Cartel Penalties Under Endogenous Detection

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Abstract

We compare the performance of cartel penalties proportional to a cartel's revenue and cartel penalties proportional to the difference between the cartel price and the competitive price (the overcharge). Prior literature has shown that when the probability of cartel detection does not depend on the cartel price, penalties based on a cartel's overcharge generate greater total surplus and consumer surplus than penalties based on a cartel's revenue. In contrast, we find that when the probability of detection depends on the cartel price, penalties based on revenue can generate greater total surplus and consumer surplus.

Keywords: Antitrust, Cartels, Cartel Penalties, Collusion

JEL Codes: L4, K2, C7

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1 Introduction

The recent discovery of a number of large and harmful cartels demonstrates that illegal price fixing remains a significant concern for antitrust authorities.[1] To deter cartels, antitrust authorities penalize firms proven guilty of illegal collusion. The structure of the penalties imposed by an antitrust authority affects both cartel formation and pricing.[2] Revenue-based and overcharge-based penalties are two often proposed types of cartel penalties. A revenue-based penalty is a multiple of cartel revenue. An overcharge-based penalty is a multiple of the difference between the collusive price and the competitive price, the overcharge. Revenue-based penalties are current practice in all major jurisdictions.[3]

Recent research (Bageri, Katsoulacos, and Spagnolo 2013; Katsoulacos, Motchenkova, and Ulph (hereafter, KMU) 2015) suggests that cartel penalties based on revenue generate a lower level of surplus than penalties based on the cartel overcharge.[4] This is the case because revenue-based penalties induce cartels to price above the monopoly level in order to reduce revenue and thereby reduce the potential penalty. Intuitively, because a cartel maximizing profits already prices on the elastic part of its demand curve, it increases price to reduce revenue. Conversely, overcharge-based penalties induce cartels to price below the monopoly level in order to reduce the measured overcharge. These studies assume a constant and exogenous probability of cartel detection.

In this study, we compare revenue-based and overcharge-based cartel penalties while allowing the probability of detection to depend on the cartel price.[5] We consider two specifications for the probability of detecting a cartel. In the first specification, the probability of detection is increasing in the cartel overcharge.[6] In the second, the probability of detection is increasing in the rate at which the cartel price increases, as in Harrington (2004, 2005). Intuitively, high prices or large price increases are more likely to attract the attention of an antitrust authority and lead to the cartel's detection. For each specification, we analyze and compare cartel pricing, deterrence, total surplus and consumer surplus under revenue-based and

[1] For example, Taro Pharmaceuticals U.S.A., Inc. and Sandoz, Inc. were fined $205.7 million and $195 million respectively for involvement in a price fixing conspiracy (*U.S. vs. Taro Pharmaceuticals U.S.A, Inc.* (E.D. Pa., No. 2:20-CR-00214-RBS 7/23/20) and *U.S. vs. Sandoz Inc.* (E.D. Pa., No. 2:20-CR-00111-RBS 3/02/20)). Starkist Co. admitted to conspiring with other major canned tuna suppliers to "fix, raise, and maintain the prices of packaged seafood products" and agreed to a fine of $100 million (*U.S. vs. Starkist Co.* (N.D. Ca., No. 3:18-CR-00513-EMC 11/14/18)).

[2] See Block, Nold, and Sidak (1981) and Bolotova, Connor, and Miller (2009) for empirical evidence that expected antitrust penalties affect the cartel price.

[3] International Competition Network Cartels Working Group, "Setting Fines for Cartels in ICN Jurisdictions", Report to the 16th Annual Conference, Porto (2017).

[4] See also: Bageri and Katsoulacos (2014), KMU (2018, 2019a, 2019b), Houba, Motchenkova, and Wen (2010), Garrod and Olczak (2018), and Dargaud, Mantovani, and Reggiani (2016).

[5] A number of cartel cases suggest that the probability of cartel detection is endogenous and depends on the cartel price. In the stainless steel cartel case, an investigation began as a result of buyers reporting a suspiciously large increase in prices to the European Commission (Levenstein, Suslow, and Oswald 2004). Anomalous pricing also created suspicions of collusion in the Nasdaq case (Christie and Schultz 1995). In the auto parts cartel, cartel members specifically set prices in order to avoid detection by buyers *(In re Automotive Parts Antitrust Litigation* (E.D. Mich,. No. 12-md-02311, 03/08/17)).

[6] See Block, Nold, and Sidak (1981), Houba, Motchenkova, and Wen (2010, 2012), Bos, Davies, and Ormosi (2015), Katsoulacos and Ulph (2013), Bos et al. (2018), and Houba, Motchenkova, and Wen (2015).

overcharge-based penalties.

We find that cartel penalties based on revenue generate a higher level of total and consumer surplus when the probability of detection is sufficiently sensitive to price under both specifications. Cartels do not charge prices above the monopoly price because such a high price is likely to raise suspicions of collusion and may cause the cartel's detection. Thus, the pricing result of KMU (2015) does not occur. Instead, cartels set low prices to avoid detection. However, a revenue-based penalty is greater than an overcharge-based penalty for low cartel prices. As a result, collusion is more difficult to sustain under revenue-based penalties. Additionally, cartels facing revenue-based penalties have a stronger incentive to further reduce price to avoid detection and the payment of larger revenue-based penalties.

