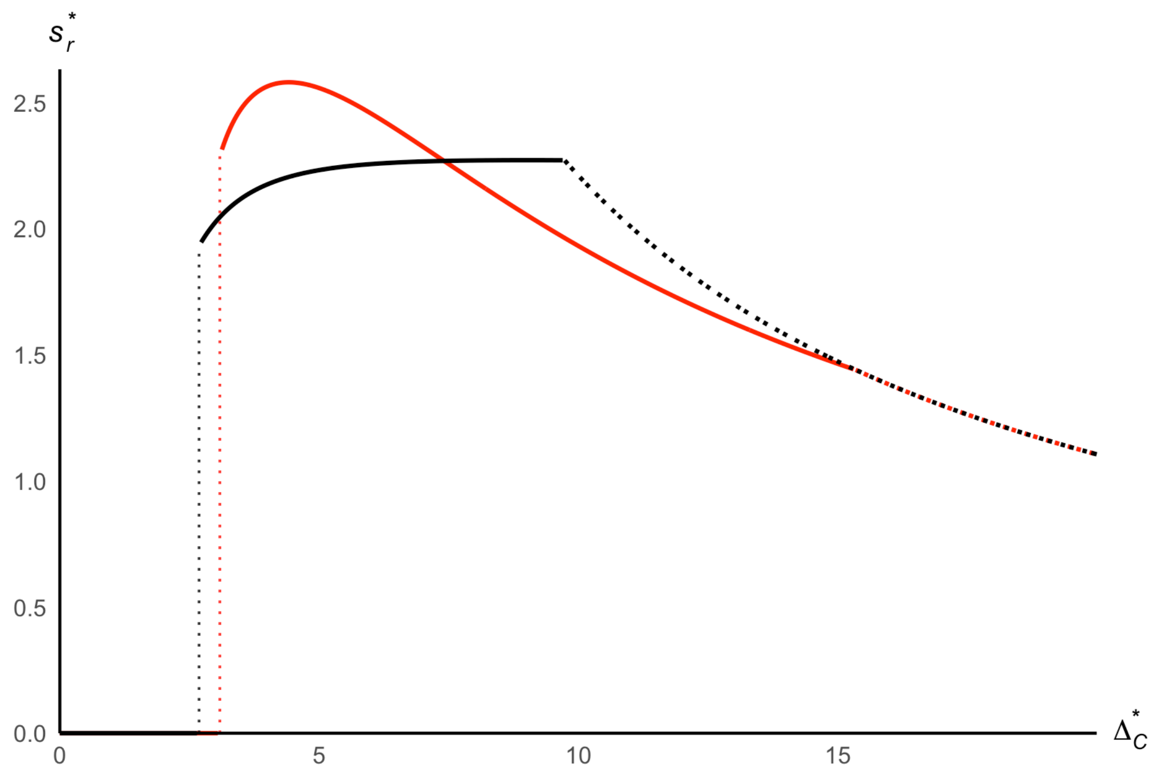

Figure 4. **The Optimal Sharing Rate (s_r^*) as $\Delta_C^* \equiv C_0^* - C_1^*$ Changes for the Uniform (black) and Truncated Normal (red) Densities.**

