Price Index Manipulation

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Abstract

In many industries, supply is procured through long-term contracts that tie purchase prices to a price index of spot market transactions. These indexes are published by third parties known as price reporting agencies (PRAs). Recent antitrust cases allege that suppliers have manipulated these indexes by deceiving PRAs or engaging in anticompetitive conduct in the spot market, prompting new regulations governing the reporting of spot transactions and the design of indexes. We study an oligopoly model of spatially differentiated producers that sell both through index-linked contracts and in the spot market that determines the index. Index-linked contracts raise Nash equilibrium prices, support higher collusive prices, and may result in suppliers refusing to deal with low-valuation buyers. However, recent regulations that prevent the concealment or misreporting of spot market transactions can raise prices and reduce buyer surplus. When buyers differ in size, the common practice of excluding small transactions from the index is suboptimal: weighting all transactions by volume yields the lowest prices and the highest buyer surplus.

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1 Introduction

In volatile product markets, prices fluctuate daily and even hourly, making it impractical for regular purchasers and their suppliers to re-negotiate a fixed price for every delivery. Instead, the two parties sign an "index-linked contract," under which the price of each delivery is tied to a price index published by an independent third party known as a price reporting agency (a "PRA"). A PRA samples spot market transactions and routinely publishes an estimate of the prevailing spot price in a market: the index. However, the spot markets underlying these indexes are often thin, with only a small number of transactions occurring in a given period. Moreover, producers selling in the spot market frequently also supply through index-linked contracts and therefore have a vested interest in raising the index. In the US natural gas industry, for example, many supply contracts set the price of each month's deliveries equal to a published index, and that index is calculated from the prices of spot sales reported by producers (Nasta, 2016; Johnson, 2017). A series of recent antitrust cases and government investigations document suppliers manipulating[1] price indexes by colluding in the spot market, reporting fake trades to the PRA, or selectively reporting trades (see Subsection 2.2 for detailed case examples).

We study a model of competition between producers that supply buyers through index-linked contracts and also sell in the spot market that determines the index. Producers are spatially differentiated and make private, personalized price offers to spot buyers. The PRA then samples spot transactions and publishes the index determining the price of contract sales. Consistent with prior literature (Xia and Sexton, 2004; Xia, Crespi and Dhuyvetter, 2019; Garrido et al., 2022*b*), we find that index-linked contracts relax competition in the spot market. Undercutting a rival to steal a spot buyer lowers the index and reduces the deviator's profits from its index-linked contracts. As a result, producers price less aggressively under index-linked contracts and Nash equilibrium prices rise. A similar effect occurs under collusion: a deviation from a collusive agreement depresses the index and reduces a deviator's profit. Index-linked contracts therefore reduce incentives to cheat, stabilize collusion, and result in higher collusive prices (Hatfield and Lowery, 2023).

The conduct documented in recent cases, however, goes beyond raising prices through competition or collusion. Firms have concealed transactions from PRAs, reported fake prices at which they did not transact, and refused to deal with certain buyers, each with the apparent aim of raising the index. We analyze these practices in a series of extensions to the baseline model, while also studying regulations and index designs intended to prevent the manipulation of price indexes, many of which have been enacted or are actively

[1] We use the term "manipulation" loosely in this article to refer to any means of artificially inflating a price index, including practices such as price-fixing, concealing transactions, dishonestly reporting transactions, or refusing to supply certain buyers. Such conduct does not necessarily imply "market manipulation" in the legal sense (Fischel and Ross, 1991; Verstein, 2014).

employed by PRAs. In Section 4, we study an extension of the baseline model wherein spot buyers differ in their valuations of the good. We find that sellers may collectively refuse to supply low-valuation buyers while charging an elevated price to high-valuation buyers, and that such equilibria arise only when index-linked contracts are pervasive. These equilibria are the result of a subtle effect: refusing to supply a subset of buyers reduces the number of spot market transactions occurring (and sampled by the PRA), which makes the index more sensitive to a deviation. This effect reduces incentives to undercut rivals and supports higher prices for high-valuation buyers, benefiting suppliers.

Recent antitrust and market manipulation cases allege firms selectively reported their spot market transactions to PRAs to increase the index.[2] We next consider an extension of the baseline model wherein sellers can conceal transactions from the PRA and prevent their inclusion in the index. When concealment is possible, a deviating seller can steal a rival's spot demand while hiding the resulting transactions from the PRA, leaving the index unchanged. Allowing concealment consequently reduces Nash equilibrium prices and, in many cases, incentives to collude. Recent regulation encourages PRAs to prevent concealment and ensure that firms submitting spot prices report data from all of their transactions. Our results identify a potential anticompetitive effect of such comprehensive-reporting requirements.