The rest of the article is organized as follows. Section 2 presents the model. Section 3 compares the two types of penalties when the probability of detection depends on the cartel overcharge. Section 4 compares the two types of penalties when the probability of detection depends on the rate at which the cartel price increases. Section 5 concludes. All proofs can be found in Appendix A.

2 Base Model

The model closely follows KMU (2015, 2020a, 2020b).[7] $N \geq 2$ symmetric firms compete in prices in each of infinitely many periods and have a common discount factor $\delta < 1$. Firms produce homogeneous products at a constant marginal cost $c > 0$. Market demand when firm i charges price p_i is $D(p_1, \ldots p_N) = N\left[1 - \underline{p}\right]$ where $\underline{p}$ is the lowest price charged by firms. Market demand is split evenly between all firms that charge the lowest price. All other firms receive zero demand. Let $D(p) = 1 - p$ denote per-firm demand when all firms charge a common price p. When all firms charge price p, the profit of each firm is $\pi(p) = D(p)\left[p - c\right]$. Demand and cost conditions do not change over time.

Industry suppliers have the opportunity to form an illegal cartel in the beginning of the initial period. If a cartel forms, firms agree to set a common price. An antitrust authority may detect and penalize a cartel. Cartels can be detected in a variety of ways. A cartel could be detected because of a report from an internal whistleblower, a complaint from a buyer, or the discovery of evidence during a merger review or separate antitrust investigation. Let $\phi(p_{t-1}, p_t)$ denote the probability of detection when the price in the current period is p_t and the price in the preceding period is p_{t-1}. We consider two different specifications for ϕ in the following sections. If a cartel is detected, we assume each firm is prosecuted, convicted and penalized with probability 1.[8]

[7] We analyze and build on the model of KMU (2015) for comparability and analytical tractability. However, as the primary forces underlying our main results are more general, our conclusions are expected to hold under other demand or cost assumptions.

[8] We consider leniency programs, which allow firms to report the cartel in exchange for reduced penalties, in the online

We compare two penalty structures: revenue-based penalties and overcharge-based penalties. Revenue-based penalties are of the form $x_R(p) = \gamma_R D(p)p$ where γ_R is a positive constant and p is the cartel price in the period of detection.[9] Overcharge-based cartel penalties are of the form $x_O(p) = \gamma_O D(p^{BF}) \left[p - p^{BF}\right]$ where p^{BF} is the but-for price and γ_O is a positive constant. The but-for price is the price that would prevail in the absence of collusion. Following KMU (2015), we assume the but-for price is the Nash equilibrium price c which implies $x_O(p) = \gamma_O Q_N [p - c]$ where $Q_N = 1 - c$. Following Motta and Polo (2003), KMU (2015), Chen and Rey (2013), and KMU (2020b), we assume a cartel reforms after detection.

A firm will defect from the cartel if the expected present discounted value of the payoff from collusion is less than the payoff from defection. After defection, the market reverts to Nash competition for all future periods. A defecting firm undercuts the cartel price and serves all demand. Defection profit for a firm when all rivals charge price p is[10]

$$\pi^D(p) = \begin{cases} ND(p)\,[p-c] & \text{if } p \leq p^m \\ ND(p^m)\,[p^m - c] & \text{if } p > p^m \end{cases}$$

where p^m is the monopoly price. We assume cartels can only be detected when all firms charge the collusive price (i.e., the cartel cannot be detected in a period where any firm defects or after the breakdown of a cartel).[11] Once a firm has defected, we assume the cartel cannot be detected.[12]

If a cartel forms, firms set a common price to maximize the expected present discounted value of per-firm profit less penalties, subject to the constraint that no cartel member wishes to defect. Cartel prices satisfy the Bellman equation

$$\begin{aligned} V_i(p_{t-1}) = \max_{p\in[c,1]} \; & \pi(p) - \phi(p_{t-1},p)x_i(p) + \delta\left[1 - \phi(p_{t-1},p)\right]V_i(p) + \delta\phi(p_{t-1},p)V_i(c) \\ \text{s.t.} \quad & \pi(p) - \phi(p_{t-1},p)x_i(p) + \delta\left[1 - \phi(p_{t-1},p)\right]V_i(p) + \delta\phi(p_{t-1},p)V_i(c) \\ & \geq \pi^D(p) \end{aligned} \tag{1}$$

where $V_i(p_{t-1})$ denotes the expected present discounted value of the payoff from collusion when the cartel price in the prior period was p_{t-1} and the penalty type is $i \in \{R, O\}$ where R denotes revenue-based penalties and O denotes overcharge-based penalties. We assume the market is competitive prior to cartel formation so

appendix (see https://www.douglascturner.com/endogenous-detection-online-appendix/). Results are qualitatively unchanged.