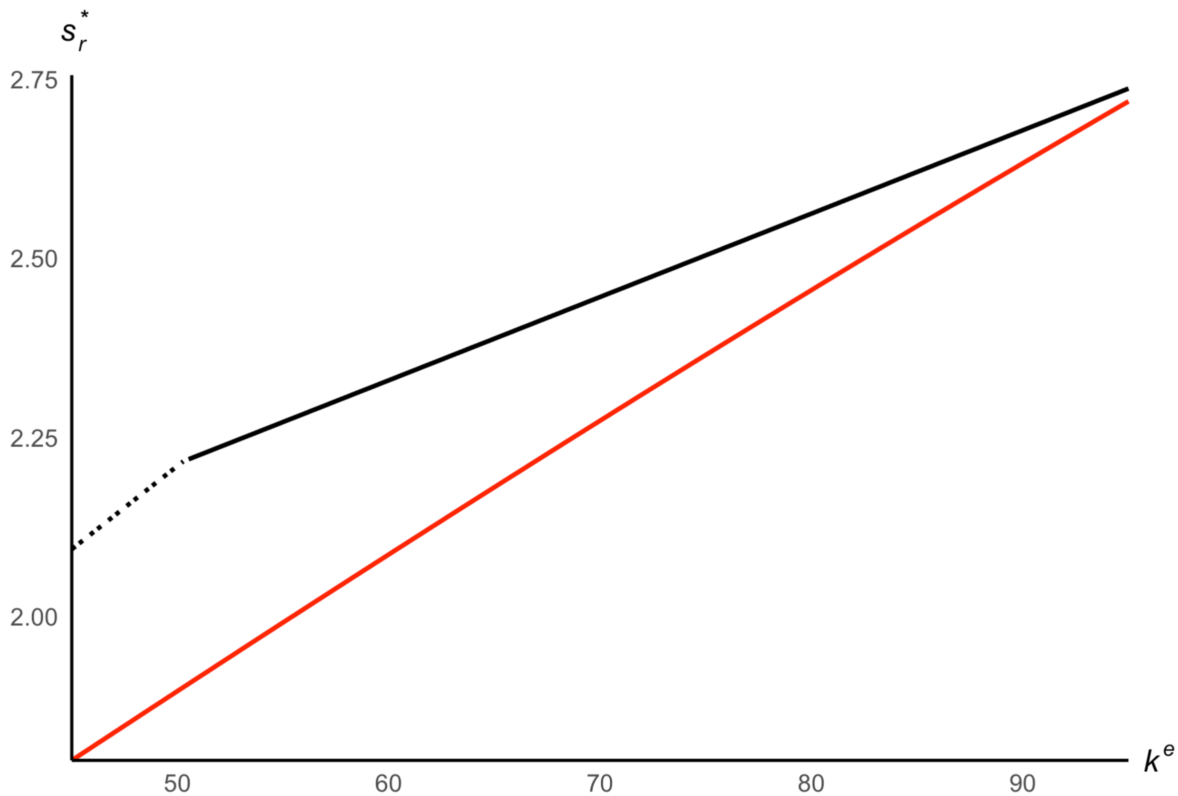

Figure B1. **The Optimal Sharing Rate (s_r^*) as k^e Changes for the Uniform (black) and Truncated Normal (red) Densities.**

