Online Appendix for "Cartel Penalties Under Endogenous Detection"

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1 Leniency and Post-Cartel Detection

Leniency is an antitrust program where a cartel member is given immunity or partial immunity from antitrust fines if they report the cartel to the antitrust authority. We consider two types of leniency, denoted Type A and Type B leniency. Type A leniency is when a cartel member reports the existence of a cartel before an investigation has begun. Type B leniency is when a cartel member reports the existence of a cartel after an investigation has begun.[1]

In the main text, we assume cartels cannot be detected during defection (i.e., in the same period that a firm undercuts the cartel price) or after defection. In practice, cartels can be uncovered, perhaps through leniency programs, after the cartel breaks down or while a firm is defecting. To capture this possibility, we assume the cartel is detected (and fined) with constant probability $\alpha_0^{post} > 0$ during a period when a cartel member defects and for $T \in \{0, 1, 2, \dots\}$ periods after the cartel breaks down.[2]

In this section, we extend the model of the main text to include Type A leniency, Type B leniency and the possibility of post-cartel detection. First, we discuss the timing of each period of the game. When a cartel is active, each period consists of four phases:[3]

1. **Pricing and Type A Leniency** Firms set prices and simultaneously decide to report the cartel (under the Type A leniency program) or not report the cartel.[4] If multiple firms report, full amnesty is

[1] See https://www.justice.gov/atr/page/file/926521/download for additional details regarding the US Department of Justice's leniency Program. See https://ec.europa.eu/competition-policy/cartels/leniency_en for information related to the European Commissions leniency program.

[2] Chen and Rey (2013), Spagnolo (2004), Harrington and Wei (2017), and Aubert, Rey, and Kovacic (2006) assume a cartel can be detected (and fined) in a period when a cartel member defects. T might be determined by the statute of limitations for cartel offenses in a particular jurisdiction.

[3] The timing structure closely follows Chen and Rey (2013).

[4] Choi and Gerlach (2012), Spagnolo (2004), and Chen and Rey (2013) make a similar timing assumption. This assumption is also supported by experimental evidence (Bigoni et al. 2012).

awarded randomly among reporting firms.[5] Each cartel member, excluding the firm receiving amnesty, is penalized with probability 1 if any firm reported the cartel.

2. **Investigation Begins** If no firm applied for Type A leniency, the antitrust authority opens an investigation with probability $\phi(p_t) = \min\left\{\alpha_0 + \alpha_1 [p_t - c]^2, 1\right\}$ (i.e., according to the level specification in the main text).[6] High cartel prices are likely to raise suspicions of collusion and initiate an investigation.[7] Alternatively, $\phi(p_t)$ represents the probability that the AA receives a "lead" which launches an investigation. The beginning of an investigation is observed by cartel members.[8]

3. **Type B Leniency** If an investigation is opened in phase 2, cartel members can apply for Type B leniency. If multiple firms apply for Type B leniency, leniency is awarded randomly among reporting firms. If any firm applies for Type B leniency, the cartel is detected, prosecuted and penalized with probability 1. The cartel member receiving Type B leniency pays a reduced penalty of $\omega x_i(p)$ where $\omega \in [0, 1)$, $i \in \{R, O\}$ and p is the cartel price. $\omega = 0$ corresponds to full amnesty and $\omega > 0$ corresponds to partial amnesty. All other firms are penalized as described in the main text.

4. **Investigation Concludes** If no firm applied for Type B leniency, the investigation leads to cartel detection with probability $\beta \in (0, 1)$.[9] Detected cartels are penalized with probability 1. If the investigation does not lead to cartel detection, the investigation is closed but may begin again in the next period.

Next, we discuss the timing of the game after the cartel breaks down (i.e., a cartel member defects). Each period consists of two phases:

1. **Pricing and Type A Leniency** In the first phase, firms set prices. Additionally, firms can report previous cartel activity in exchange for full amnesty from fines. If any firm reported the cartel, then each cartel member, excluding the firm receiving amnesty, is penalized with probability 1.

2. **Detection and Penalization** The second phase occurs only if the cartel was active within the last T periods and has not previously been detected or reported. The cartel's previous illegal activity is

[5] Chen and Rey (2013), Harrington (2008), and Chen and Harrington (2007) also make this assumption.

[6] In this section, we restrict attention to the level specification for ease of exposition. We conjecture that results also hold under the changes specification as the main effects of leniency and post-cartel detection persist.

[7] See Emons (2020). Motta and Polo (2003) and Chen and Rey (2013) also assume an investigation is opened, with a certain probability, after firms set prices.

[8] Chen and Rey (2013) make the same assumption.

[9] The probability of an investigation resulting in detection typically depends on the AA's ability to collect hard evidence (for example, through inspections, raiding offices, questioning whistleblowers, wiretaps etc.) which does not typically depend on the cartel price (Motta and Polo 2003; Chen and Rey 2013). Put differently, "It is the act of communicating to coordinate behavior that is illegal (or taken as evidence of illegality), and not the actual prices that are charged." (Bos et al. 2018).

detected and penalized with probability $\alpha_0^{post} > 0$. If detected, cartel members are penalized as in the main text.[10]

Following Motta and Polo (2003), we consider two potential collusive strategies: one where firms never apply for either type of leniency and one where firms simultaneously apply for Type B leniency if an investigation opens and do not apply for Type A leniency.[11] First, we consider the cartel's problem in the absence of leniency programs for comparison purposes.

1.1 No Leniency

If no leniency program exists, then the model is similar to the main text except for two differences. First, there is a probability of post-cartel detection. Second, investigations are successful with probability β.[12] The cartel's Bellman equation is

$$\begin{aligned} V_i = \max_{p \in [c,1]} & \; \pi(p) - \phi(p)\beta x_i(p) + \delta V_i \\ \text{s.t.} \quad & \pi(p) - \phi(p)\beta x_i(p) + \delta V_i \\ & \geq \pi_D(p) - \alpha_0^{post} x_i(p) \cdots - \delta^T \alpha_0^{post} x_i(p) \end{aligned} \tag{1}$$

where V_i denotes the expected present discounted value of the payoff from collusion when the penalty type is $i \in \{R, O\}$. Note that the possibility of post-cartel detection reduces the expected discounted present value of the payoff from defection which increases the sustainability of collusion.

1.2 Collusion with Reporting after an Investigation Opens

Suppose cartels collude by applying for Type B leniency if an investigation opens (i.e., all firms apply simultaneously) and not applying for Type A leniency. Formally, cartels set a collusive price p in every period unless any firm has 1) charged a price other than p in any prior period, 2) applied for Type A leniency in any prior period or 3) not applied for Type B leniency after an investigation opened in any prior period.[13] If any firm defects (by charging a price other than the agreed cartel price in phase 1, by applying for Type A leniency in phase 1 or not applying for Type B leniency in phase 3), then all firms charge Nash equilibrium prices in all future periods (i.e., grim trigger strategies).[14]

[10] Cartel penalties are based on the cartel price in the last period of collusion, as in the main text.

[11] Motta and Polo (2003) refer to these two strategies as "Collude and Not Reveal" and "Collude and Reveal". Chen and Rey (2013) also consider these two strategies which they denote "Normal Collusion" and "Collude and Report in Case of Investigation". Emons (2020) also considers these two collusive strategies.

[12] This probability was subsumed into $\phi(p)$ in the main text.

[13] As we will show, no firm will have an incentive to deviate by not applying for Type B leniency when its rivals do.

[14] Formally, the cartel's punishment strategy could involve leniency applications. However, in practice, it is unnecessary to specify when and if the cartel applies for leniency after a firm defects because a defecting firm will always simultaneously apply

Collusion is sustainable if firms do not wish to defect from this strategy in either phase 1 or phase 3. First, consider phase 1. When defecting, a firm always wishes to also apply for Type A leniency to avoid the possibility of post-cartel detection. Thus, firms earn a discounted present value of $\pi_D(p)$ (where p is the cartel price) when defecting, as in the main text. Firms do not defect in phase 1 if the expected present discounted value of collusion is greater than or equal to $\pi_D(p)$. Next, consider phase 3. If all N firms apply for Type B leniency, the expected penalty is $\frac{N-1}{N}x_i(p) + \frac{1}{N}\omega x_i(p)$. If a firm instead does not apply for Type B leniency while its rivals do, its expected penalty is $x_i(p)$. Thus, a firm would never wish to deviate in phase 3 and the constraint in phase 1 determines the sustainability of collusion.

The cartel price satisfies the Bellman equation

$$\begin{aligned} V_i = \max_{p \in [c,1]} \; & \pi(p) - \phi(p)\left(\frac{N-1}{N} + \frac{\omega}{N}\right) x_i(p) + \delta V_i \\ \text{s.t.} \quad & \pi(p) - \phi(p)\left(\frac{N-1}{N} + \frac{\omega}{N}\right) x_i(p) + \delta V_i \\ & \geq \pi_D(p) \end{aligned} \tag{2}$$

where V_i denotes the expected present discounted value of the payoff from collusion when the penalty type is $i \in \{R, O\}$. A cartel forms if collusion is sustainable (i.e., there exists a price that satisfies the constraint in Equation (2)) and collusion is profitable (i.e., $V_i > 0$).

The cartel's problem in Equation (2) is very similar to that of the main text. The penalty multiplier in the main text, γ_i, is simply replaced by $\left(\frac{N-1}{N} + \frac{\omega}{N}\right)\gamma_i$. Thus, the primary results of the main text hold if γ_i is replaced with $\gamma_i\left(\frac{N-1}{N} + \frac{\omega}{N}\right)$ for $i \in \{O, R\}$.

1.3 Collusion with No Reporting after an Investigation Opens

Suppose that cartels collude by never applying for either type of leniency. Formally, cartels set a collusive price p in every period unless any firm has 1) charged a price other than p in any prior period, 2) applied for Type A leniency in any prior period or 3) applied for Type B leniency after an investigation opened in any prior period. If any firm defects (by charging a price other than the agreed cartel price in phase 1, by applying for Type A leniency in phase 1 or applying for Type B leniency in phase 3), then all firms charge Nash equilibrium prices in all future periods (i.e., grim trigger strategies).

Collusion is sustainable if firms do not wish to defect in phase 1 or phase 3. First, consider phase 1. When defecting, a firm always wishes to simultaneously apply for Type A leniency to avoid the possibility of post-cartel detection. Thus, firms earn a discounted present value of $\pi_D(p)$ when defecting as in the main

for Type A leniency in phase 1. Thus, the cartel cannot use leniency applications to punish defecting firms because the cartel will already be detected after any firm has defected.

text. Next, consider phase 3. A cartel member defects in phase 3 by applying for Type B leniency when other cartel members do not. The discounted present value from this strategy, from phase 3 onwards, is $-\omega x_i(p)$. The discounted present value from following the collusive strategy and not applying for Type B leniency is $-\beta x_i(p) + \delta V_i$. Thus, cartels do not deviate if $-\beta x_i(p) + \delta V_i \geq -\omega x_i(p)$ or $\delta V_i \geq (\beta - \omega) x_i(p)$.

Cartel prices satisfy the Bellman equation

$$\begin{aligned} V_i = & \max_{p \in [c,1]} \pi(p) - \phi(p)\beta x_i(p) + \delta V_i \\ \text{s.t.} \quad & \pi(p) - \phi(p)\beta x_i(p) + \delta V_i \\ & \geq \pi_D(p) \\ \text{and } & \delta V_i \geq (\beta - \omega) x_i(p). \end{aligned} \tag{3}$$

The constraint $\pi(p) - \phi(p)\beta x_i(p) + \delta V_i \geq \pi_D(p)$ (hereafter, the phase 1 constraint) ensures that no firm wishes to defect in phase 1 (by undercutting the cartel price). The constraint $\delta V_i \geq (\beta - \omega)x_i(p)$ (hereafter, the phase 3 constraint) ensures no cartel member wishes to defect in phase 3 (by applying for Type B leniency after an investigation begins). Intuitively, collusion must be sufficiently profitable that the possibility of reduced fines from Type B leniency does not tempt firms to deviate from the agreement and apply for amnesty.

The Bellman Equation in (3) differs from the main text in two ways. First, there is an additional constraint to ensure no firm applies for Type B leniency in phase 3. Second, investigations are successful (absent leniency applications) with probability β. This has an effect of reducing the penalty multiplier multiplicatively from γ_i to $\beta\gamma_i$ for $i \in \{O, R\}$.

A cartel forms if there exists a price that satisfies the constraints in Equation (3) and ensures collusion is profitable (i.e., $V_i > 0$). There are two cases to consider: $\omega \geq \beta$ and $\omega < \beta$.

1.3.1 Case 1: $\omega \geq \beta$

Suppose $\omega \geq \beta$. If collusion is profitable, then the phase 3 constraint is immediately satisfied: $V_i > 0 \implies \delta V_i > 0 \geq (\beta - \omega)x_i(p)$. Thus, the phase 3 constraint does not bind and the cartel's problem in (3) is equivalent to the cartel's problem in the main text with the simple modification that γ_i is replaced with $\gamma_i\beta$ for $i \in \{O, R\}$. Thus, the cartel's problem is essentially unchanged and the results of the main text hold.

1.3.2 Case 2: $\omega < \beta$

Suppose $\omega < \beta$. In this subsection, we examine cartel formation under both penalty types when $\omega < \beta$. We show that the cartel formation results of the main text hold (with slight modifications to parameters and critical discount factors) which implies that the surplus results (i.e., the optimality of revenue-based penalties when α_1 is sufficiently high) also hold.

Before presenting results, two assumptions from the main text must be modified to account for the probability of successful investigation β. Specifically, we now assume $\alpha_0\beta\gamma_O < 1$, $\alpha_0\beta\gamma_R < 1 - c$, $\gamma_O\beta > 1$ and $\gamma_R\beta > 1$.[15] Under these assumptions, $p_O \in \left(c, c + \sqrt{\frac{1-\alpha_0}{\alpha_1}}\right)$ and $p_R \in \left(c, c + \sqrt{\frac{1-\alpha_0}{\alpha_1}}\right)$ if a cartel forms. The proof follows immediately from the steps in the proof of Lemma 1 in Appendix A.

First, we consider overcharge-based penalties.