[9] We assume penalties depend only on the price in the period of detection and do not depend on the length of time that a cartel has operated. See Akyapi and Turner (2021) for an exploration of cartel penalties which depend on duration.

[10] We write defection profit as a function of the cartel price p. If $p \leq p^m$, the defecting firm infinitesimally undercuts the cartel price and the defecting firm's price is $p - \epsilon$. If $p > p^m$, the defecting firm charges a price of p^m.

[11] Motta and Polo (2003), KMU (2015), and Dargaud, Mantovani, and Reggiani (2016) make a similar assumption.

[12] This reflects the fact that, once a cartel has dissolved, evidence of collusion is less likely to be uncovered. Additionally, as the market is now competitive, buyers or other observers are less likely to form or report suspicions of collusion. We explore the possibility of detection after cartel breakdown in the online appendix (https://www.douglascturner.com/endogenous-detection-online-appendix/).

the initial price is the Nash equilibrium price c.[13] We refer to the inequality in equation (1) as an incentive compatibility constraint.

Consumers have discount factor $\delta < 1$. The expected present discounted value of consumer surplus (total surplus) under penalty type $i \in \{O, R\}$ when a cartel forms is denoted CS_i (TS_i). CS_i and TS_i are defined and derived in Appendix B.

A cartel forms if and only if collusion is sustainable and profitable. Collusion is profitable if $V_i(c) > 0$. Collusion is sustainable if there exists a sequence of prices which satisfy the constraint in (1) in every period. As δ increases, the constraint in (1) is more easily satisfied. The critical discount factor δ_i is the discount factor such that a cartel forms under penalty type $i \in \{O, R\}$ for all $\delta > \delta_i$ and does not form under penalty type $i \in \{O, R\}$ if $\delta < \delta_i$.[14] If a cartel does not form, firms engage in Nash competition in all future periods, earning zero profit.

3 Level Detection Specification

In this section, the probability of detection depends on the cartel overcharge. A large overcharge is more likely to attract the attention of an antitrust authority, either directly or through the complaints of buyers or other industry observers, and cause the detection of the cartel.

Let

$$\phi\left(\cdot\,, p_t\right) = \min\left\{\alpha_0 + \alpha_1 \left[p_t - c\right]^2, 1\right\} \tag{2}$$

where $\alpha_1 > 0$ measures the sensitivity of the probability of detection to the cartel overcharge.[15] This specification is not a function of p_{t-1}, the cartel price in the prior period, and depends only on the level of the cartel price in the current period. For example, in industries where a high overcharge is considered anomalous and indicative of collusion, α_1 is high. On the other hand, in industries where a high overcharge is unlikely to attract the attention of an antitrust authority and lead to detection, α_1 is low. $\alpha_0 > 0$ represents the probability of detection that does not depend on the cartel price.[16]

Houba, Motchenkova, and Wen (2010, 2012, 2015) also analyze a specification where the probability of detection is increasing in the cartel price but do not compare revenue and overcharge-based penalties.[17] We

[13]When a cartel reforms after detection, we again consider the initial price to be c. This reflects the expectation of competitive market conditions after detection. A large deviation from marginal cost pricing, or a high value of $\phi(c, p)$, indicates a cartel may have reformed. Alternatively, we could assume the market operates competitively for one period after detection and then a cartel reforms. Results are robust to this alternative assumption.

[14]If a cartel does not form for any discount factor, we set $\delta_i = 1$ as a convention.

[15]The overcharge is squared in Equation (2) for consistency with prior literature (Harrington 2005; Bos, Davies, and Ormosi 2015) and analytical tractability (i.e., concavity of the maximization problem).

[16]Certain mechanisms of cartel detection, such as internal whistleblowers, random auditing or the uncovering of evidence in a separate antitrust investigation, are likely unrelated to the cartel price. α_0 captures the probability of detection due to these mechanisms.

[17]See also: Block, Nold, and Sidak (1981), Bos, Davies, and Ormosi (2015), Katsoulacos and Ulph (2013), and Bos et al.

assume $\gamma_R > 1$ and $\gamma_O > 1$ to ensure penalties are large enough that cartels do not choose a price that would cause immediate detection.[18] We also assume $\alpha_0\gamma_R < 1 - c$ and $\alpha_0\gamma_O < 1$ to ensure expected penalties are not so large as to preclude all collusion.[19]

3.1 Cartel Formation under Price Level Specification

In this section, we analyze the critical discount factor under revenue and overcharge-based cartel penalties when the probability of detection is given by Equation (2). A cartel forms if collusion is sustainable (i.e., no cartel member wishes to defect in any period) and profitable.

Theorem 1. $\delta_O = \frac{N-1}{N} + \frac{\alpha_0\gamma_O}{N}$

$\delta_O < 1$ because $\alpha_0\gamma_O < 1$ by assumption. A cartel forms for any $\delta > \delta_O$. The critical discount factor under overcharge-based penalties does not depend on α_1.