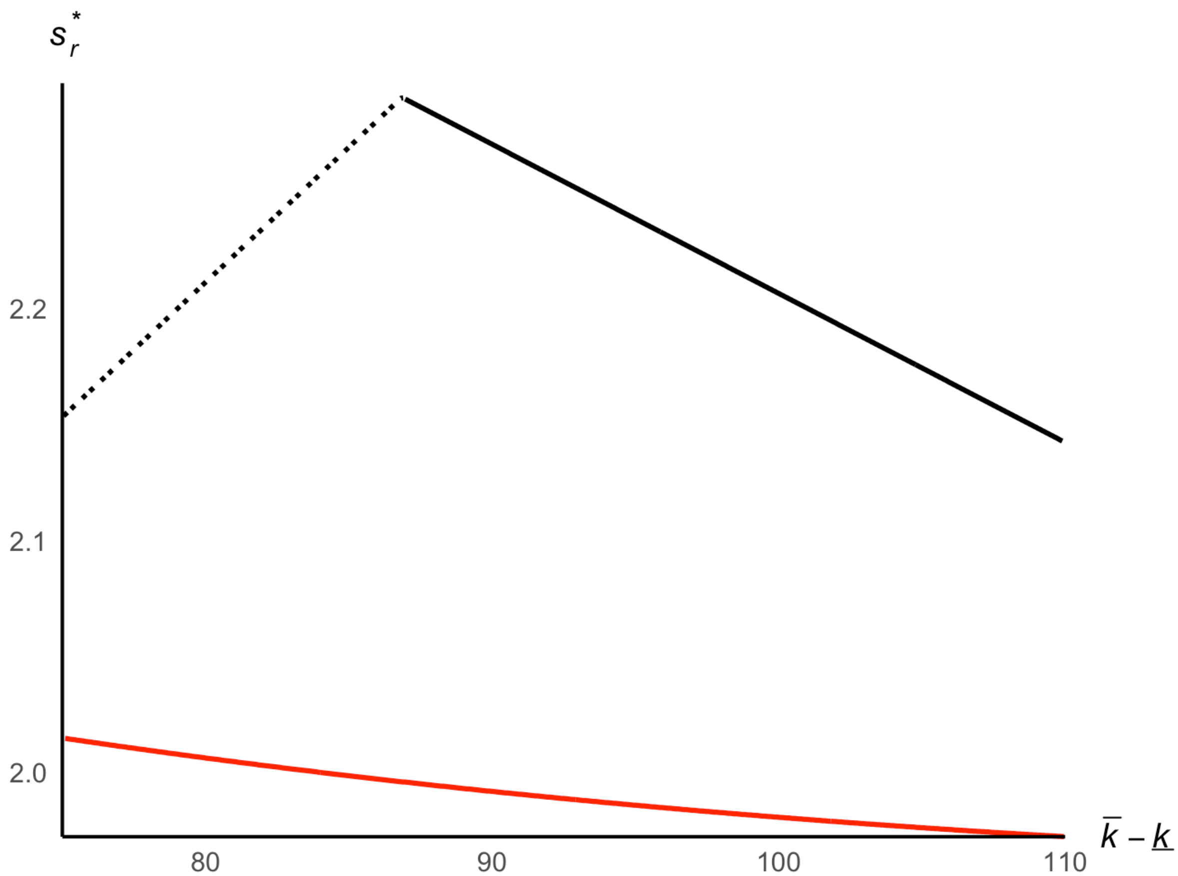

Figure B2. **The Optimal Sharing Rate (s_r^*) as $\overline{k} - \underline{k}$ Changes for the Uniform (black) and Truncated Normal (red) Densities.**