Theorem. $\delta_O = \max\left\{\frac{(\beta-\omega)\gamma_O}{1-\gamma_O\alpha_0\beta+(\beta-\omega)\gamma_O}, \frac{N-1}{N} + \frac{\alpha_0\beta\gamma_O}{N}\right\}$

Proof. We show that a cartel forms when $\delta > \delta_O$ and does not form when $\delta \leq \delta_O$. A cartel forms if collusion is sustainable and profitable. For collusion to be sustainable, collusion must be incentive compatible (i.e., the two constraints in equation (3) must hold). Note that the cartel price p satisfies both $c < p < c + \sqrt{\frac{1-\alpha_0}{\alpha_1}}$ and $p \leq p^m$.[16] The phase 1 constraint is satisfied at a price $p = c + \epsilon$ where $p \leq p^m$ and $p \in \left(c, c + \sqrt{\frac{1-\alpha_0}{\alpha_1}}\right)$ if

$$\begin{aligned}
N\pi(p) &\leq \frac{1}{1-\delta}\pi(p) - \frac{\alpha_0\beta x_O(p)}{1-\delta} - \frac{\alpha_1\beta(p-c)^2 x_O(p)}{1-\delta} \\
1-\delta &\leq \frac{1}{N} - \frac{\alpha_0\beta x_O(p)}{N\pi(p)} - \frac{\alpha_1\beta(p-c)^2 x_O(p)}{N\pi(p)} \\
\delta &\geq \frac{N-1}{N} + \frac{\alpha_0\beta x_O(p)}{N\pi(p)} + \frac{\alpha_1\beta(p-c)^2 x_O(p)}{N\pi(p)} \\
\delta &\geq \frac{N-1}{N} + \frac{\alpha_0\beta\gamma_O Q_N\epsilon}{N\epsilon D(c+\epsilon)} + \frac{\alpha_1\beta\epsilon^2\gamma_O Q_N\epsilon}{N\epsilon D(c+\epsilon)} \\
\delta &\geq \frac{N-1}{N} + \frac{\alpha_0\beta\gamma_O Q_N}{ND(c+\epsilon)} + \frac{\alpha_1\beta\epsilon^2\gamma_O Q_N}{ND(c+\epsilon)} \\
\delta &\geq \frac{N-1}{N} + \frac{\alpha_0\beta\gamma_O\left(1-c-\epsilon+\epsilon\right)}{ND(c+\epsilon)} + \frac{\alpha_1\beta\epsilon^2\gamma_O Q_N}{ND(c+\epsilon)} \\
\delta &\geq \frac{N-1}{N} + \frac{\alpha_0\beta\gamma_O D(c+\epsilon)}{ND(c+\epsilon)} + \epsilon\frac{\alpha_0\beta\gamma_O}{ND(c+\epsilon)} + \frac{\alpha_1\beta\epsilon^2\gamma_O Q_N}{ND(c+\epsilon)} \\
\delta &\geq \frac{N-1}{N} + \frac{\alpha_0\beta\gamma_O}{N} + \epsilon\frac{\alpha_0\beta\gamma_O}{ND(c+\epsilon)} + \epsilon^2\frac{\alpha_1\beta\gamma_O Q_N}{ND(c+\epsilon)}.
\end{aligned} \tag{4}$$

[15] These assumptions serve the same purpose as in the main text. $\alpha_0\beta\gamma_O < 1$ and $\alpha_0\beta\gamma_R < 1 - c$ ensure that a cartel forms, for some parameter values. $\gamma_O\beta > 1$ and $\gamma_R\beta > 1$ ensure that penalties are large enough that cartels do not choose a price that would cause certain investigation/detection.

[16] The optimal price under overcharge-based penalties does not exceed the monopoly price. To see this, suppose the optimal cartel price is $p > p^m$. The cartel could increase profit and reduce the expected penalty by reducing the cartel price to p^m. Additionally, the payoff from defection is unchanged. Thus, $p > p^m$ is not optimal.

If $\delta > \frac{N-1}{N} + \frac{\alpha_0 \beta \gamma_O}{N}$, then there exists a sufficiently small $\epsilon > 0$ such that inequality (4) holds and the phase 1 constraint is satisfied. The phase 3 constraint is satisfied at a price $p = c + \epsilon$ if

$$\delta\left[\frac{1}{1-\delta}\pi(p) - \frac{\alpha_0 \beta x_O(p)}{1-\delta} - \frac{\alpha_1 (p-c)^2 \beta x_O(p)}{1-\delta}\right] \geq (\beta - \omega) x_O(p)$$

$$\frac{\delta}{1-\delta}\left[\frac{(1-p)(p-c)}{(p-c)(1-c)} - \alpha_0 \beta \gamma_O - \alpha_1 (p-c)^2 \beta \gamma_O\right] \geq (\beta - \omega)\, \gamma_O$$

$$\frac{\delta}{1-\delta}\left[\frac{1-c-\epsilon}{1-c} - \alpha_0 \beta \gamma_O - \alpha_1 \beta \epsilon^2 \gamma_O\right] \geq (\beta - \omega)\, \gamma_O$$

$$\frac{\delta}{1-\delta} \geq \frac{(\beta-\omega)\,\gamma_O}{\frac{1-c-\epsilon}{1-c} - \alpha_0\beta\gamma_O - \alpha_1 \epsilon^2 \beta \gamma_O}.$$

$$\delta \geq \frac{(\beta-\omega)\,\gamma_O}{\frac{1-c-\epsilon}{1-c} - \alpha_0\beta\gamma_O - \alpha_1 \epsilon^2 \beta \gamma_O + (\beta-\omega)\,\gamma_O} \tag{5}$$

If $\delta > \frac{(\beta-\omega)\gamma_O}{1-\gamma_O \alpha_0 \beta + (\beta-\omega)\gamma_O}$, then there exists a sufficiently small $\epsilon > 0$ such that inequality (5) holds and the phase 3 constraint is satisfied. Thus, both constraints are satisfied at a sufficiently low price. If the phase 3 constraint is satisfied for some $p > c$, then collusion is profitable: $\delta V_O \geq (\beta - \omega) x_O(p) > 0 \implies V_O > 0$.

Therefore, collusion is profitable and sustainable if $\delta > \delta_O = \max\left\{\frac{(\beta-\omega)\gamma_O}{1-\gamma_O\alpha_0\beta+(\beta-\omega)\gamma_O}, \frac{N-1}{N} + \frac{\alpha_0\beta\gamma_O}{N}\right\}$. Note that $\beta\gamma_O\alpha_0 < 1$ implies $\delta_O < 1$. If $\delta \leq \delta_O$, collusion is unsustainable for all $\epsilon \in \left(0, \sqrt{\frac{1-\alpha_0}{\alpha_1}}\right)$ and unprofitable for $\epsilon = 0$ and $\epsilon \geq \sqrt{\frac{1-\alpha_0}{\alpha_1}}$. Thus, a cartel does not form. □

Next, consider revenue-based penalties.

Theorem. $\delta_R \to 1$ *as* $\alpha_1 \to \infty$

Proof. A cartel forms if collusion is sustainable (i.e., both phase 1 and phase 3 constraints are satisfied) and profitable. We show collusion is unprofitable for all $\delta < 1$ if $\alpha_1 > \frac{(1-\alpha_0)(1-\alpha_0\beta\gamma_R)^2}{(\beta\alpha_0\gamma_R c)^2}$. Therefore, a cartel does not form and $\delta_R = 1$ if $\alpha_1 > \frac{(1-\alpha_0)(1-\alpha_0\beta\gamma_R)^2}{(\beta\alpha_0\gamma_R c)^2}$. For collusion to be profitable, $\pi(p) - \alpha_0 \beta x_R(p) > 0$ must hold for some price $p \in \left(c, c+\frac{\sqrt{1-\alpha_0}}{\sqrt{\alpha_1}}\right)$.[17] Note that

$$
\begin{aligned}
\pi(p) - \alpha_0 \beta x_R(p) &= D(p)(p-c) - \alpha_0 \beta \gamma_R D(p) p \\
&= D(p)\left[p\left[1 - \alpha_0 \gamma_R \beta\right] - c\right] \\
&\leq D(p)\left[\left[c + \frac{\sqrt{1-\alpha_0}}{\sqrt{\alpha_1}}\right]\left[1 - \alpha_0 \beta \gamma_R\right] - c\right] \\
&= D(p)\left[c - \alpha_0 \beta \gamma_R c + \frac{\sqrt{1-\alpha_0}}{\sqrt{\alpha_1}}\left[1 - \alpha_0 \beta \gamma_R\right] - c\right] \\
&= D(p)\left[-\alpha_0 \beta \gamma_R c + \frac{\sqrt{1-\alpha_0}}{\sqrt{\alpha_1}}\left[1 - \alpha_0 \beta \gamma_R\right]\right] \\
&\leq D(p)\left[-\alpha_0 \beta \gamma_R c + \frac{\alpha_0 \beta \gamma_R c}{1 - \alpha_0 \beta \gamma_R}\left[1 - \alpha_0 \beta \gamma_R\right]\right] \\
&= 0
\end{aligned}
$$

for all $p \in \left(c, c+\frac{\sqrt{1-\alpha_0}}{\sqrt{\alpha_1}}\right)$ where the last inequality follows from $\alpha_1 > \frac{(1-\alpha_0)(1-\alpha_0\beta\gamma_R)^2}{(\alpha_0\beta\gamma_R c)^2}$. Thus, collusion is unprofitable and a cartel does not form (i.e., $\delta_R = 1$) if $\alpha_1 > \frac{(1-\alpha_0)(1-\alpha_0\beta\gamma_R)^2}{(\alpha_0\beta\gamma_R c)^2}$ which implies $\delta_R \to 1$ as $\alpha_1 \to \infty$. □

Results imply that, when α_1 is sufficiently high and $\delta > \delta_O$, a cartel forms under overcharge-based penalties and does not form under revenue-based penalties. Additionally, when α_1 is sufficiently high and $\delta \leq \delta_O$, a cartel does not form under either penalty type. Thus, revenue-based penalties result in a greater level of total and consumer surplus when α_1 is sufficiently large, as in the main text.

1.4 Discussion

In summary, we find that the results of the main text continue to hold for both collusive strategies. Intuitively, the presence of a Type B leniency program increases firms' incentives to report the cartel after an investigation commences. This raises the likelihood of successful cartel detection and prosecution. However, high cartel prices are still likely to cause suspicions of collusion and launch an investigation when α_1 is high. Thus, cartels face incentives to reduce price in order to avoid creating suspicions of collusion and triggering an investigation, as in the main text. When cartels set low prices, revenue-based penalties exceed overcharge-based penalties and, as a result, revenue-based penalties are a more effective penalty when α_1 is high. Thus, the comparison between revenue-based and overcharge-based penalties in the main text continues to hold.

Type A leniency makes collusion harder to sustain because Type A leniency increases the discounted presented value of defection by allowing the cartel to report the cartel and evade post-cartel detection

[17] If $\pi(p) - \alpha_0 \beta x_R(p) \leq 0$ for all $p \in \left(c, c+\frac{\sqrt{1-\alpha_0}}{\sqrt{\alpha_1}}\right)$, then $W_R(p) = \frac{1}{1-\delta}\left[\pi(p) - \phi\beta x_R(p)\right] \leq 0$ for all $p \in \left(c, c+\frac{\sqrt{1-\alpha_0}}{\sqrt{\alpha_1}}\right)$ (by $\phi \geq \alpha_0$) and collusion is unprofitable.

and penalization (Spagnolo (2004) refers to this effect as the “protection from fines effect”). However, this effect does not alter the cartel’s pricing incentives nor does it change the comparison between revenue and overcharge-based penalties.

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2 Value Function Iteration Algorithm

We want to find the policy function of prices, i.e., the best choice of p_t given p_{t-1}, that satisfies the following maximization problem. Denote this policy function as $p_t = h(p_{t-1})$.

$$
\begin{aligned}
V_i(p_{t-1}) = \max_{p\in[c,1]} \pi(p) - \phi(p_{t-1},p)x_i(p) + \delta\left[1-\phi(p_{t-1},p)\right]V_i(p) + \delta\phi(p_{t-1},p)V_i(c) \\
\text{s.t.} \quad \pi(p) - \phi(p_{t-1},p)x_i(p) + \delta\left[1-\phi(p_{t-1},p)\right]V_i(p) + \delta\phi(p_{t-1},p)V_i(c) \\
\geq \pi^D(p)
\end{aligned}
$$

1. Begin with a guess for the value function. Using the constraint, we set the initial guess

$$V^0(p_i) = \pi_D(p_i) \quad \forall i = 1,...,n$$

   where n is the number of points considered for the price grid $p_i \in [0,1]$. For example, if $n = 11$, the price grid will be: $p_i \in \{0, .1, .2, .3, .4, .5, .6, .7, .8, .9, 1\}$. We set $n = 2,000$ in our applications.

2. Given p_i and $V^0(p_i)$ calculate $V^1(p_i)$ $\forall i = 1,...,n$ as follows:

$$V^1(p_i) = \pi(p_j) - \phi(p_i,p_j)x(p_j) + \delta\left[1-\phi(p_i,p_j)\right]V^0(p_j) + \delta\phi(p_i,p_j)V(c) \quad \forall j = 1,...,n$$

   Note that given p_i and p_j, we can calculate $\phi(p_i,p_j) = \alpha_0 + \alpha_1(p_j - p_i)^2$ and the penalty $x(p_j)$.

3. If $V^1(p_i) \geq \pi^D(p_i)$ set $V^1(p_i) = V^1(p_i)$, otherwise set $V^1(p_i) = \pi_D(p_i)$ $\forall i = 1,...,n$.

4. For a given p_i, find the index j^* such that

$$
\begin{aligned}
&\pi(p_{j^*}) - \phi(p_i,p_{j^*})x(p_{j^*}) + \delta\left[1-\phi(p_i,p_{j^*})\right]V^0(p_{j^*}) + \delta\phi(p_i,p_{j^*})V(c) \\
&\qquad > \pi(p_j) - \phi(p_i,p_j)x(p_j) + \delta\left[1-\phi(p_i,p_j)\right]V^0(p_j) + \delta\phi(p_i,p_j)V(c) \quad \forall j = 1,...,n
\end{aligned}
$$

   $\forall i = 1,...,n$.

   Note that p_{j^*} is the optimal price choice given p_i (i.e., the policy function for prices). Denote this policy function as $h(p_i)$.

5. Let $\|V^0 - V^1\|_\infty$ denote the largest absolute value of the difference between the respective elements of V^0 and V^1. If $\|V^0 - V^1\|_\infty < \epsilon$ stop the iteration and return the value function V^1 and the policy

function $h(p_i)$. Otherwise set $V^0 = V^1$ and return to Step 2. We set $\epsilon = .0001$ in our applications.

3 Changes Specification Robustness Analysis

Under specification 2 in the main text, prices are lower under revenue-based penalties in early periods and lower under overcharge-based penalties in later periods. In this appendix, we explore the robustness of this result.

Specifically, we compute the cartel price path under revenue-based penalties, $\{p_t^R\}_{t=1}^{\infty}$, and the cartel price path under overcharge-based penalties, $\{p_t^O\}_{t=1}^{\infty}$, in a variety of alternative parameter configurations under specification 2. Each alternative parameter configuration is a modification of the baseline setting where we change one of α_0, δ, c, N, γ_R or γ_O while keeping the other parameters fixed at their values from the baseline setting. We primarily restrict attention to parameter values where a cartel forms under both penalty types, as in the baseline setting. We plot the difference between the price under revenue-based penalties and the price under overcharge-based penalties, $p_t^R - p_t^O$, across time for a range of α_1 values.

Figures 1-6 present $p_t^R - p_t^O$ for a variety of alternative values for δ. Table 1 summarizes results and presents the average price in the first 5 periods (denoted $\bar{p}^i_{1-5}$) under each penalty type $i \in \{R, O\}$ and the average price in periods 95-100 (denoted $\bar{p}^i_{95-100}$) under each penalty type. Figures 7-11 present $p_t^R - p_t^O$ for a variety of alternative values for α_0. Table 2 summarizes results. Figures 12-14 present $p_t^R - p_t^O$ for a variety of alternative values for c. Table 3 summarizes results.

Figures 15-19 present $p_t^R - p_t^O$ for a variety of alternative values for N. Table 4 summarizes results. Figures 20-24 present $p_t^R - p_t^O$ for a variety of alternative values for γ_R. Table 6 summarizes results. Figures 25-29 present $p_t^R - p_t^O$ for a variety of alternative values for γ_O. Table 5 summarizes results.