As α_1 increases, cartels must reduce the cartel price to avoid detection (i.e., $p \to c$). However, a reduction in the cartel price does not make collusion unprofitable. This is the case because while a reduction in the cartel price diminishes profit, it also causes a commensurate drop in the overcharge-based penalty. Cartel profit approaches 0 as $p \to c$ but, as illustrated in Figure 1, overcharge based penalties also approach 0 as $p \to c$. This is the case because both a cartel's profit and overcharge-based penalty are tied to the same outcome—the markup $p - c$. In other words, the benefit (cartel profit) and cost (the expected penalty) of collusion decrease in tandem as the cartel price declines. Thus, although the payoff from collusion declines as $\alpha_1 \to \infty$, it remains positive if the cartel charges a sufficiently low price.

Turning to sustainability, a similar intuition holds. As $\alpha_1 \to \infty$, the cartel price falls which reduces profits. However, the payoff from defection is also reduced proportionately. Thus, incentives to defect are limited as $\alpha_1 \to \infty$, and collusion remains sustainable provided firms are sufficiently patient (i.e., $\delta > \delta_O$). Therefore, the critical discount factor above which a cartel forms, δ_O, does not depend on α_1.

Theorem 2. $\delta_R \to 1$ *monotonically as* $\alpha_1 \to \infty$.[20]

Under revenue-based penalties, the critical discount factor increases as the sensitivity of the probability of detection to the cartel price increases. When the probability of detection is highly sensitive to price, cartels must set low prices to avoid detection (as was the case under overcharge-based penalties). However,

(2018).

[18] If $\gamma_R < 1$ or $\gamma_O < 1$, cartels could profitably collude by ignoring the possibility of raising suspicions of collusion and setting prices such that the cartel is detected with certainty in every period (i.e., a price greater than $c + \sqrt{\frac{1-\alpha_0}{\alpha_1}}$). Immediate detection after one period of collusion is inconsistent with empirical evidence (Levenstein and Suslow 2006). Additionally, the examples in footnote 5 suggest cartels do not ignore the possibility of raising suspicions of collusion.

[19] The continued detection of illegal cartels suggests this is a reasonable assumption.

[20] Recall that no cartel forms for any δ when $\delta_R = 1$.

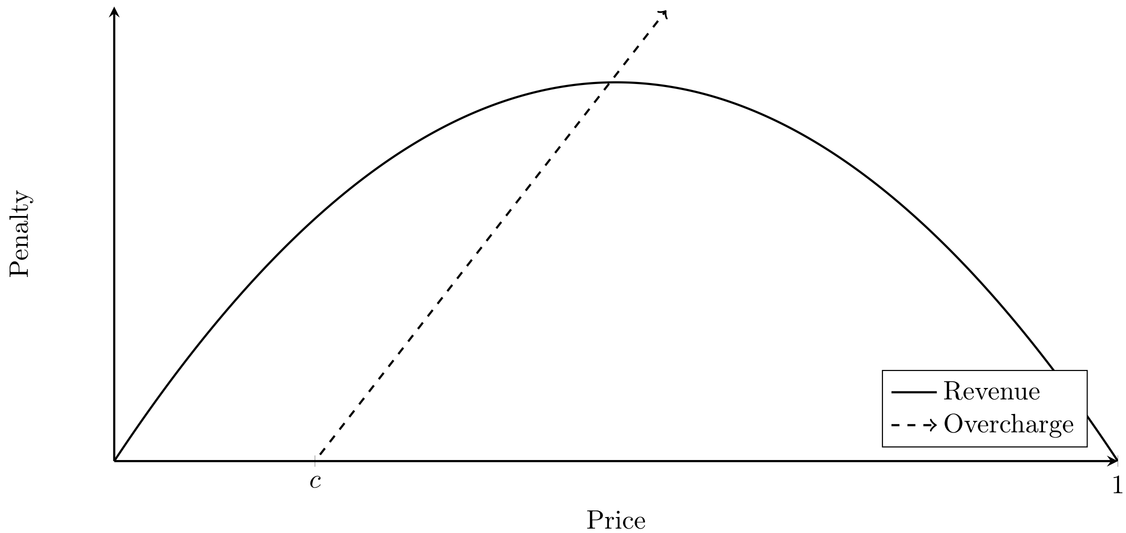

Figure 1: Revenue-based Penalty ($\gamma_R p(1-p)$) and Overcharge-based Penalty ($\gamma_O(p-c)$)

in contrast to the setting with overcharge-based penalties, collusion is unprofitable under revenue-based penalties when the cartel price is low.