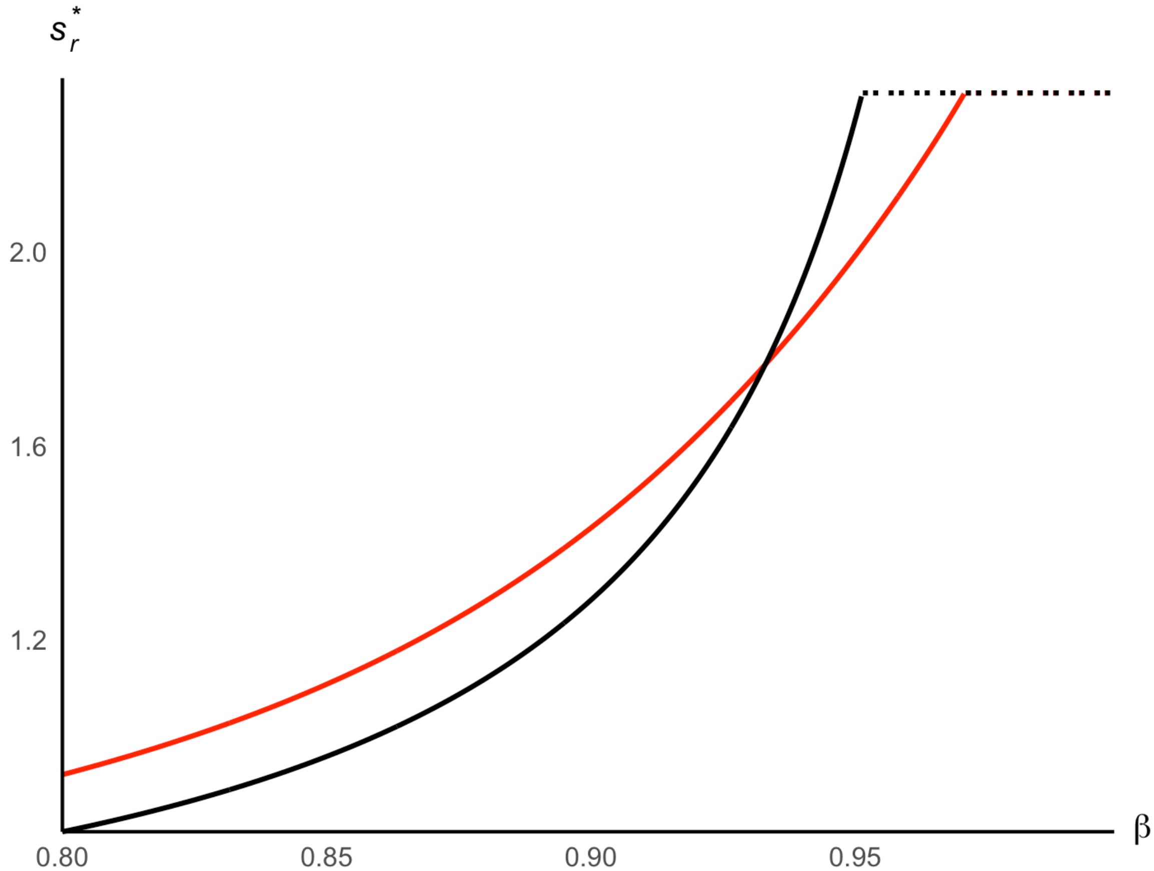

Figure B3. **The Optimal Sharing Rate (s_r^*) as β Changes for the Uniform (black) and Truncated Normal (red) Densities.**