In all cases where a cartel forms under both penalty types, $p_t^R - p_t^O$ is negative in early periods and positive in later periods when α_1 is sufficiently high. Therefore, prices are lower under revenue-based penalties in early periods of collusion and lower under overcharge-based penalties in later periods of collusion when α_1 is sufficiently high.

Table 1: Alternative Parameter Values: δ

δ	α_1	$\bar{p}^R_{1-5}$	$\bar{p}^O_{1-5}$	$\bar{p}^R_{95-100}$	$\bar{p}^O_{95-100}$
0.7	5	0.2592	0.2808	0.566	0.481
0.7	10	0.2148	0.2463	0.565	0.4805
0.7	15	0.182	0.2267	0.564	0.48
0.75	5	0.2637	0.2828	0.566	0.481
0.75	10	0.2195	0.2487	0.565	0.4805
0.75	15	0.1974	0.2295	0.5645	0.48
0.8	5	0.2682	0.2851	0.566	0.481
0.8	10	0.2244	0.2513	0.5655	0.4805
0.8	15	0.2019	0.2322	0.565	0.4805
0.85	5	0.273	0.2873	0.566	0.481
0.85	10	0.2297	0.2541	0.5655	0.481
0.85	15	0.2072	0.2348	0.565	0.4805
0.9	5	0.2781	0.2895	0.5665	0.481
0.9	10	0.2352	0.2568	0.566	0.481
0.9	15	0.2131	0.2381	0.5655	0.4805
0.95	5	0.2832	0.2919	0.5665	0.481
0.95	10	0.2416	0.2599	0.566	0.481
0.95	15	0.2194	0.2413	0.566	0.481

Table 2: ALTERNATIVE PARAMETER VALUES: α_0

α_0	α_1	$\bar{p}^R_{1-5}$	$\bar{p}^O_{1-5}$	$\bar{p}^R_{95-100}$	$\bar{p}^O_{95-100}$
0.01	2.5	0.334	0.3464	0.5525	0.536
0.01	5	0.2901	0.3108	0.5525	0.536
0.01	7.5	0.2651	0.2888	0.5525	0.536
0.01	10	0.248	0.2734	0.552	0.536
0.03	2.5	0.3293	0.3333	0.5585	0.5085
0.03	5	0.2845	0.3003	0.5585	0.5085
0.03	7.5	0.259	0.2798	0.5585	0.5085
0.03	10	0.242	0.2653	0.558	0.5085
0.05	2.5	0.3241	0.3203	0.5665	0.4815
0.05	5	0.2781	0.2895	0.5665	0.481
0.05	7.5	0.2526	0.2704	0.566	0.481
0.05	10	0.2352	0.2568	0.566	0.481
0.07	2.5	0.3181	0.307	0.5765	0.454
0.07	5	0.2707	0.2786	0.5765	0.4535
0.07	7.5	0.2452	0.2609	0.576	0.4535
0.07	10	0.228	0.2481	0.576	0.4535
0.09	2.5	0.3108	0.2936	0.5905	0.4265
0.09	5	0.2628	0.2677	0.5905	0.426
0.09	7.5	0.2369	0.2514	0.59	0.426
0.09	10	0.2201	0.2396	0.5895	0.426

Table 3: ALTERNATIVE PARAMETER VALUES: c

c	α_1	$\bar{p}^R_{1-5}$	$\bar{p}^O_{1-5}$	$\bar{p}^R_{95-100}$	$\bar{p}^O_{95-100}$
0.1	5	0.2781	0.2895	0.5665	0.481
0.1	10	0.2352	0.2568	0.566	0.481
0.1	15	0.2131	0.2381	0.5655	0.4805
0.2	5	0.3616	0.378	0.633	0.539
0.2	10	0.3199	0.3492	0.6325	0.5385
0.2	15	0.2985	0.3325	0.632	0.5385
0.3	5	0.4484	0.4652	0.6995	0.5965
0.3	10[a]	0.3	0.4405	0.3	0.5965
0.3	15[b]	0.3	0.4255	0.3	0.596

[a] α_1 is sufficiently high that a cartel does not form under revenue-based penalties for this parameter value.
[b] α_1 is sufficiently high that a cartel does not form under revenue-based penalties for this parameter value.

Table 4: ALTERNATIVE PARAMETER VALUES: N

N	α_1	$\bar{p}^R_{1-5}$	$\bar{p}^O_{1-5}$	$\bar{p}^R_{95-100}$	$\bar{p}^O_{95-100}$
2	2.5	0.3241	0.3203	0.5665	0.4815
2	5	0.2781	0.2895	0.5665	0.481
2	7.5	0.2526	0.2704	0.566	0.481
2	10	0.2352	0.2568	0.566	0.481
3	2.5	0.3241	0.3203	0.5665	0.4815
3	5	0.2781	0.2895	0.5665	0.481
3	7.5	0.2526	0.2704	0.566	0.481
3	10	0.2352	0.2568	0.566	0.481
4	2.5	0.3241	0.3203	0.5665	0.4815
4	5	0.2781	0.2895	0.5665	0.481
4	7.5	0.2526	0.2704	0.566	0.481
4	10	0.2352	0.2568	0.566	0.481
5	2.5	0.3241	0.3203	0.5665	0.4815
5	5	0.2781	0.2895	0.5665	0.481
5	7.5	0.2526	0.2704	0.566	0.481
5	10	0.2352	0.2568	0.566	0.481
6	2.5	0.3241	0.3203	0.5665	0.4815
6	5	0.2781	0.2895	0.5665	0.481
6	7.5[a]	0.1	0.2704	0.1	0.481
6	10[b]	0.1	0.2568	0.1	0.481

[a] α_1 is sufficiently high that a cartel does not form under revenue-based penalties for this parameter value.
[b] α_1 is sufficiently high that a cartel does not form under revenue-based penalties for this parameter value.

Table 5: Alternative Parameter Values: γ_O

γ_O	α_1	$\bar{p}^R_{1-5}$	$\bar{p}^O_{1-5}$	$\bar{p}^R_{95-100}$	$\bar{p}^O_{95-100}$
2	10	0.2352	0.2789	0.566	0.5045
2	20	0.1987	0.2429	0.565	0.504
2	30	0.1804	0.2233	0.564	0.504
2	40	0.1692	0.2102	0.563	0.5035
3	10	0.2352	0.2579	0.566	0.482
3	20	0.1987	0.2258	0.565	0.4815
3	30	0.1804	0.2086	0.564	0.481
3	40	0.1692	0.1972	0.563	0.4805
4	10	0.2352	0.2416	0.566	0.4595
4	20	0.1987	0.2129	0.565	0.459
4	30	0.1804	0.1973	0.564	0.458
4	40	0.1692	0.1873	0.563	0.4575
5	10	0.2352	0.2282	0.566	0.437
5	20	0.1987	0.2022	0.565	0.436
5	30	0.1804	0.1881	0.564	0.4355
5	40	0.1692	0.1789	0.563	0.435
6	10	0.2352	0.2169	0.566	0.4145
6	20	0.1987	0.1929	0.565	0.4135
6	30	0.1804	0.1805	0.564	0.413
6	40	0.1692	0.1723	0.563	0.412

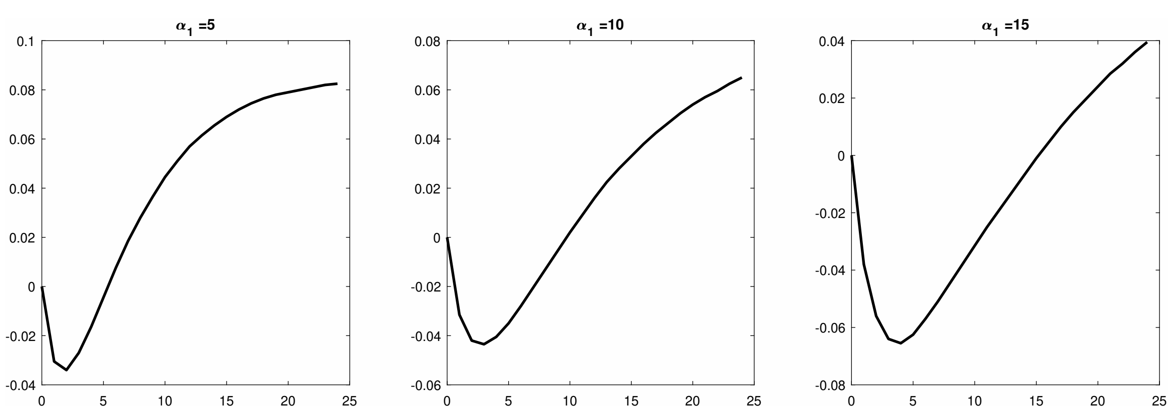

Figure 1: $p^R_t - p^O_t$ for $\delta = .7$

Table 6: ALTERNATIVE PARAMETER VALUES: γ_R

γ_R	α_1	$\bar{p}^R_{1-5}$	$\bar{p}^O_{1-5}$	$\bar{p}^R_{95-100}$	$\bar{p}^O_{95-100}$
2	10	0.2844	0.2568	0.555	0.481
2	20	0.2396	0.2251	0.555	0.4805
2	30	0.2164	0.2078	0.5545	0.48
2	40	0.2013	0.1966	0.554	0.4795
2	50	0.1908	0.1884	0.5535	0.4785
2	60	0.183	0.1822	0.553	0.478
3	10	0.2638	0.2568	0.5585	0.481
3	20	0.2224	0.2251	0.558	0.4805
3	30	0.2012	0.2078	0.557	0.48
3	40	0.1877	0.1966	0.5565	0.4795
3	50	0.1785	0.1884	0.556	0.4785
3	60	0.1708	0.1822	0.555	0.478
4	10	0.2479	0.2568	0.562	0.481
4	20	0.2091	0.2251	0.561	0.4805
4	30	0.1897	0.2078	0.5605	0.48
4	40	0.1774	0.1966	0.5595	0.4795
4	50	0.1687	0.1884	0.559	0.4785
4	60	0.1623	0.1822	0.558	0.478
5	10	0.2352	0.2568	0.566	0.481
5	20	0.1987	0.2251	0.565	0.4805
5	30	0.1804	0.2078	0.564	0.48
5	40	0.1692	0.1966	0.563	0.4795
5	50	0.1612	0.1884	0.5619	0.4785
5	60	0.1551	0.1822	0.5555	0.478
6	10	0.2246	0.2568	0.5705	0.481
6	20	0.1896	0.2251	0.5695	0.4805
6	30	0.1726	0.2078	0.568	0.48
6	40	0.1622	0.1966	0.567	0.4795
6	50	0.1544	0.1884	0.5605	0.4785
6	60[a]	0.1	0.1822	0.1	0.478

[a] α_1 is sufficiently high that a cartel does not form under revenue-based penalties for this parameter value.