To understand why, observe that the per-period profit from collusion (i.e., $\pi(p)$) approaches 0 as $p \to c$. However the penalty to be paid if detected (i.e., $\gamma_R pD(p)$) approaches $\gamma_R cD(c) > 0$ as $p \to c$ (see Figure 1) because, even when the cartel price is low, revenue is strictly positive. Thus, the expected penalty is greater than the per-period profit from collusion when the cartel price is sufficiently low (which occurs when α_1 is sufficiently large) which implies collusion is unprofitable and a cartel does not form.[21]

Intuitively, revenue-based penalties more effectively penalize cartels setting low prices because, as shown in Figure 1, revenue-based penalties are greater than overcharge-based penalties for low cartel prices. However, overcharge-based penalties are typically more effective at penalizing cartels setting high prices because, as shown in Figure 1, overcharge-based penalties exceed revenue-based penalties for high cartel prices. Crucially, when the probability of detection is highly sensitive to the cartel price, cartels must set low prices to avoid detection. Therefore, revenue-based penalties are superior and deter the formation of a greater number of cartels.

3.2 Cartel Pricing under Price Level Specification

In this section, we assume a cartel forms under both penalty types and compare cartel prices under revenue and overcharge-based penalties. Note that the cartel's problem under the ϕ specification in equation (2) is

[21] Theorem 2 can also be understood by analyzing the ratio of a cartel's penalty to its per-period profit: $\frac{x_R(p)}{\pi(p)} = \frac{\gamma_R D(p)p}{D(p)(p-c)} = \frac{\gamma_R p}{p-c}$. As $\alpha_1 \to \infty$ and $p \to c$, this ratio approaches ∞ which implies that the penalty from collusion becomes infinitely larger then per-period profit.

identical in every period because the probability of detection does not depend on the cartel price in the prior period p_{t-1}. Therefore, the cartel sets the same price in each period.

Let p_O denote the price under overcharge-based penalties and let p_R denote the price under revenue-based penalties.[22] KMU (2015) show that $p_O < p^m < p_R$ if $\alpha_1 = 0$. Intuitively, cartels increase price above the monopoly price in order to reduce revenue under revenue-based penalties and set a price below the monopoly price in order to reduce the overcharge under overcharge-based penalties. However, we show in this section that cartel prices may be lower under revenue-based penalties when $\alpha_1 > 0$.

Theorem 3. *There exists an* $\bar{\alpha}_1^L$ *such that* $p_R < p_O$ *if* $\alpha_1 > \bar{\alpha}_1^L$ *and a cartel forms under both penalty types.*

Theorem 3 implies that prices are lower under revenue-based penalties than under overcharge-based penalties when a cartel forms under both penalty types and the probability of detection is sufficiently sensitive to the cartel overcharge.

As KMU (2015) show, cartels set a high price under revenue-based penalties (i.e., $p_R > p^m$) when $\alpha_1 = 0$. However, when α_1 is sufficiently large, such a high price is likely to raise suspicions of collusion and cause the cartel's detection. Instead, cartels set low prices when α_1 is high to reduce the likelihood of detection. When cartel prices are low (i.e., close to marginal cost), revenue-based penalties are strictly larger than overcharge-based penalties (see Figure 1). Faced with the possibility of a relatively large penalty if detected, cartels under revenue-based penalties further reduce price to avoid detection and penalization.

Intuitively, the sensitivity of the probability of detection to the price level compels cartels to set prices in a region (i.e., prices relatively close to marginal cost) where revenue-based penalties are large. Recognizing this, cartels facing revenue-based penalties are more fearful of detection and, as a result, reduce their price below the price set under overcharge-based penalties to disguise collusion and avoid detection.

3.3 Surplus under Price Level Specification

In this section, we compare total surplus and consumer surplus under overcharge-based penalties and revenue-based penalties. There are two cases to consider: $\delta \leq \delta_O$ and $\delta > \delta_O$. First, we consider the case of $\delta \leq \delta_O$. Theorem 2 implies that $\delta_R > \delta_O$ if α_1 is sufficiently high. Therefore, no cartel forms under either penalty type and both penalties generate the same level of total and consumer surplus when $\delta \leq \delta_O$ and the probability of detection is sufficiently sensitive to the cartel price.

Next, we consider the case of $\delta > \delta_O$. When $\delta > \delta_O$, a cartel forms under overcharge-based penalties. However, a cartel does not form under revenue-based penalties when α_1 is sufficiently high, by Theorem 2.

[22] Recall that the price is c if a cartel does not form. Also, note that the cartel price must exceed marginal cost if a cartel forms because collusion must be profitable.

Therefore, both total and consumer surplus are greater under revenue-based penalties than under overcharge-based penalties when $\delta > \delta_O$ and the probability of detection is sufficiently sensitive to the cartel price.

Theorem 4. *When $\delta > \delta_O$, there exists an $\tilde{\alpha}_1^L$ such that $CS_R > CS_O$ and $TS_R > TS_O$ if $\alpha_1 > \tilde{\alpha}_1^L$. When $\delta \leq \delta_O$, there exists an $\tilde{\alpha}_1^L$ such that $CS_R = CS_O$ and $TS_R = TS_O$ if $\alpha_1 > \tilde{\alpha}_1^L$.*

In summary, the optimal penalty type depends on α_1. As KMU (2015) show, overcharge-based penalties generate a greater level of surplus when the probability of detection does not depend on the cartel overcharge ($\alpha_1 = 0$). Conversely, surplus is greater under revenue-based penalties when the probability of detection is sufficiently sensitive to the cartel overcharge.