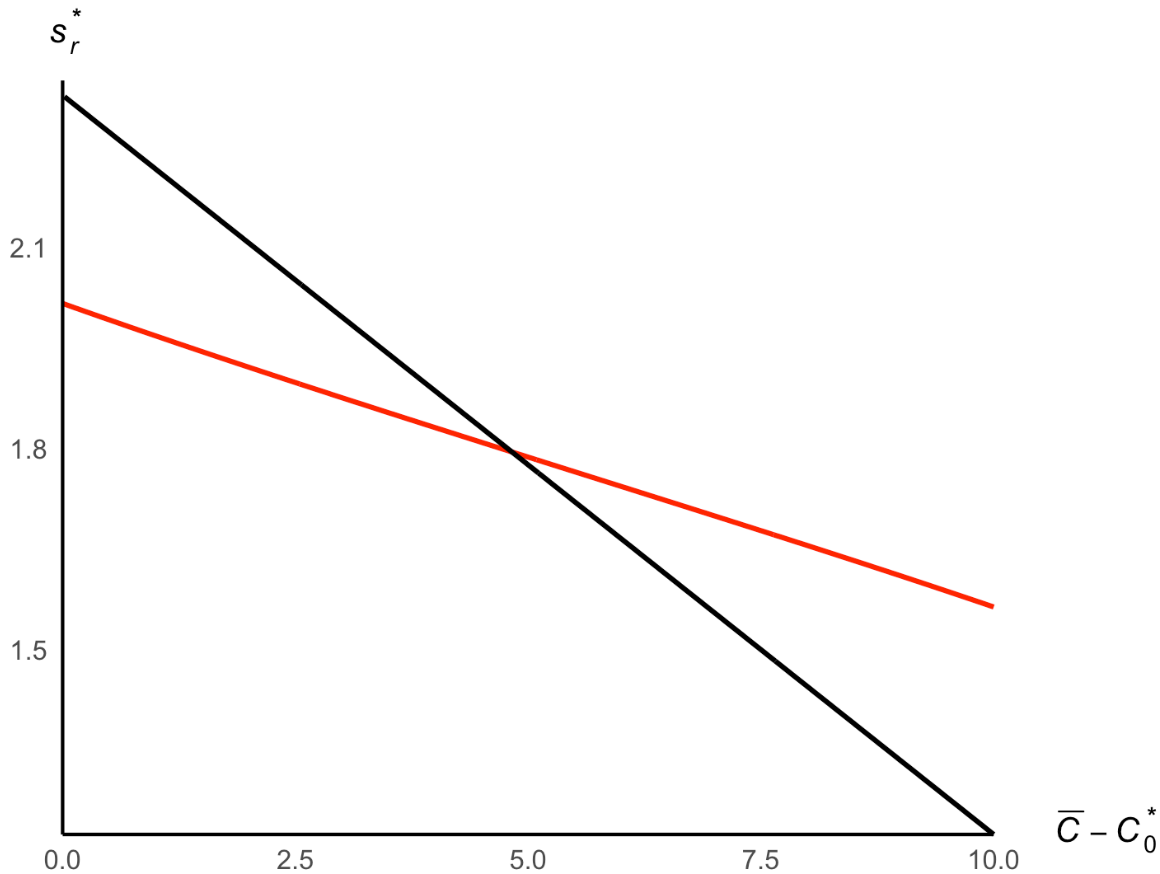

Figure B4. **The Optimal Sharing Rate (s_r^*) as $\overline{C} - C_0^*$ Changes for the Uniform (black) and Truncated Normal (red) Densities.**