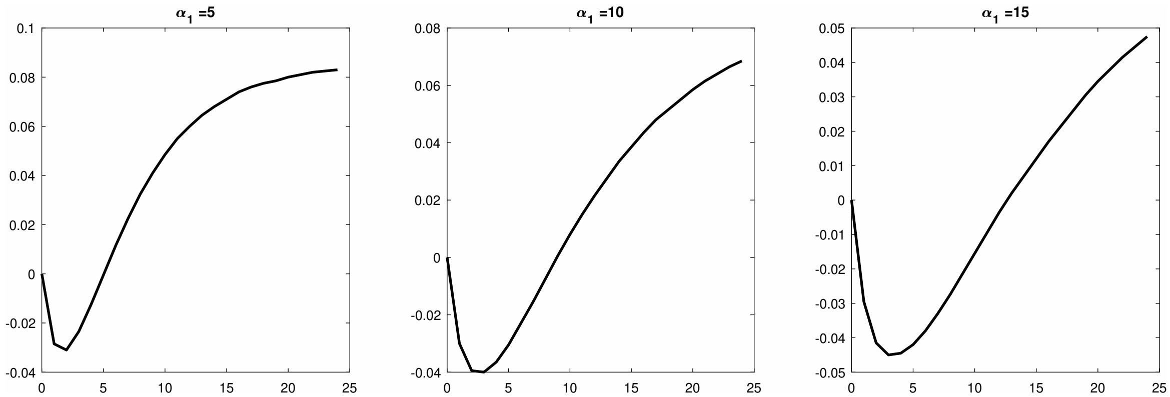

Figure 2: $p_t^R - p_t^O$ for $\delta = .75$

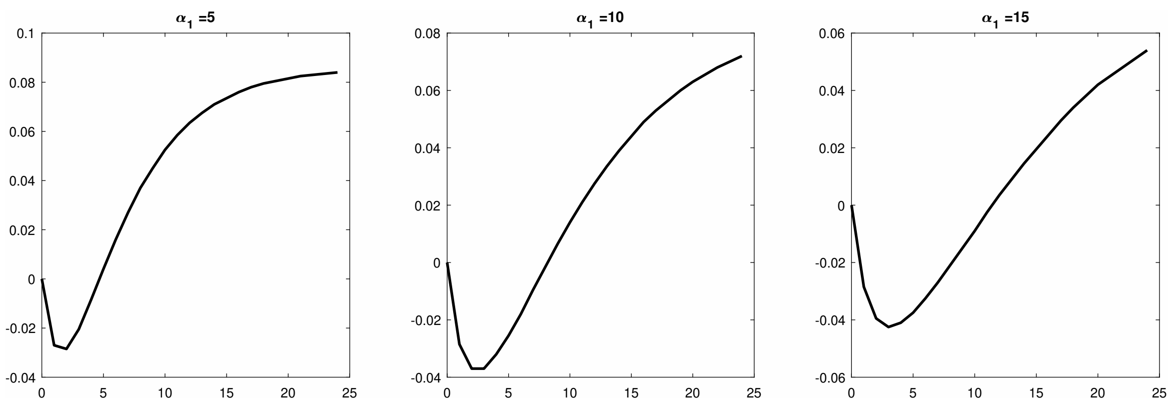

Figure 3: $p_t^R - p_t^O$ for $\delta = .8$

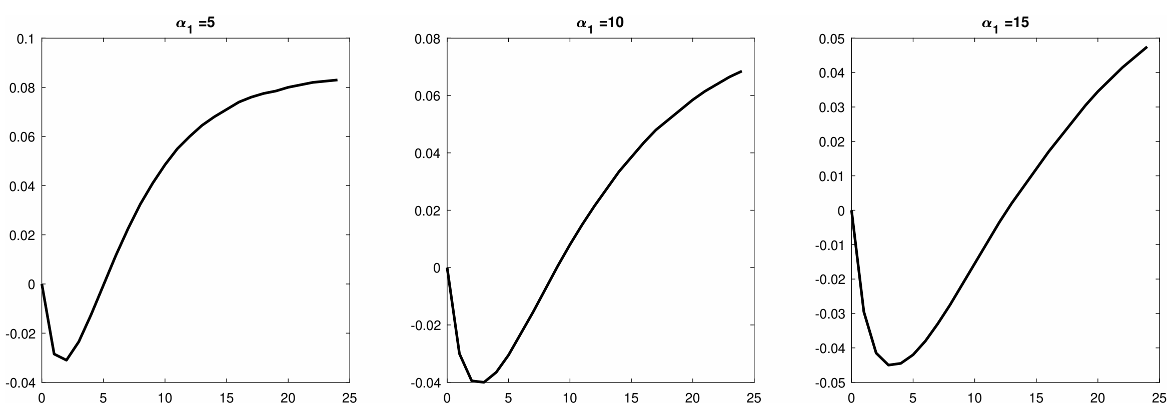

Figure 4: $p_t^R - p_t^O$ for $\delta = .85$

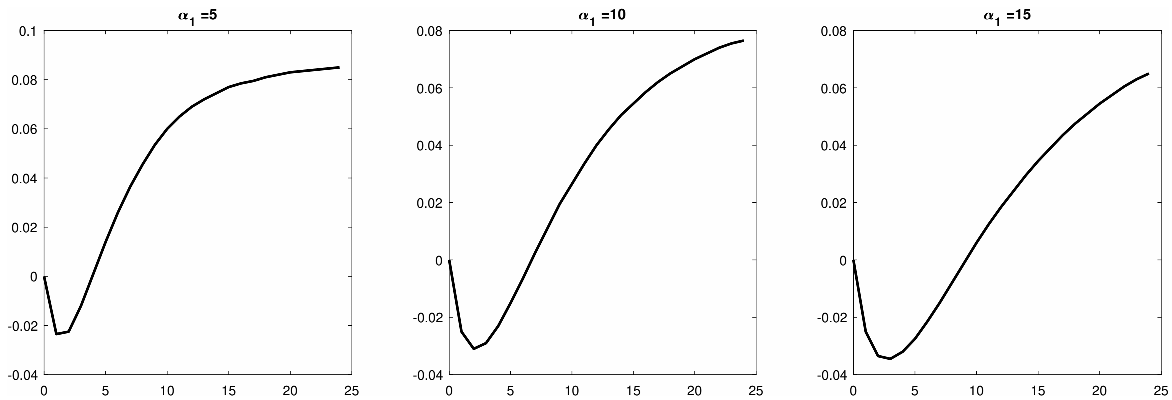

Figure 5: $p_t^R - p_t^O$ for $\delta = .9$

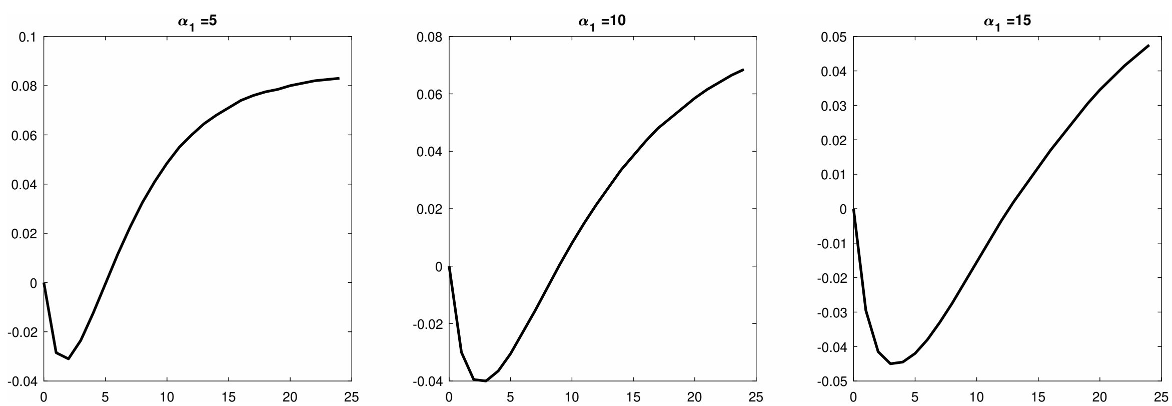

Figure 6: $p_t^R - p_t^O$ for $\delta = .95$

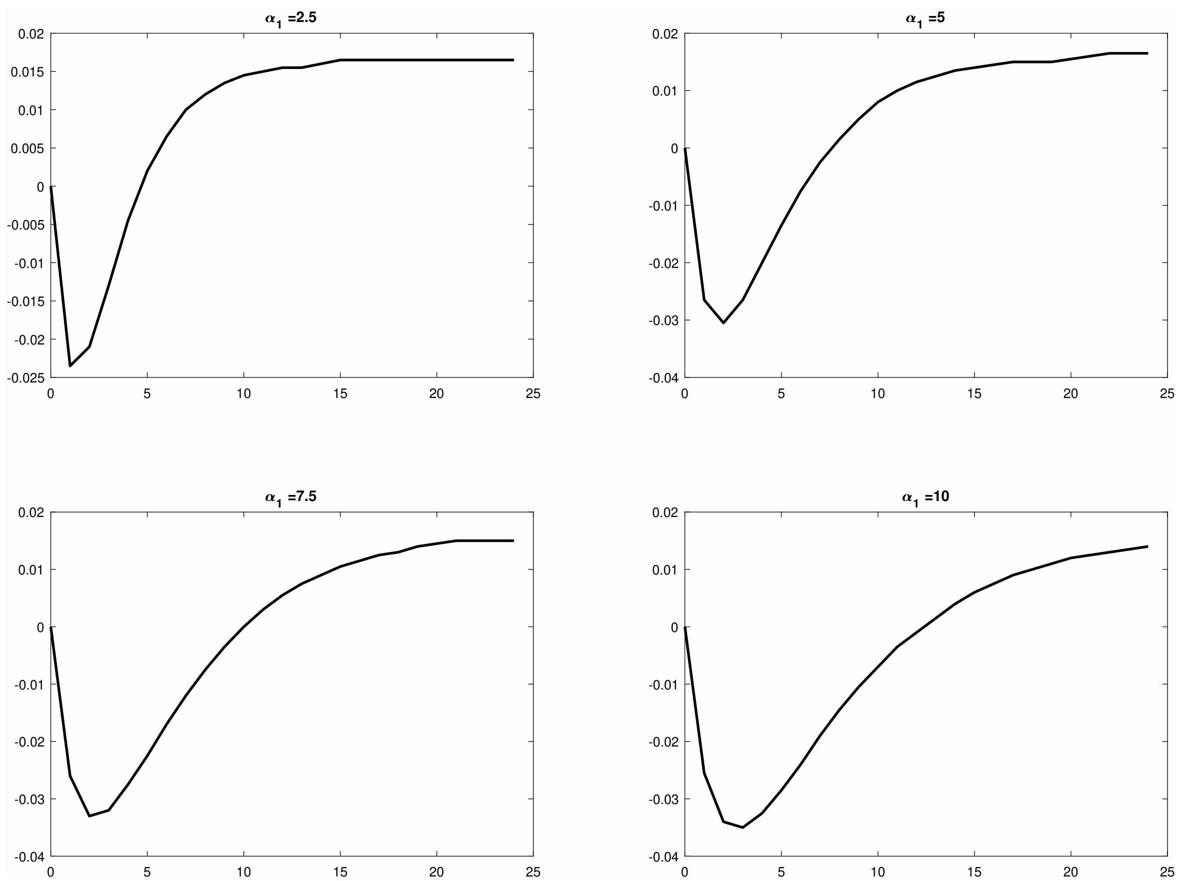

Figure 7: $p_t^R - p_t^O$ for $\alpha_0 = .01$

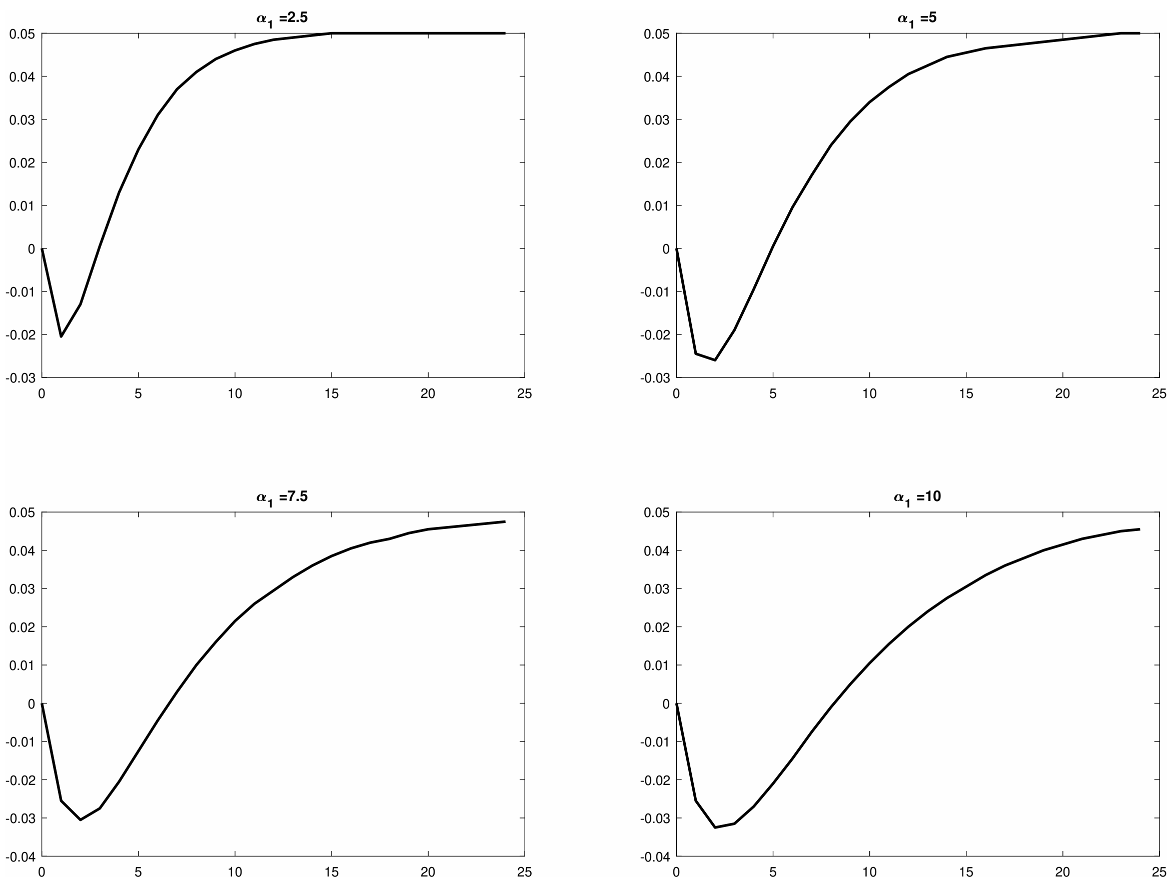

Figure 8: $p_t^R - p_t^O$ for $\alpha_0 = .03$