4 Price Change Specification

In this section, the probability of detection depends on the difference between the current period price and the price in the preceding period. We assume large price increases are more likely to lead to detection than smaller increases. As noted above, a large increase in price might create suspicions of collusion among buyers or other industry observers which may be reported to a competition authority.[23] Alternatively, a large and anomalous price increase could directly attract the attention of competition authorities.

Let

$$\phi(p_{t-1}, p_t) = \begin{cases} \min\left\{\alpha_0 + \alpha_1 \left[p_t - p_{t-1}\right]^2, 1\right\} & \text{if } p_t > p_{t-1} \\ \alpha_0 & \text{if } p_t \leq p_{t-1} \end{cases} \tag{3}$$

where $\alpha_1 \geq 0$ represents the sensitivity of the probability of detection to the price increase.[24] α_0 represents the probability of detection when the cartel price is constant or decreasing. Harrington (2004, 2005) first introduced and analyzed a similar specification, but these studies do not consider revenue-based penalties. As in the previous section, we assume $\gamma_R > 1$ and $\gamma_O > 1$ to ensure a cartel does not choose a price that would cause detection with probability 1. Additionally, we assume $\alpha_0\gamma_R < 1 - c$ and $\alpha_0\gamma_O < 1$ to ensure expected penalties are not so large as to preclude all collusion.

4.1 Cartel Formation under Price Change Specification

In this section, we analyze the critical discount factor under revenue and overcharge-based cartel penalties when the probability of detection is given by Equation (3). A cartel forms if collusion is sustainable (i.e., no cartel member wishes to defect in any period) and profitable.

[23] See Harrington (2004, 2005) for further discussion.

[24] The change in price is squared in Equation (3) for consistency with prior literature (Harrington 2004, 2005) and analytical tractability (i.e., concavity of the maximization problem).

Theorem 5. $\delta_O = \frac{N-1}{N} + \frac{\alpha_0 \gamma_O}{N}$

$\delta_O < 1$ because $\alpha_0 \gamma_O < 1$ by assumption. A cartel forms for any $\delta > \delta_O$. As was the case under the price level specification, the critical discount factor under overcharge-based penalties does not depend on α_1.

As α_1 increases, cartels must increase the cartel price more gradually to avoid detection (i.e., $p_t \to p_{t-1}$). Raising the cartel price more gradually effectively reduces the cartel price in each period. However, the more gradual price increase does not render collusion unprofitable. This is the case because while increasing the cartel price more gradually diminishes profit, it also causes a commensurate drop in the overcharge-based penalty the cartel faces in each period. In other words, the benefit (cartel profit) and cost (the expected penalty) of collusion decrease in tandem as the cartel price rises more slowly. Thus, while the payoff from collusion declines as $\alpha_1 \to \infty$, the payoff from collusion remains positive if the cartel increases price at a sufficiently slow rate.

Turning to sustainability, a similar intuition holds. As $\alpha_1 \to \infty$, the cartel price falls in each period, which reduces profit. However, the payoff from defection is also reduced proportionately. Thus, incentives to defect are limited as $\alpha_1 \to \infty$ and collusion remains sustainable if firms are sufficiently patient (i.e., $\delta > \delta_O$). The critical discount factor above which a cartel forms, δ_O, does not depend on α_1.

Theorem 6. $\delta_R \to 1$ *monotonically as* $\alpha_1 \to \infty$.[25]

Under revenue-based penalties, the critical discount factor is increasing in the sensitivity of the probability of detection to changes in the cartel price. When the probability of detection is highly sensitive to changes in the cartel price, cartels must increase price slowly in order to avoid detection (as was the case under overcharge-based penalties). However, unlike under overcharge-based penalties, collusion becomes unprofitable under revenue-based penalties when the cartel is forced to increase price slowly.

To see this, first note that gradually increasing the cartel price effectively reduces the price in each period. Next, recall that the per-period profit from collusion (i.e., $\pi(p_t)$) approaches 0 as $p_t \to c$. However the penalty (i.e., $\gamma_R p_t D(p_t)$) approaches $\gamma_R c D(c) > 0$ as $p_t \to c$ (see Figure 1) because, even when the cartel price is low, revenue is strictly positive. Thus, the expected penalty is greater than the per-period profit from collusion when the cartel price is sufficiently low.[26]

As $\alpha_1 \to \infty$, the cartel is forced to price in a region where the expected penalty exceeds cartel profit (i.e., low prices) in an increasingly large number of periods. Rapidly increasing price to a level where collusion is profitable would cause detection. Thus, as α_1 increases, the cartel incurs a per-period loss in more and more periods and, for sufficiently large α_1, collusion is altogether unprofitable and a cartel does not form.

[25] Recall that no cartel forms for any δ when $\delta_R = 1$.