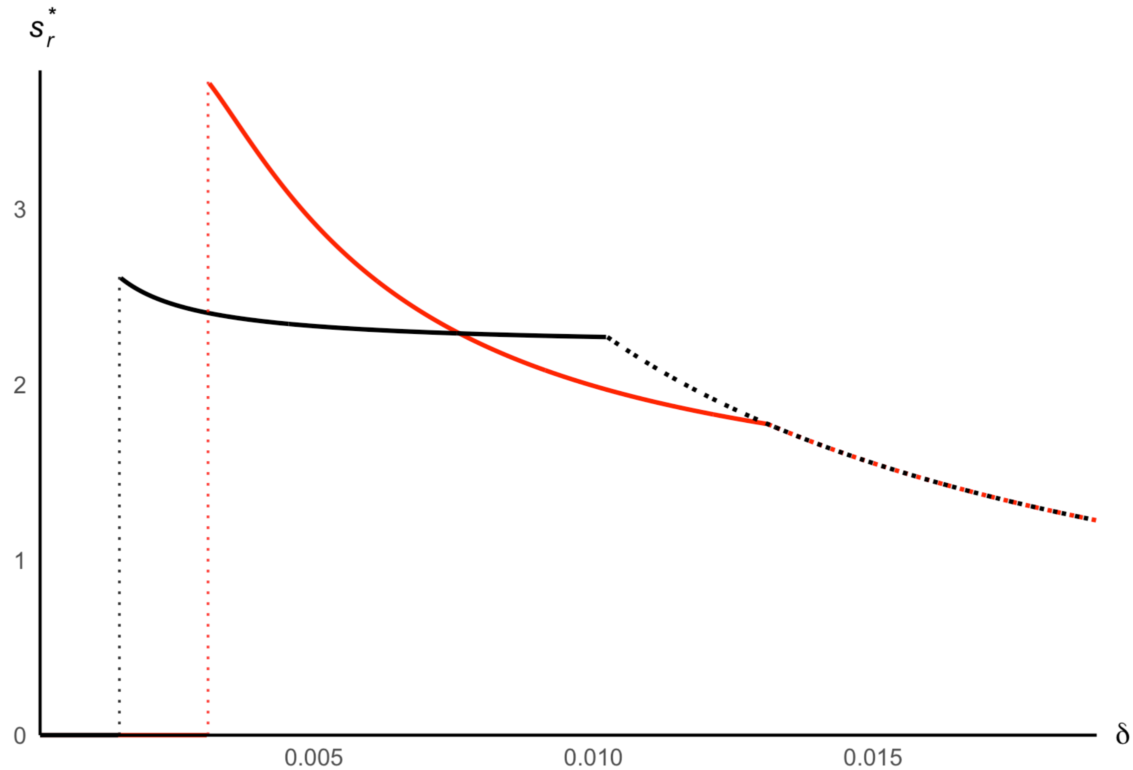

Figure B5. **The Optimal Sharing Rate (s_r^*) as δ Changes for the Uniform (black) and Truncated Normal (red) Densities.**

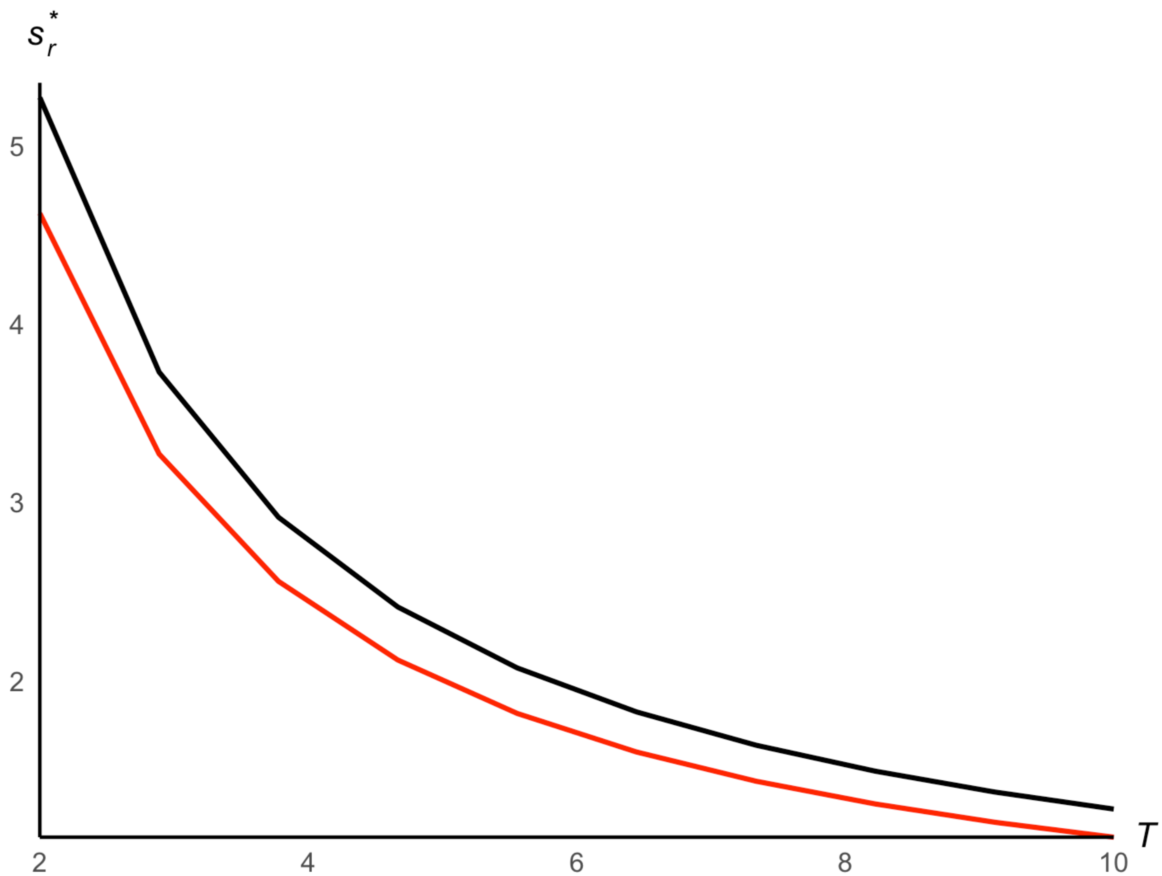

Figure B6. **The Optimal Sharing Rate (s_r^*) as T Changes for the Uniform (black) and Truncated Normal (red) Densities.**