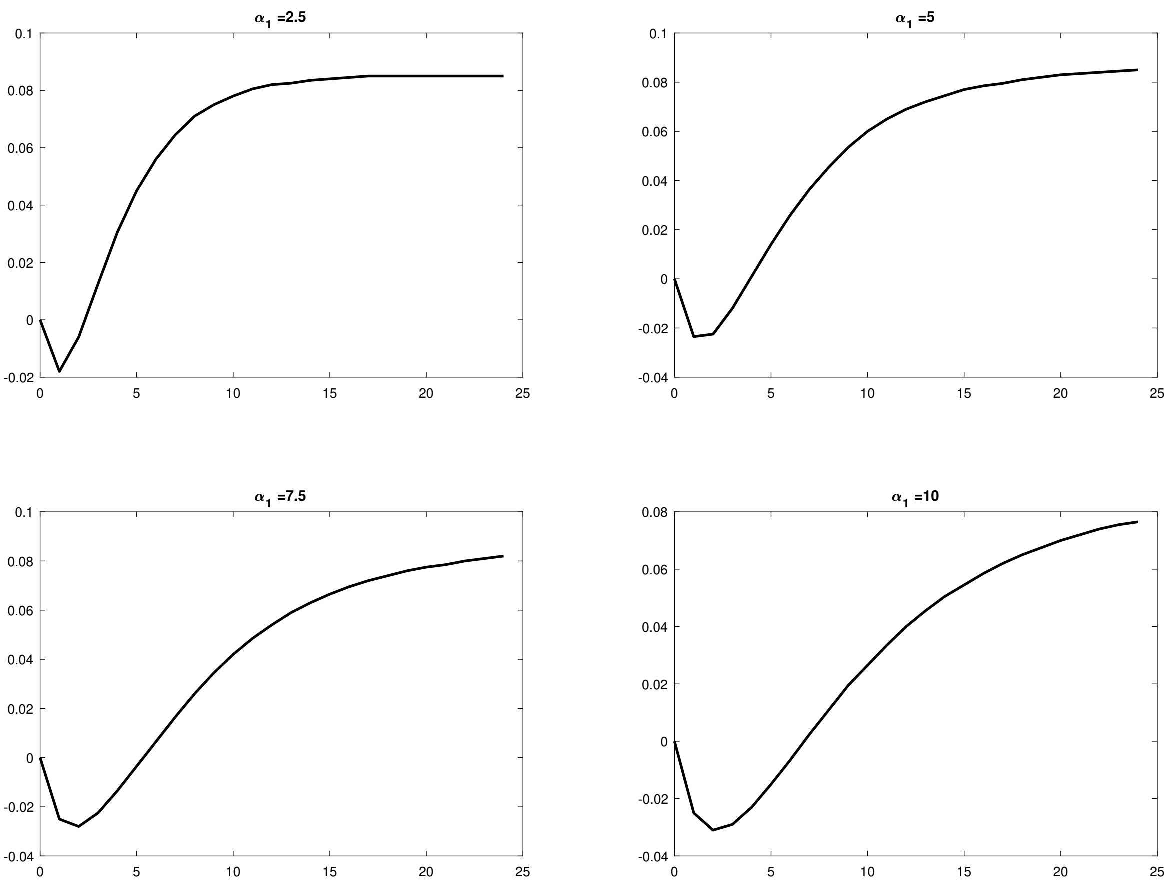

Figure 9: $p_t^R - p_t^O$ for $\alpha_0 = .05$

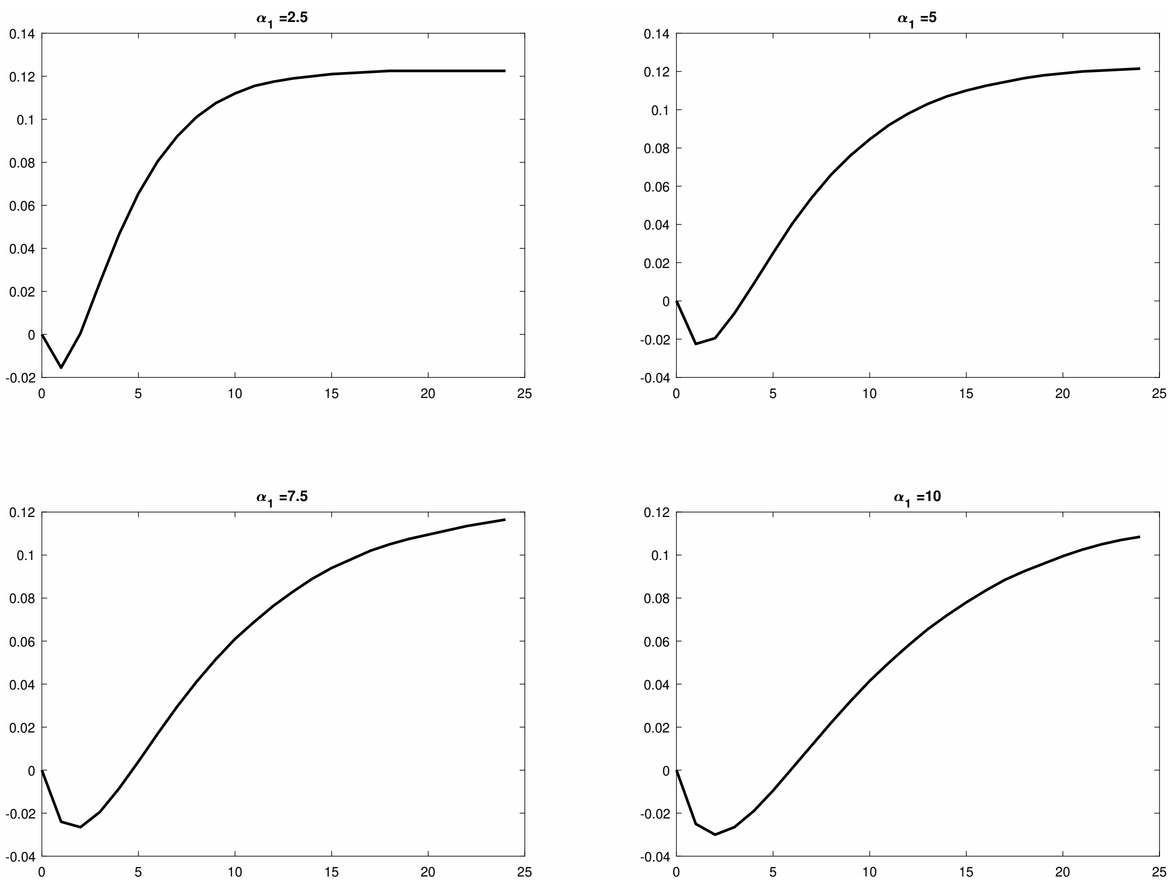

Figure 10: $p_t^R - p_t^O$ for $\alpha_0 = .07$

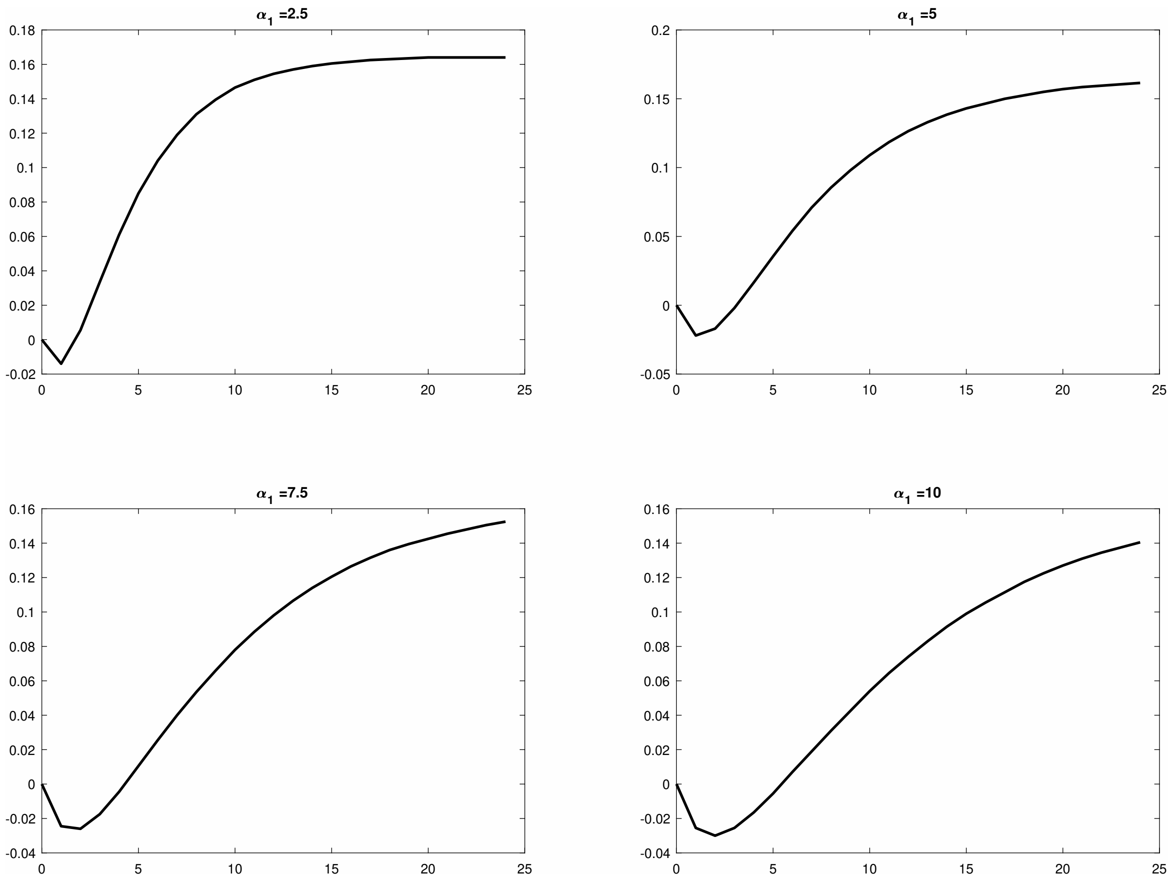

Figure 11: $p_t^R - p_t^O$ for $\alpha_0 = .09$

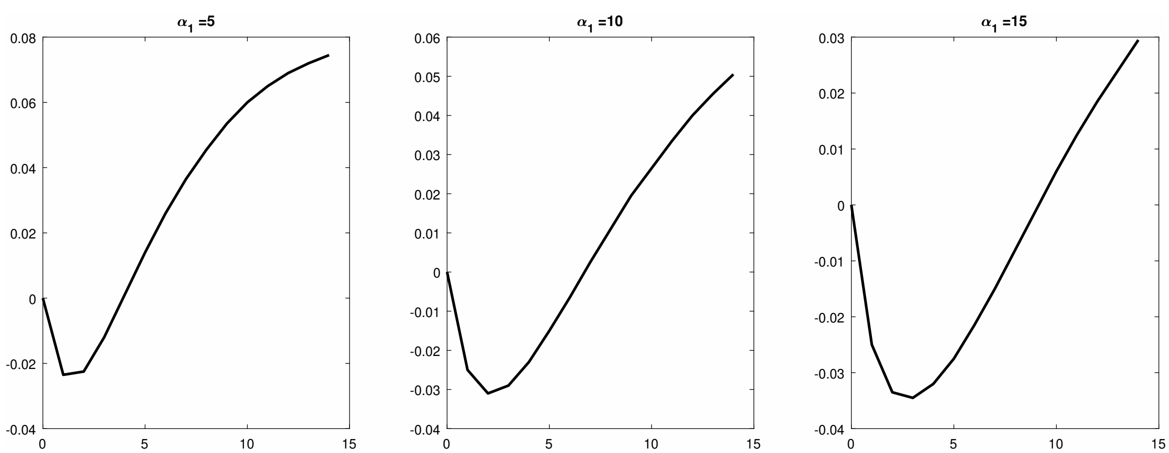

Figure 12: $p_t^R - p_t^O$ for $c = .1$

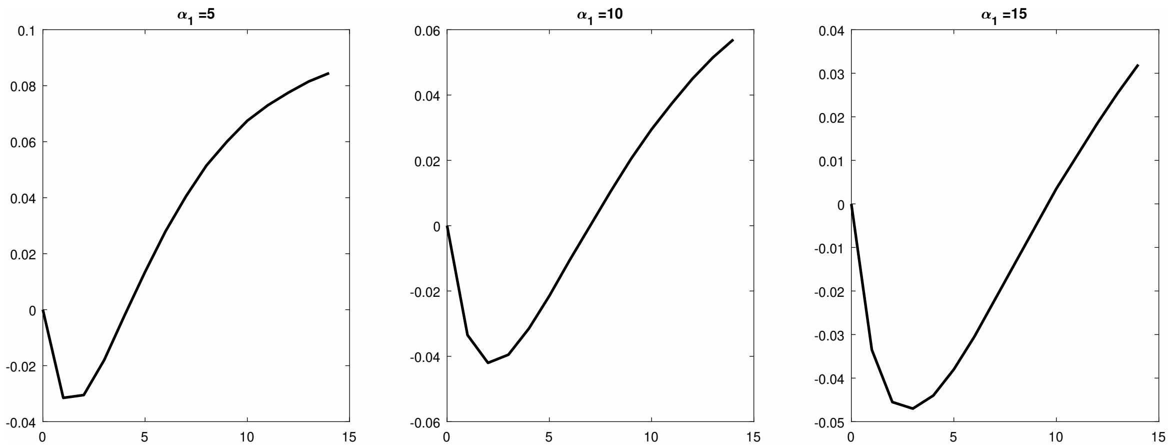

Figure 13: $p_t^R - p_t^O$ for $c = .2$

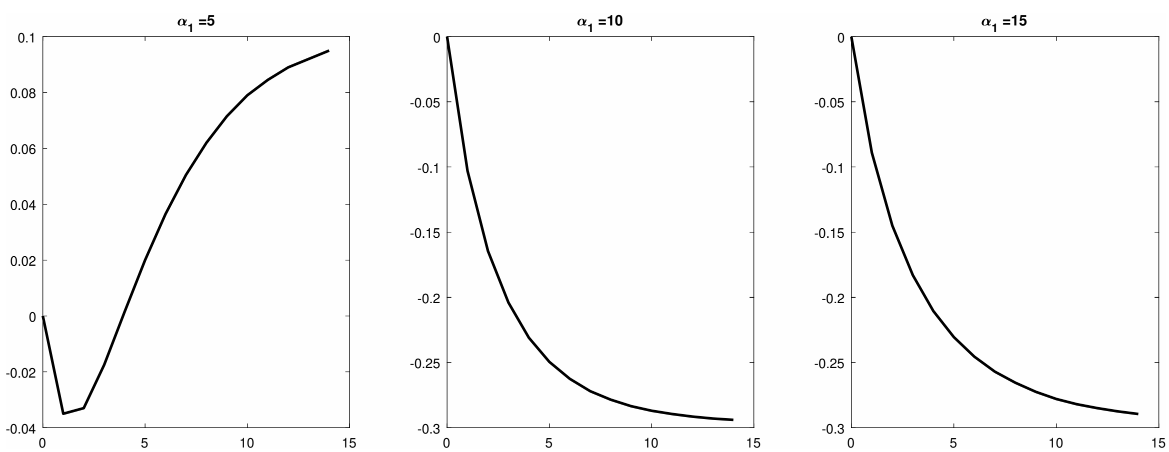

Figure 14: $p_t^R - p_t^O$ for $c = .3$. A cartel does not form under revenue-based penalties when $\alpha_1 = 10$ or $\alpha_1 = 15$.