[26] Per-period profit in period t is $\pi(p_t)$ and the expected penalty in period t is $\phi(p_{t-1}, p_t) x_R(p_t)$. As $p_t, p_{t-1} \to c$, $\pi(p_t) \to \pi(c) = 0$ and $\phi(p_{t-1}, p_t) x_R(p_t) \to \alpha_0 x_R(c) > 0$.

4.2 Cartel Pricing under Price Change Specification

In this section, we assume a cartel forms under both penalty types and compare cartel prices under revenue and overcharge-based penalties. Under the ϕ specification in (3), the Bellman equation in (1) does not readily generate tractable closed-form solutions or analytical results. However, cartel prices under each penalty type can be determined through numerical solutions. Specifically, we solve the Bellman equation in (1) using value function iteration.[27] We first analyze this issue in an illustrative baseline setting. Next, we consider modifications of the baseline setting to assess the robustness of our findings.

Baseline Setting. $\delta = .9$, $c = .1$, $N = 2$, $\gamma_R = 5$, $\gamma_O = 3.05$, $\alpha_0 = .05$

The baseline setting represents a duopoly. γ_O and γ_R are chosen to ensure the cartel penalty is the same under the two penalty types when each member of the cartel charges the monopoly price.[28] We consider a range of α_1 values. Let p_i^t denote the cartel price under penalty type $i \in \{O, R\}$ in period t. Figure 2 shows that the cartel price is constant and higher under revenue-based penalties than overcharge-based penalties when $\alpha_1 = 0$, as shown in KMU (2015). However, Figure 2 also shows that cartel prices are lower under revenue-based penalties in early periods and lower under overcharge-based penalties in later periods when $\alpha_1 = 5$, $\alpha_1 = 10$ or $\alpha_1 = 15$.

To understand these results, first note cartels increase price gradually as bigger price movements are likely to attract attention and lead to the cartel's detection. As Figure 1 shows, revenue-based penalties exceed overcharge-based penalties when the cartel price is low. Thus, a cartel detected in one of the first few periods of collusion pays a large penalty under revenue-based penalties and a smaller penalty under overcharge-based penalties. Recognizing this, cartels facing revenue-based penalties increase the cartel price more slowly to disguise collusion and avoid the payment of relatively high penalties. Conversely, cartels facing overcharge-based penalties are less wary of rapidly increasing their price and raising suspicions of collusion because they face a smaller penalty. These considerations cause the cartel price to be lower under revenue-based penalties in early periods of collusion.

In later periods of collusion, $p_t^R > p_t^O$. This occurs because the cartel price converges to the price derived in KMU (2015) (i.e., the cartel price when $\alpha_1 = 0$) as $t \to \infty$.[29] Increasing the cartel price any further would reduce the payoff from collusion. Thus, $p_t^R > p_t^O$ in later periods when the cartel price has converged to its limiting value. We demonstrate the robustness of our results to alternative parameter configurations in the

[27] See the online appendix (https://www.douglascturner.com/endogenous-detection-online-appendix/) for details.

[28] The monopoly price is .55. The overcharge-based cartel penalty when $p = .55$ is $3.05(.55 - .1)D(.1) \approx 1.24$. The revenue-based cartel penalty when $p = .55$ is $5(.55)D(.55) \approx 1.24$.

[29] Harrington (2004) makes a similar observation regarding the limiting (or steady-state) cartel price (see Theorem 2 in Harrington (2004)). Recall that, as KMU (2015) showed, the cartel price under revenue-based penalties is greater than the cartel price under overcharge-based penalties when the probability of detection is constant.

online appendix.[30]