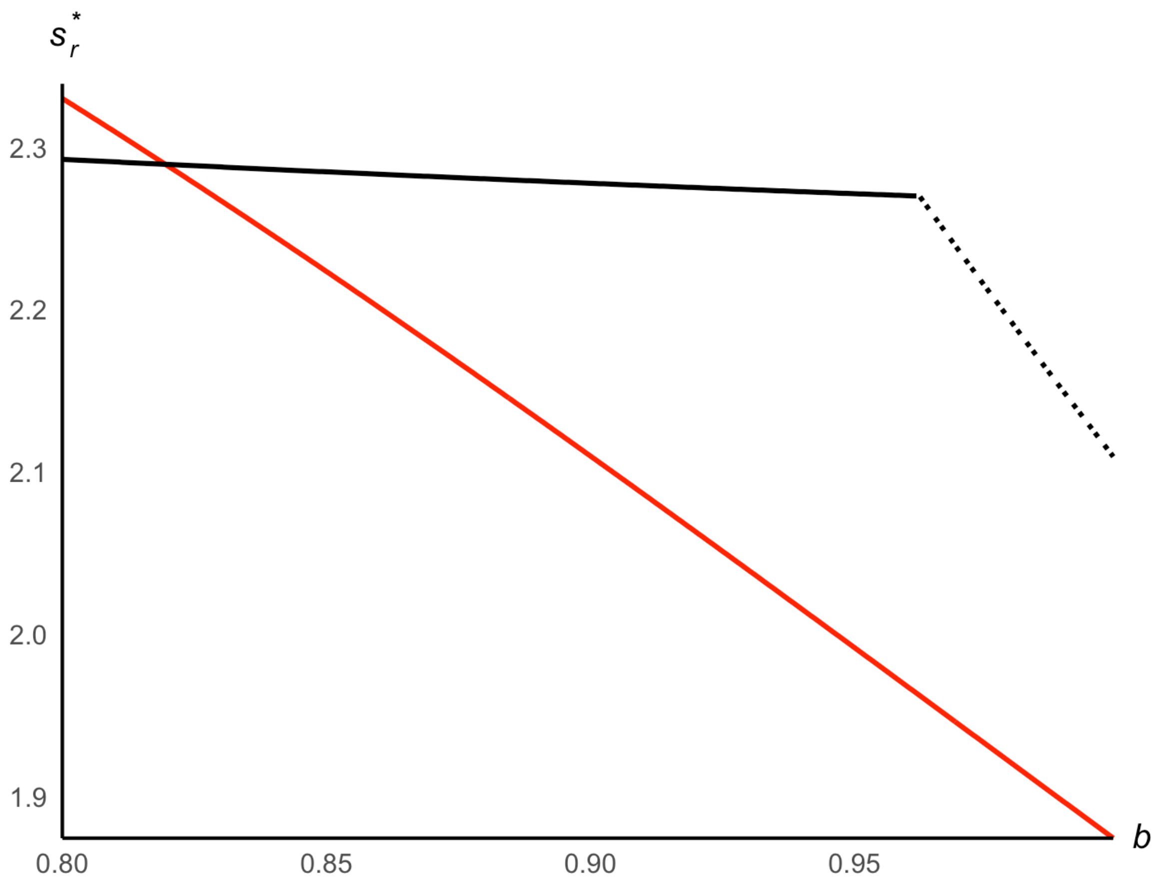

Figure B7. **The Optimal Sharing Rate (s_r^*) as b Changes for the Uniform (black) and Truncated Normal (red) Densities.**