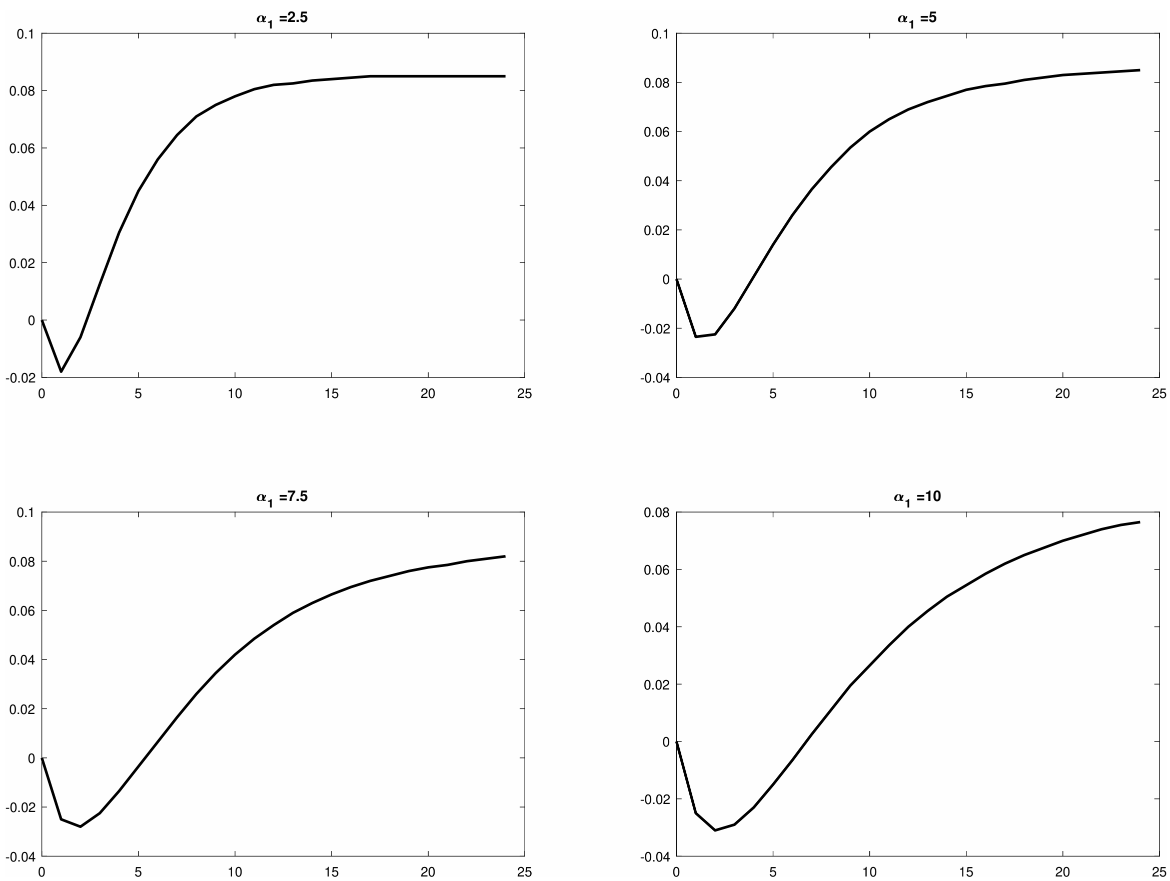

Figure 15: $p_t^R - p_t^O$ for $N = 2$

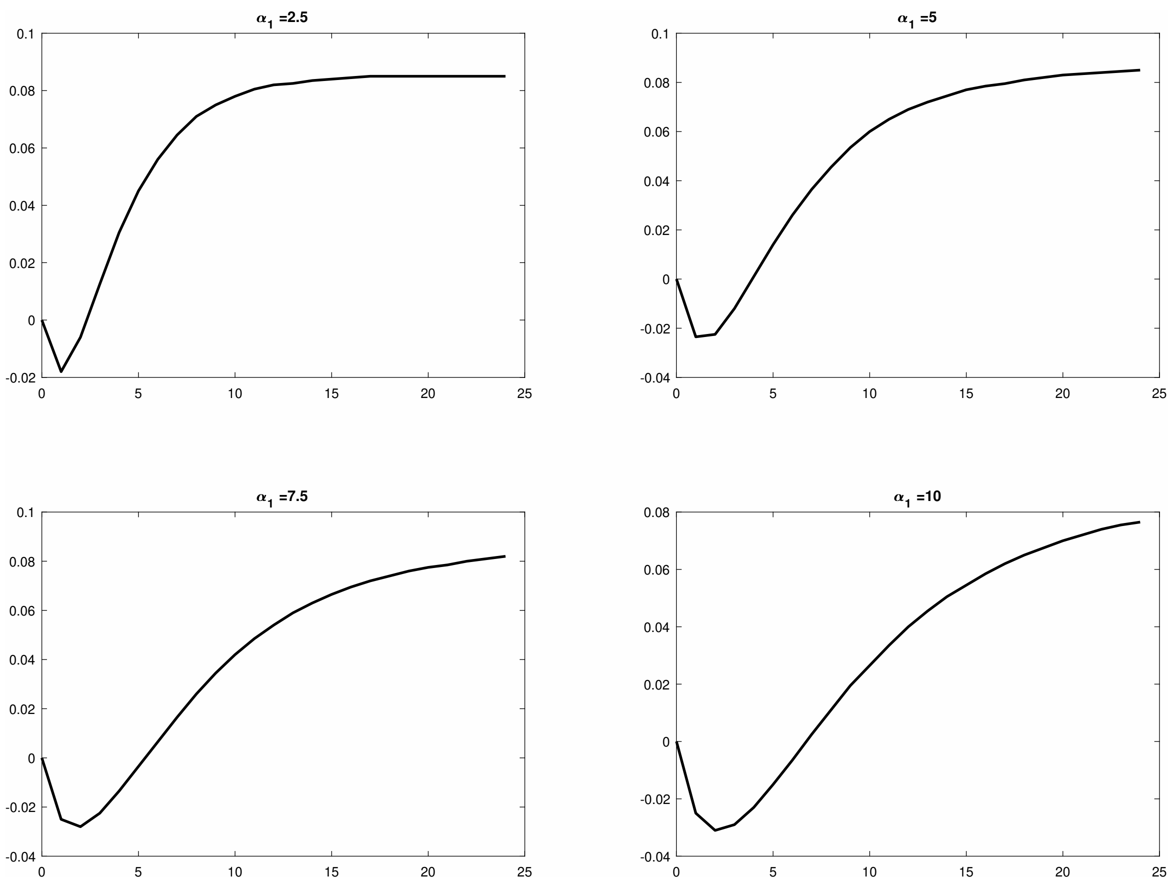

Figure 16: $p_t^R - p_t^O$ for $N = 3$

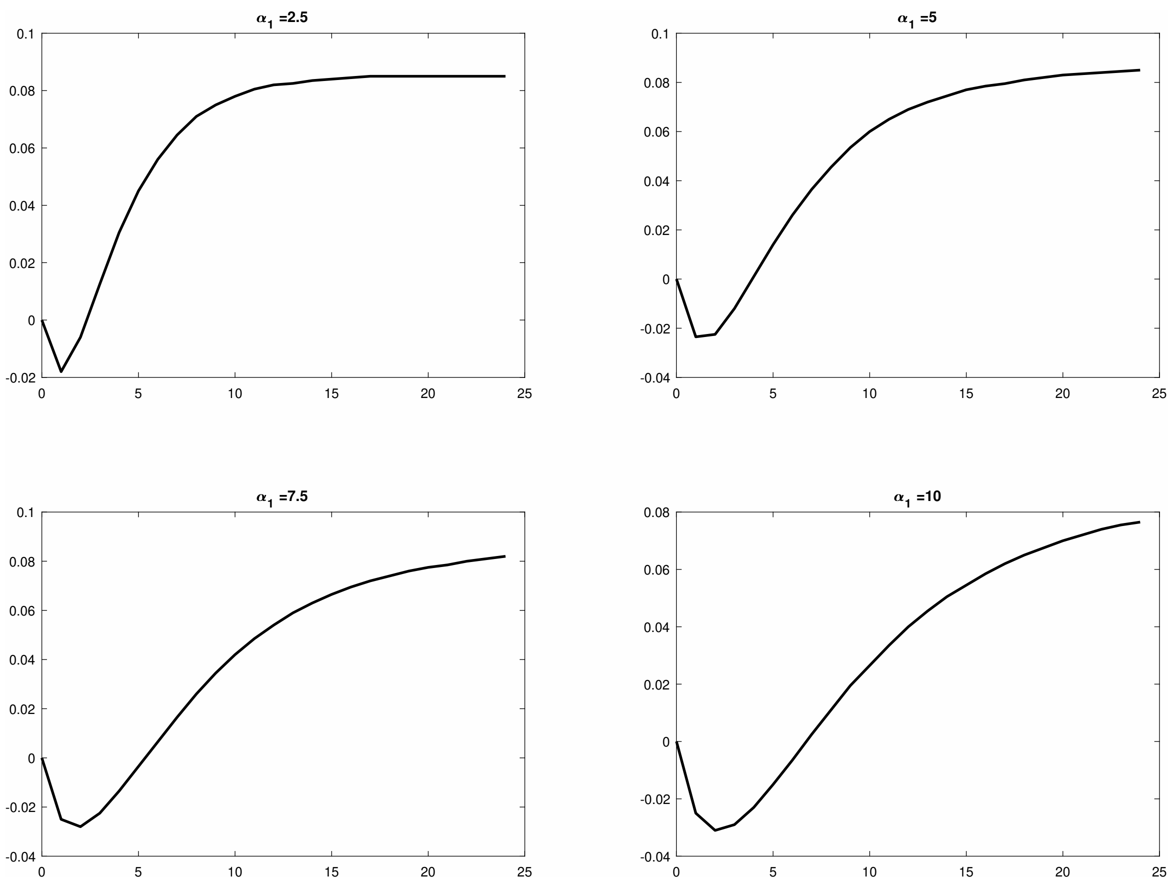

Figure 17: $p_t^R - p_t^O$ for $N = 4$

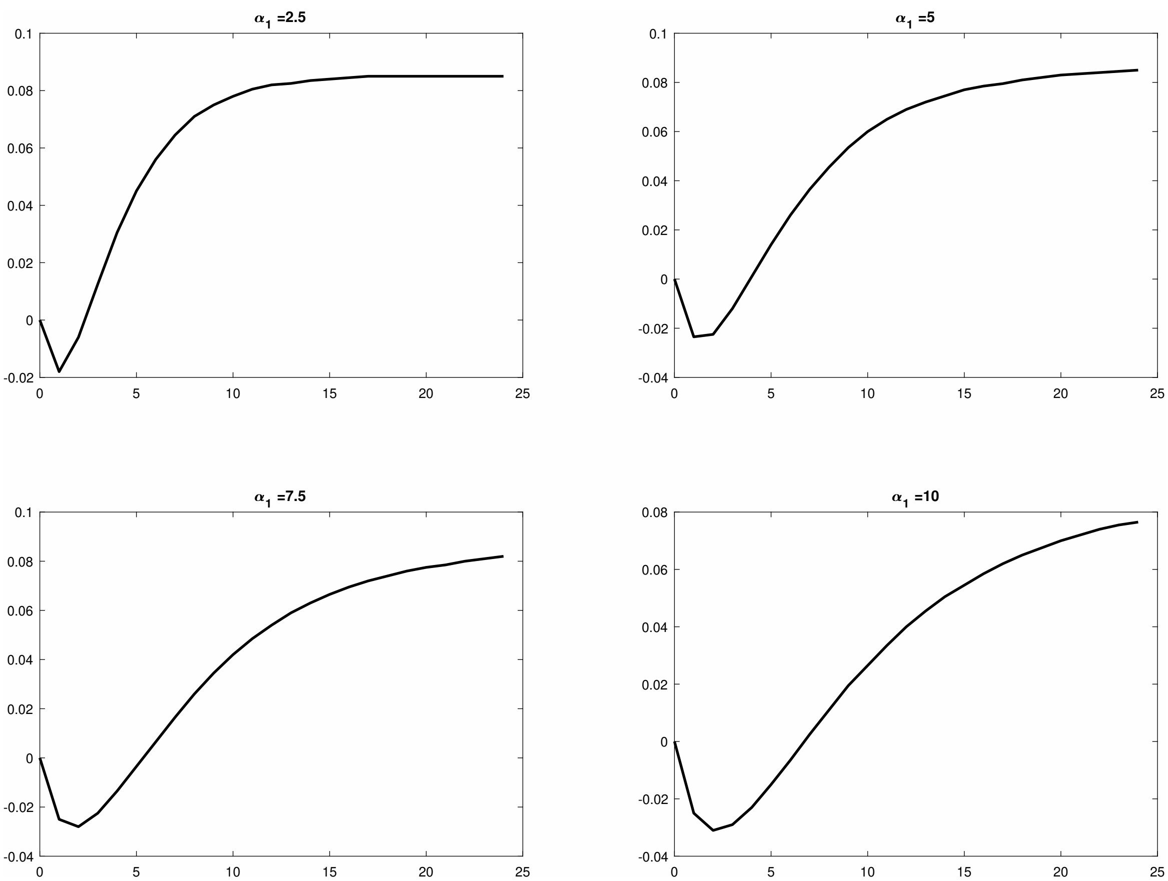

Figure 18: $p_t^R - p_t^O$ for $N = 5$

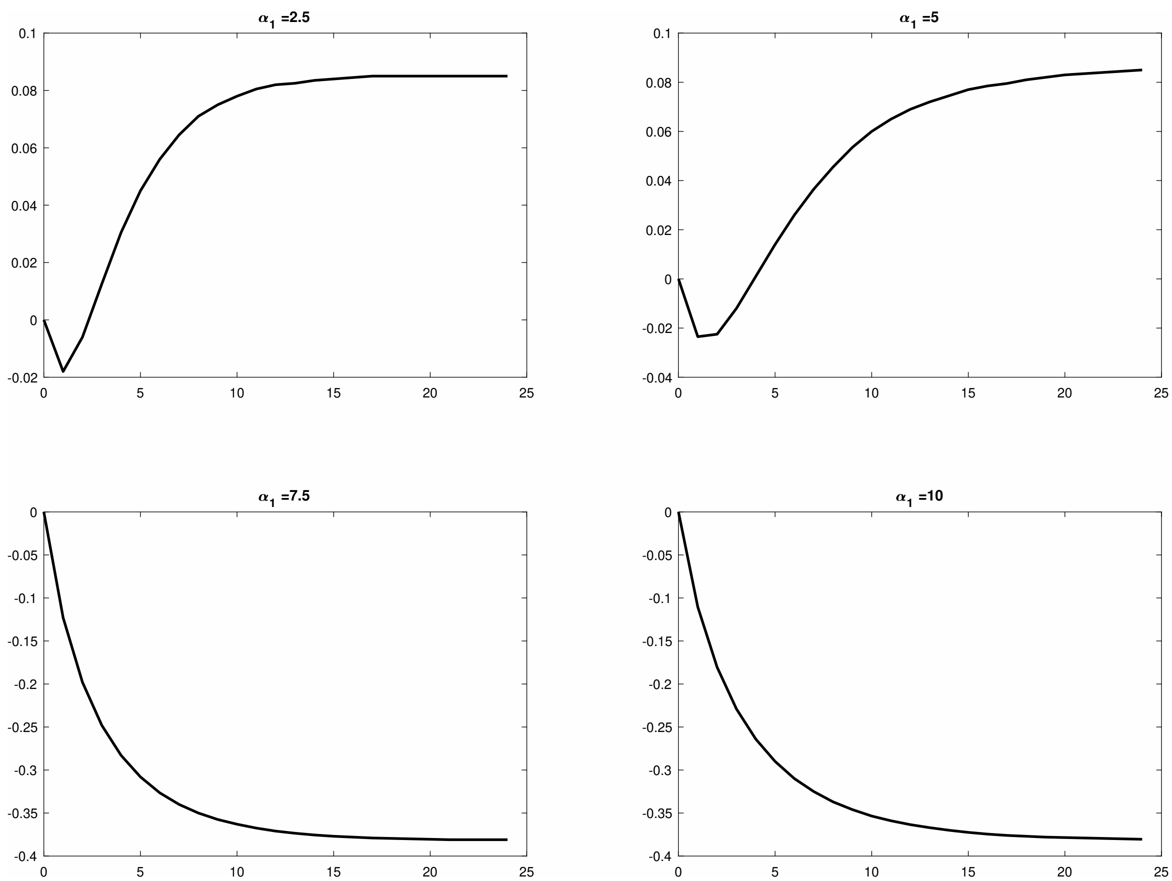

Figure 19: $p_t^R - p_t^O$ for $N = 6$. A cartel does not form under revenue-based penalties when $\alpha_1 = 7.5$ or $\alpha_1 = 10$.