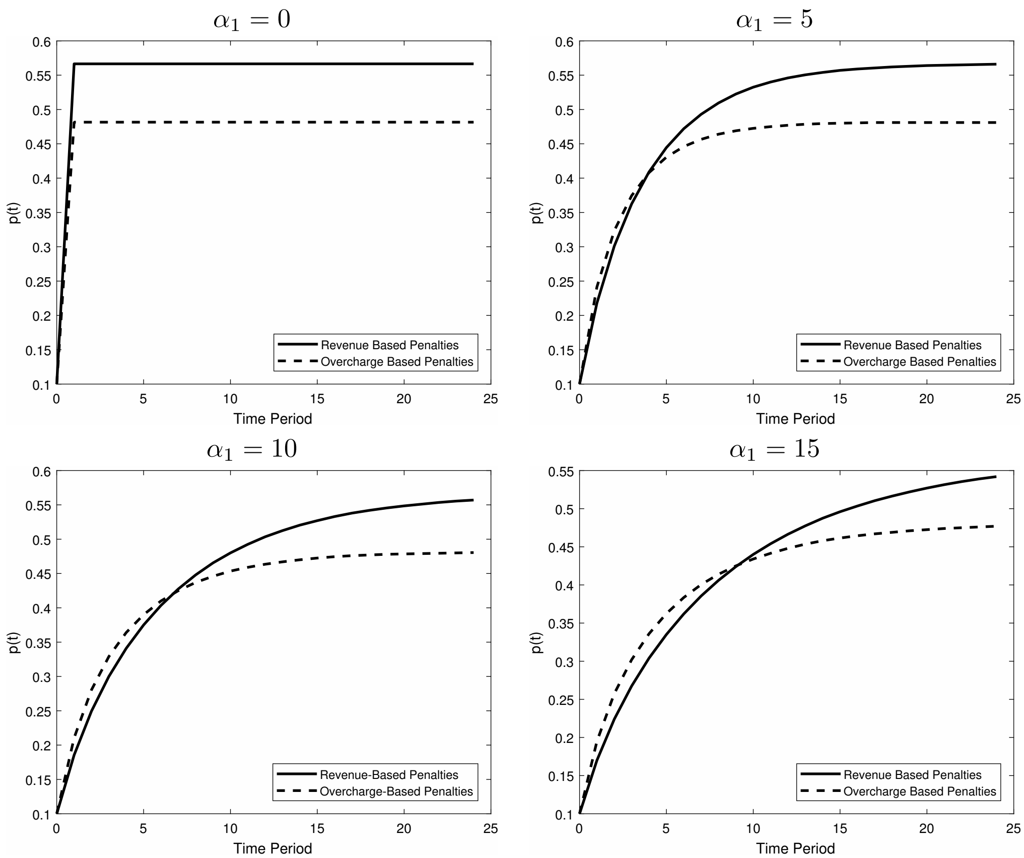

Figure 2: Cartel Prices in the Baseline Setting

In summary, the following conclusion arises in every numerical solution we conducted.

Conclusion 1. If α_1 is sufficiently large and a cartel forms under both penalty types, then $p_t^R < p_t^O$ in early periods and $p_t^R > p_t^O$ in later periods.

4.3 Surplus under Price Change Specification

In this section, we compare total and consumer surplus under overcharge-based penalties and revenue-based penalties. There are two cases to consider: $\delta \leq \delta_O$ and $\delta > \delta_O$. First, consider the case of $\delta \leq \delta_O$. Theorem 6 implies that $\delta_R > \delta_O$ when α_1 is sufficiently high. Therefore, no cartel forms under either penalty type and both penalties generate the same level of total and consumer surplus when $\delta \leq \delta_O$ and the probability of detection is sufficiently sensitive to the change in cartel price.

If $\delta > \delta_O$, a cartel forms under overcharge-based penalties. However, a cartel does not form under revenue-based penalties when α_1 is sufficiently high, by Theorem 6. Therefore, both total and consumer

[30] See https://www.douglascturner.com/endogenous-detection-online-appendix/.

surplus are greater under revenue-based penalties than under overcharge-based penalties when $\delta > \delta_O$ and the probability of detection is sufficiently sensitive to the change in cartel price.[31]

Theorem 7. *When $\delta > \delta_O$, there exists an $\tilde{\alpha}_1^C$ such that $CS_R > CS_O$ and $TS_R > TS_O$ if $\alpha_1 > \tilde{\alpha}_1^C$. When $\delta \leq \delta_O$, there exists an $\tilde{\alpha}_1^C$ such that $CS_R = CS_O$ and $TS_R = TS_O$ if $\alpha_1 > \tilde{\alpha}_1^C$.*

As under the price level specification, the optimal penalty type depends on α_1. As KMU (2015) show, overcharge-based penalties induce a greater level of surplus when the probability of detection does not depend on the cartel price ($\alpha_1 = 0$). Conversely, surplus is greater under revenue-based penalties when the probability of detection is sufficiently sensitive to price increases.

5 Conclusion

We have examined revenue and overcharge-based cartel penalties when the probability of detection depends on the cartel price. We have compared the two penalty structures on the basis of both consumer surplus and total surplus. First, we consider a model in which the probability of detection depends on the cartel overcharge. Next, we analyze a model in which the probability of detection depends on the rate at which the cartel price increases. Under both models, we obtain a similar result: when the probability of detection is sufficiently sensitive to the cartel price, both consumer surplus and total surplus are higher under revenue-based penalties.

In summary, which penalty structure secures a higher level of surplus depends crucially on the detection process. In industries where a large price increase or a high cartel overcharge is considered anomalous, indicative of collusion and likely to lead to detection, revenue-based cartel penalties generate a higher level of surplus. For example, in a stable industry which does not experience significant price variation when competitive, a large price increase is likely to attract the attention of an antitrust authority and lead to detection. In this case, revenue-based penalties are optimal.[32]

In industries where a large price increase or high overcharge is not likely to lead to detection, overcharge-based penalties generate a higher level of surplus. For instance, in a turbulent industry which experiences regular price variation even when competitive, a large price increase is unlikely to raise suspicions of collusion. In this case, overcharge-based penalties are optimal.

[31] The proof of Theorem 7 does not depend on the numerical results of Section 4.2, see Appendix A.

[32] Revenue-based penalties have an additional advantage. Revenue-based penalties are less costly to calculate than alternative penalty types, because they require no information about a firm's marginal cost or but-for price. Information on firm revenue is often readily available to investigators.

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