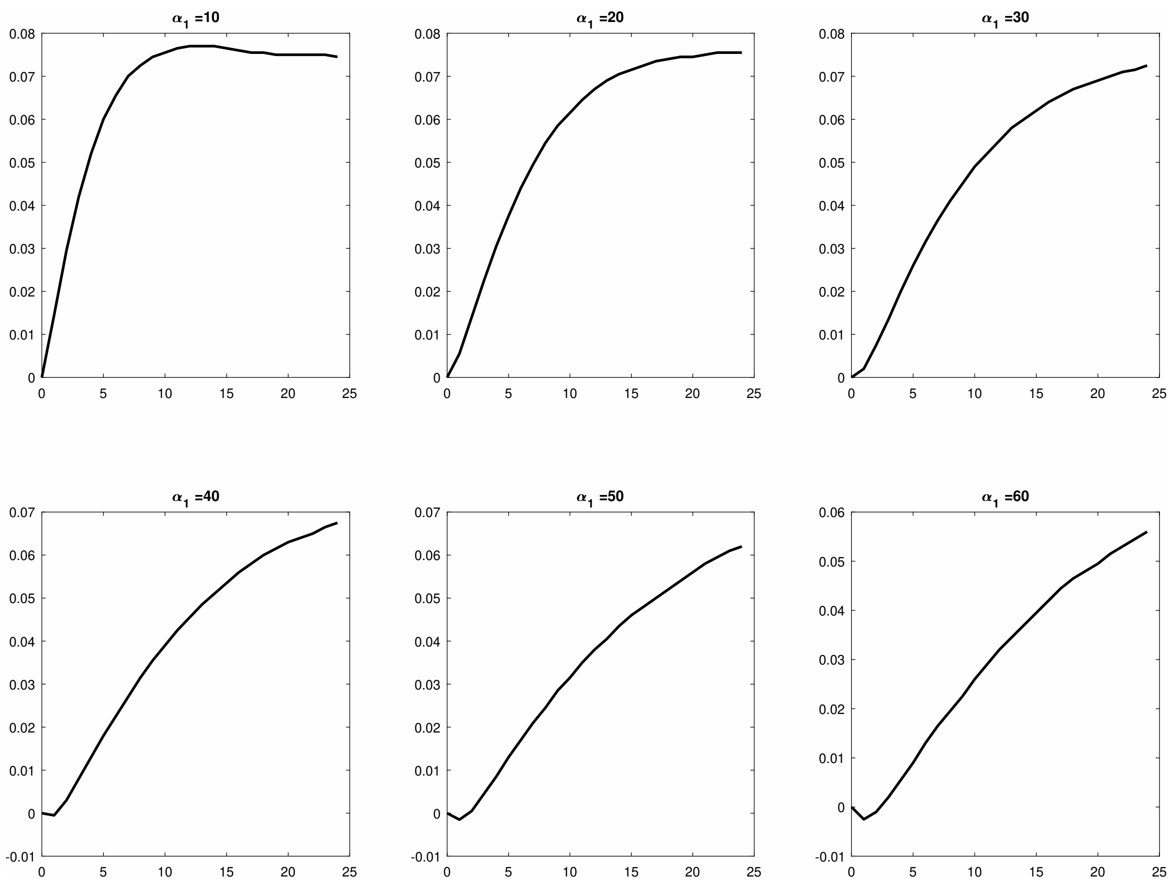

Figure 20: $p_t^R - p_t^O$ for $\gamma_R = 2$

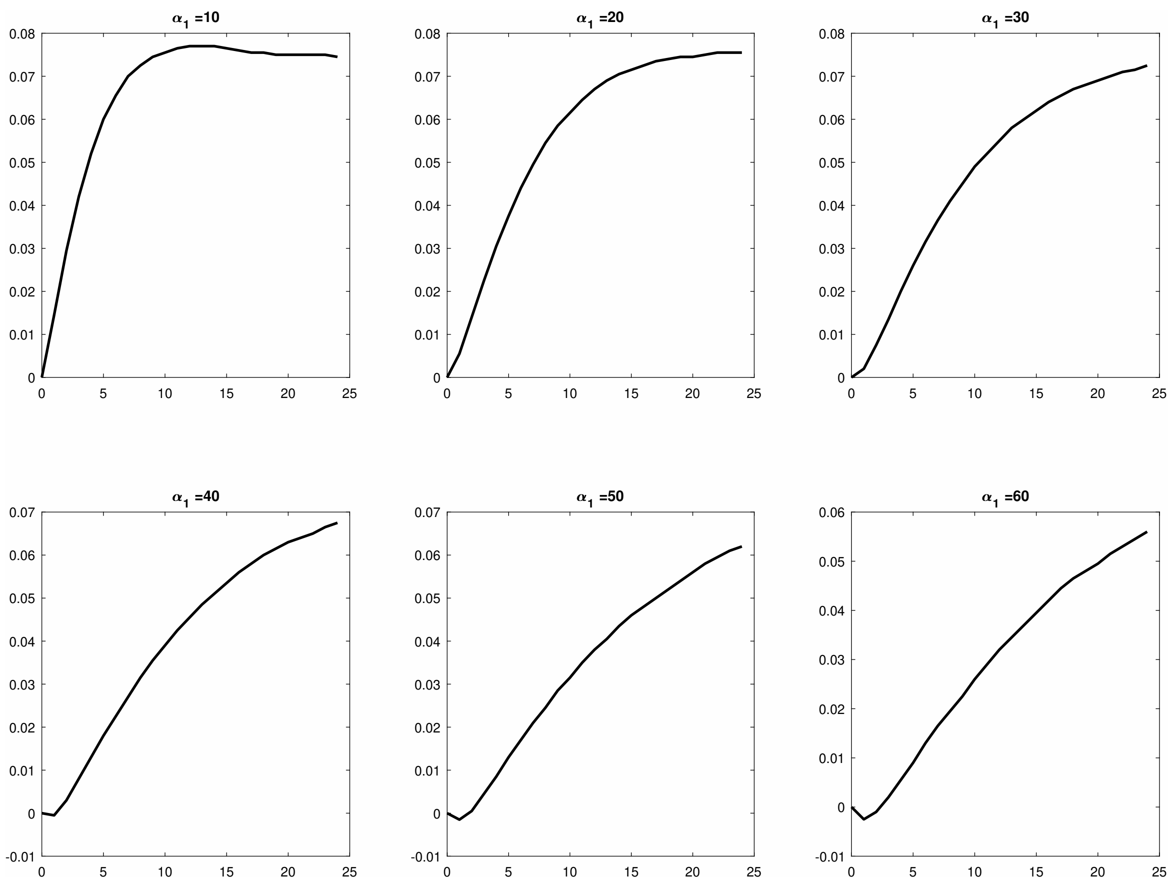

Figure 21: $p_t^R - p_t^O$ for $\gamma_R = 3$

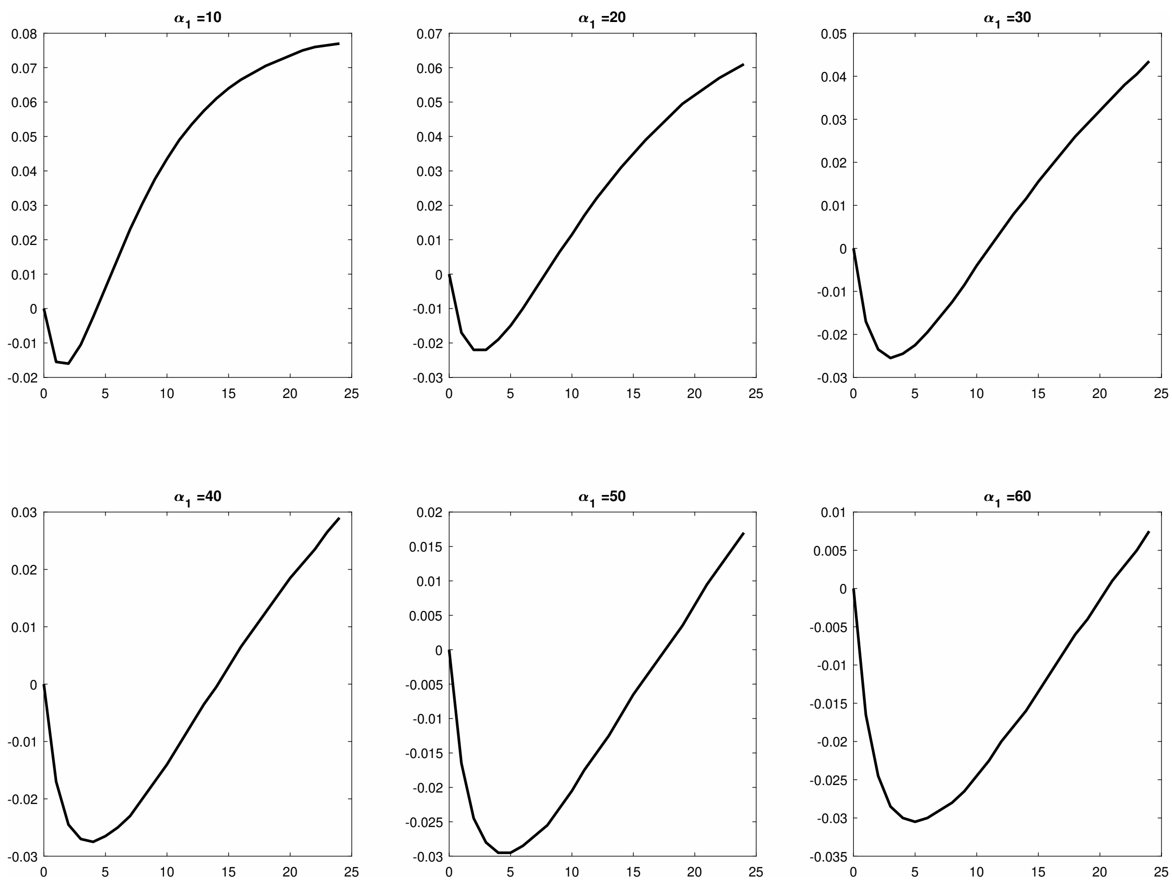

Figure 22: $p_t^R - p_t^O$ for $\gamma_R = 4$

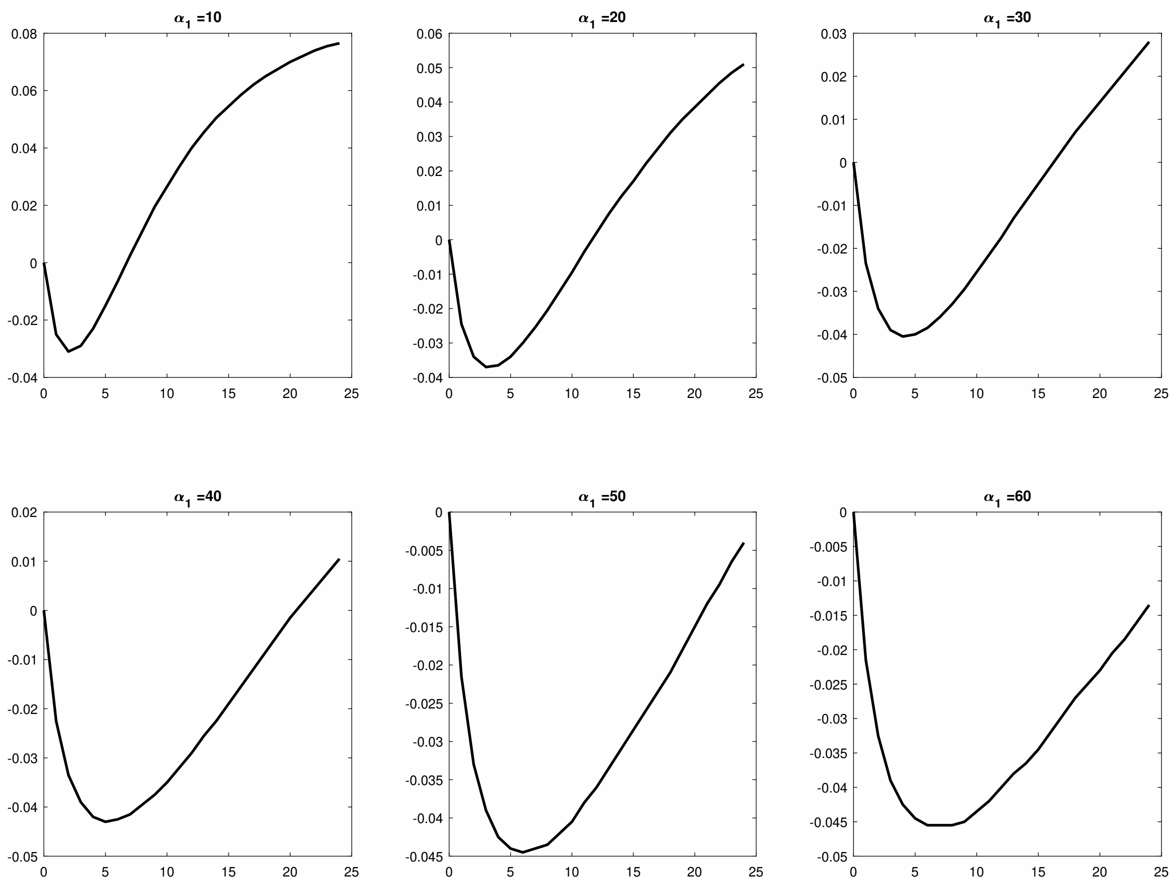

Figure 23: $p_t^R - p_t^O$ for $\gamma_R = 5$

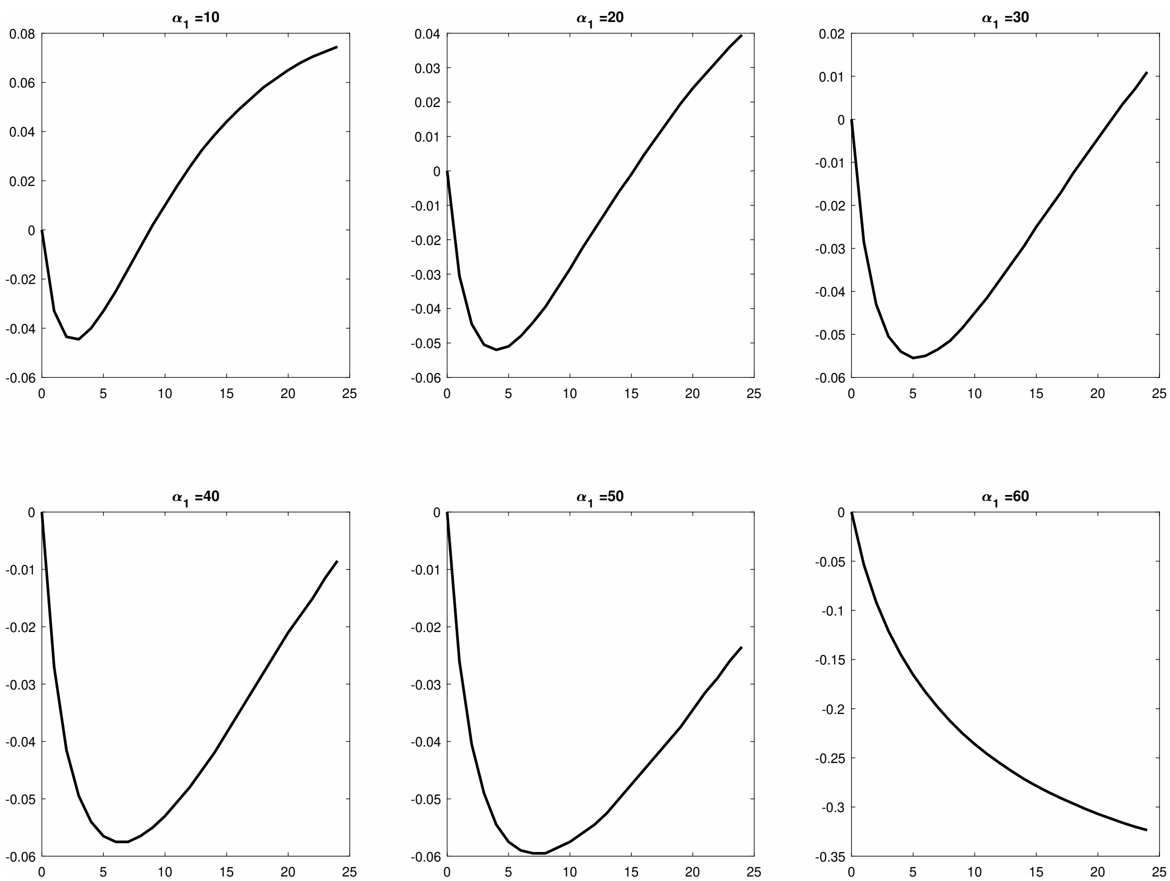

Figure 24: $p_t^R - p_t^O$ for $\gamma_R = 6$. A cartel does not form under revenue-based penalties when $\alpha_1 = 60$.

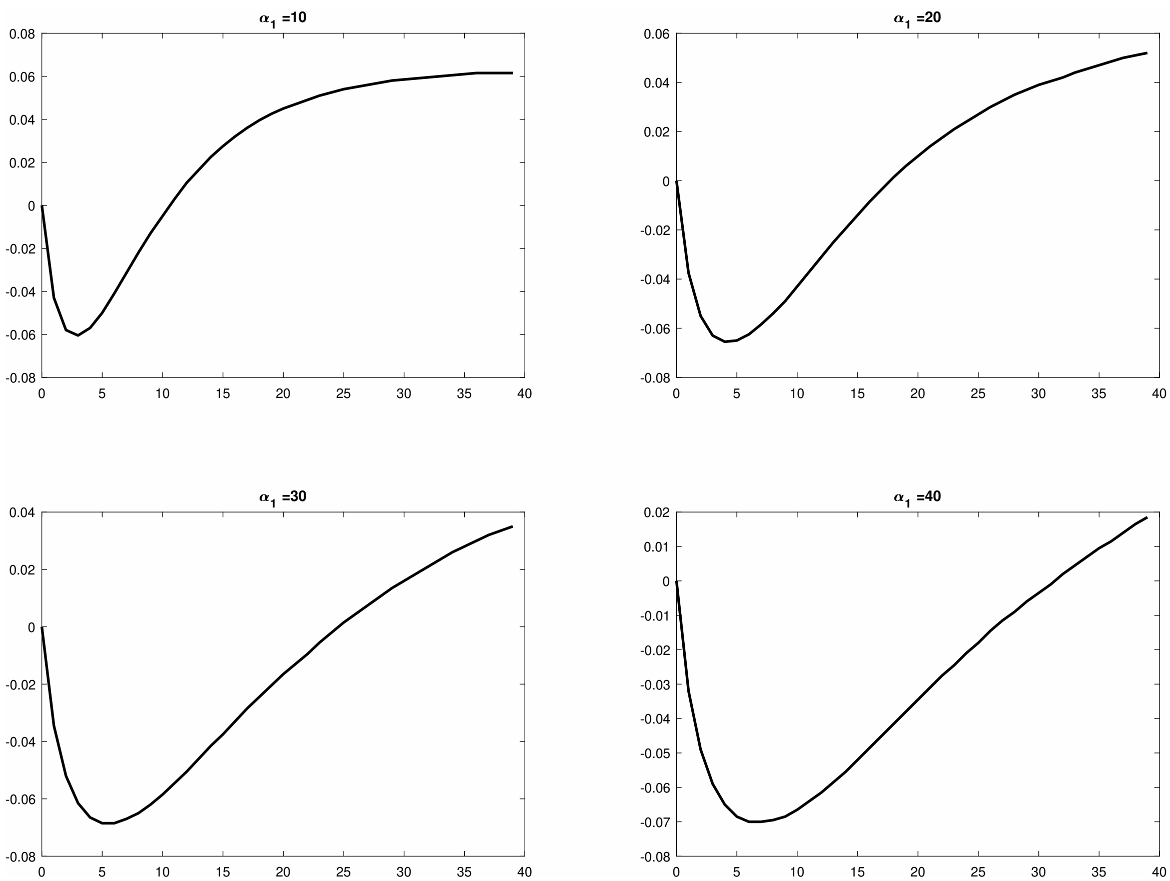

Figure 25: $p_t^R - p_t^O$ for $\gamma_O = 2$

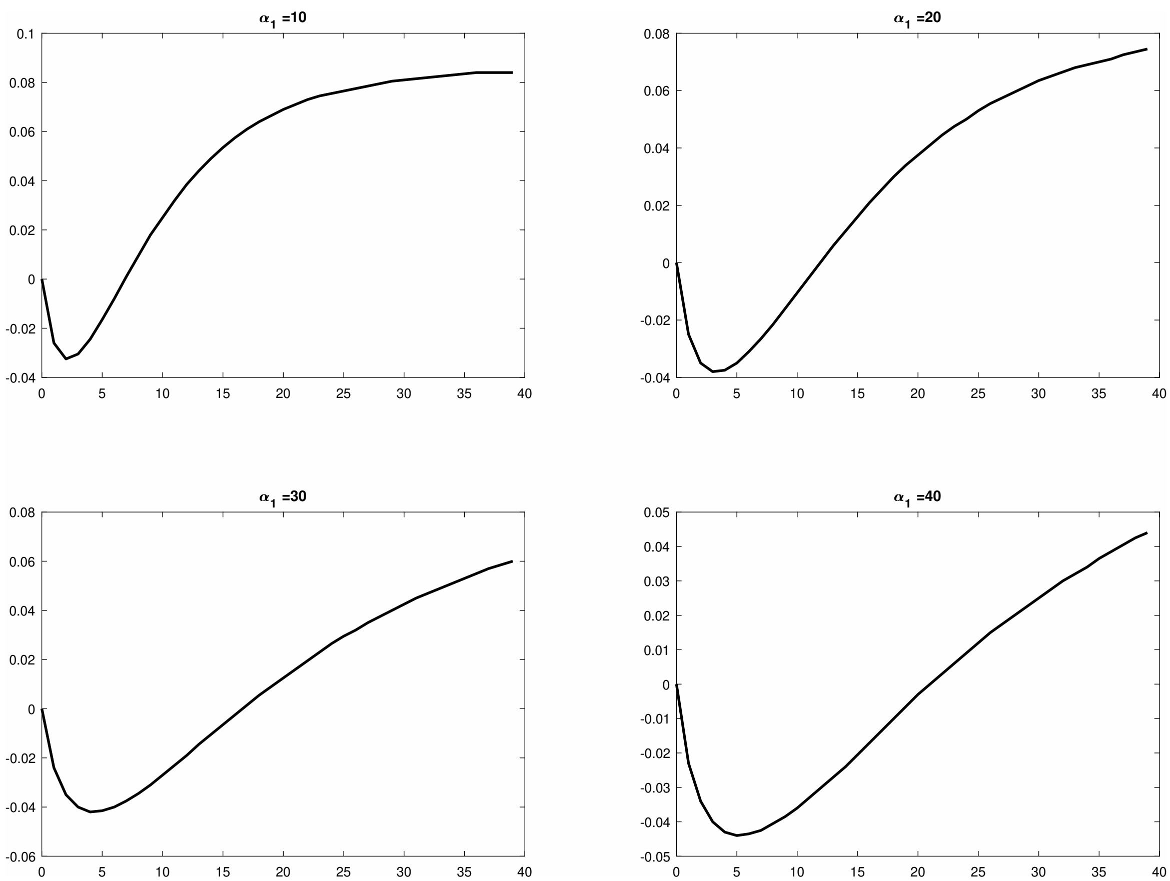

Figure 26: $p_t^R - p_t^O$ for $\gamma_O = 3$

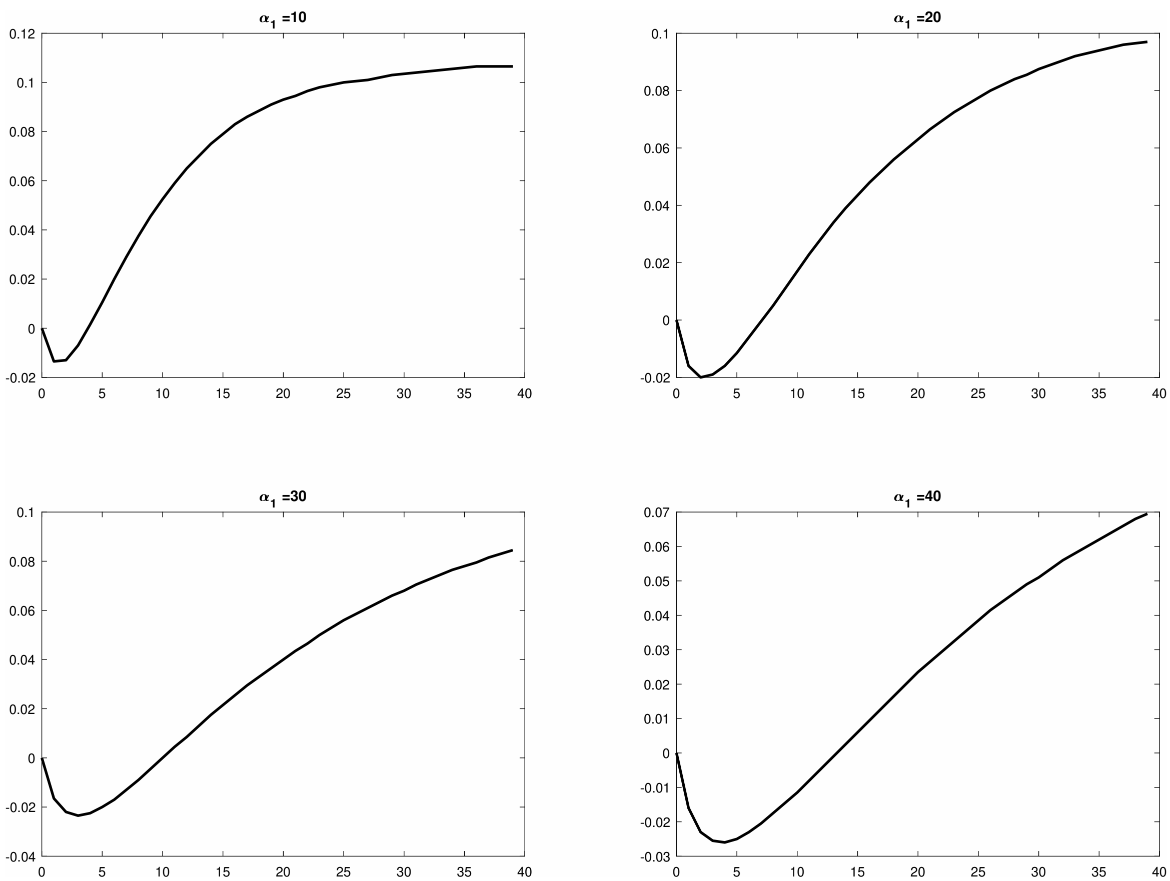

Figure 27: $p_t^R - p_t^O$ for $\gamma_O = 4$

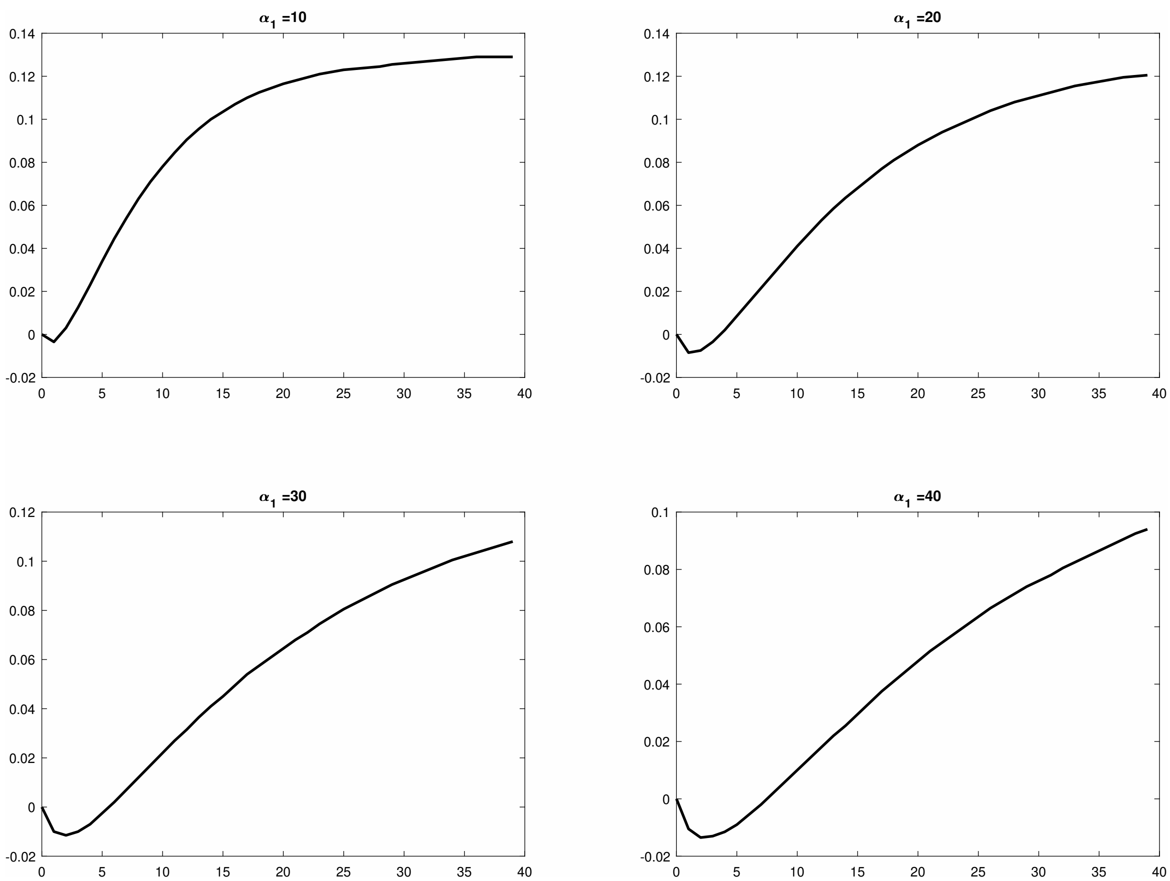

Figure 28: $p_t^R - p_t^O$ for $\gamma_O = 5$

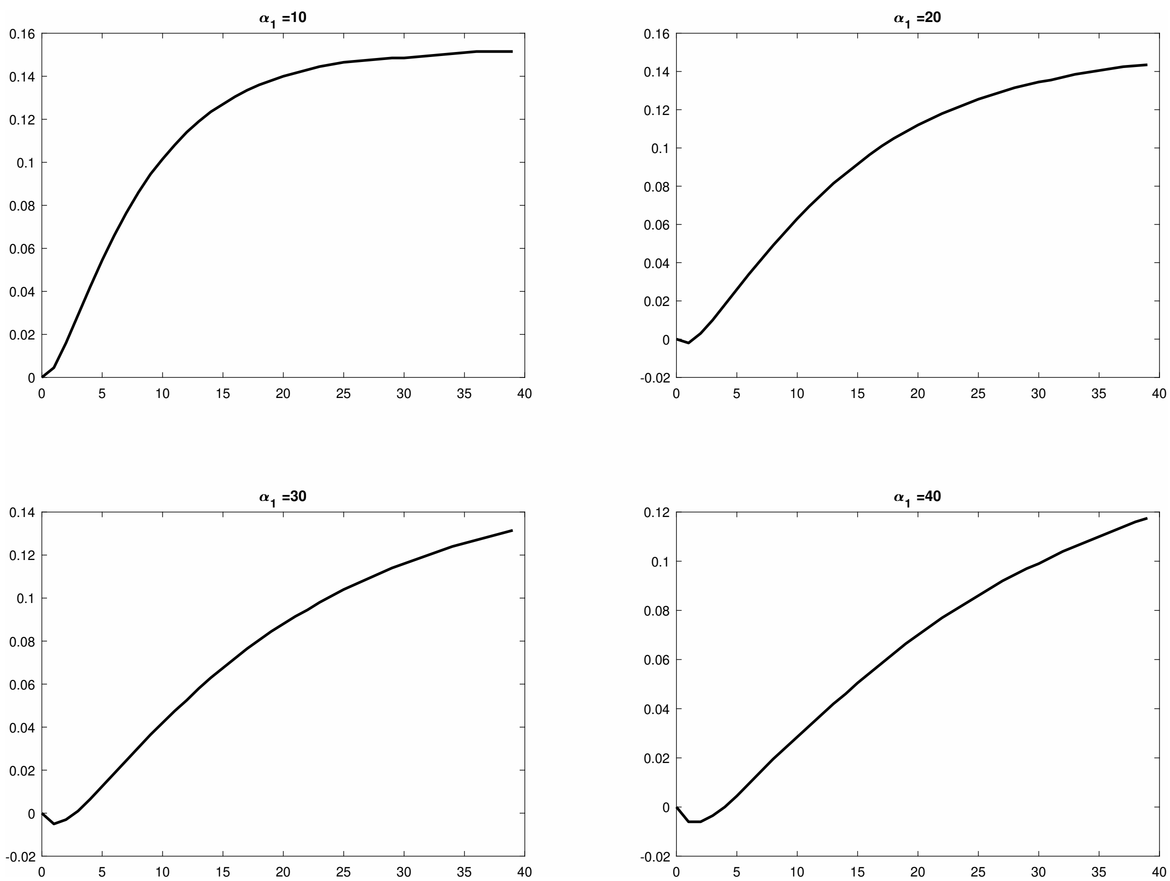

Figure 29: $p_t^R - p_t^O$ for $\gamma_O = 6$