Sellers have also manipulated indexes by dishonestly reporting their spot transactions.[3] In a third extension (Section 6), sellers may report a price to the PRA that differs from their actual average transaction price. Perhaps surprisingly, sellers' ability to lie to the PRA by reporting an inflated spot price can lower Nash equilibrium spot prices, lower the expected value of the index, reduce producer profit, and increase buyer surplus. This is the case as the downward impact of undercutting a rival on the index can be offset by reporting an untruthful, elevated price. This mechanism intensifies Nash competition and lowers prices. An increase in the probability that lying is detected, or in the penalty for dishonest reporting, can raise prices, increase seller profit, and reduce buyer surplus. Recent regulatory proposals encourage PRAs to exert additional effort to detect and penalize untruthful price reports. Our results suggest that this recommendation may, in some circumstances, harm buyers.

Finally, we examine the optimal design of an index when buyers differ in their demand for the good (Section 7). Sales to small buyers present a natural instrument for distorting the index. When all transactions are included in the index and weighted equally, the gain from stealing a small buyer is modest relative to the transaction's downward impact on the index. Sellers therefore have little incentive to undercut one another for small buyers, and the prices of small transactions are easily inflated. For this reason, many PRAs exclude

[2]See, for example, ongoing caustic soda litigation (Precious Plate v. Olin, 2019) and a recent gasoline market manipulation case in California (Cal. AG, 2020).

[3]See, for example, allegations of index manipulation in the broiler chicken (In re Broiler Chicken, 2016) and natural gas (Johnson, 2017) industries.

small transactions from the index's determination. In a model with small and large buyers, we find that exclusion is not optimal in the sense of yielding the lowest prices and highest buyer surplus. Excluding small transactions shifts weight onto large ones, increasing sellers' incentives to distort the index through those sales. Weighting transactions in proportion to their size, without excluding any spot sales, maximizes buyer surplus.

Related Literature Johnson (2017) provides a detailed and comprehensive review of PRAs, their activities, and PRA regulation. He notes that "[a]cademic coverage of the price reporting agencies (PRAs) and the benchmarks that they provide has been limited," which may reflect "the generally low profile that PRAs maintain."[4]

Xia and Sexton (2004); Xia, Crespi and Dhuyvetter (2019); Garrido et al. (2022*b*) and Moschini and Smith (2026) identify and examine anticompetitive effects of index-linked contracts (known as "Top of the Market Pricing" or "Alternative Marketing Arrangements") in cattle procurement markets.[5] They find that cattle buyers have an incentive to depress spot market prices in order to reduce the price paid through these contracts. Garrido et al. (2022*b*) also provide empirical evidence of this effect, documenting that the emergence of index-linked contracts is associated with lower spot prices for cattle. Hatfield and Lowery (2023) show theoretically that index-linked contracts can facilitate collusion between meat packers (i.e., cattle buyers), increasing their profits and harming cattle producers. While their model is tailored to the beef-processing industry, their insights regarding the impact of index-linked contracts on incentives to collude are more general.

As in the preceding studies, index-linked contracts relax spot market competition and facilitate collusion in our model (see the baseline model in Section 3). However, unlike the preceding articles, this article focuses on practices (documented in recent antitrust cases) that distort the index beyond what conventional competition or collusion in the spot market would produce. Specifically, we examine conduct intended to artificially manipulate price indexes such as the dishonest reporting of transactions, concealing transactions, or refusing to deal with certain buyers. Additionally, we explore the efficacy and design of regulation intended to prevent index manipulation of this kind.

Forward positions, much like index-linked contracts, alter producers' incentives in the spot market (Allaz and Vila, 1993; Mahenc and Salanié, 2004; Liski and Montero, 2006; Green and Le Coq, 2010). Crucially, forward contracts fix the price of future sales at the time the contract is signed while index-linked contracts peg the purchase price to the value of a price index published by a third party, which is not fixed and varies over the length of the contract. An insightful literature in finance studies the manipulation of financial

[4]Boyd (2020) makes a similar observation, noting that "[g]iven their importance, it is remarkable how little we know about [price indexes]—how they are made, how they are used, and how they can be manipulated."

[5]See, also, MacDonald (2006).

indexes (Duffie and Dworczak, 2021; Baldauf, Frei and Mollner, 2022; Zhang, 2022). These studies employ models that differ markedly from ours and are tailored toward the manipulation of financial benchmarks by traders holding derivative positions (such as swaps, futures, and options), rather than index distortion by oligopoly producers. Additionally, our focus is the impact of index-linked contracts and related regulation on buyer surplus, while these studies focus on the accuracy of the benchmark as an estimator of fair market value and the settlement-price risk that manipulation imposes on hedgers and traders.

Section 2 provides background information on PRAs, index-linked contracts, the industries PRAs serve, and recent index manipulation cases. The base model is presented in Section 3. Sections 4, 5, and 6 analyze refusals to deal, the concealment of transactions, and the dishonest reporting of transactions, respectively. The case of spot buyers who differ in their demand for the product is studied in Section 7. Section 8 concludes. Proofs of formal conclusions are contained in Appendix A. Extended proofs and robustness checks are presented in the Technical Appendix.

2 Background

2.1 PRAs and Index-Linked Contracts

In many industries, the majority of supply is procured through long-term contracts referencing an estimate of the market price provided by a third party (Johnson, 2017), known as an index-linked contract. Index-linked contracts are prevalent in, for example, the natural gas, styrene, ethylene, aluminum, cattle, steel, broiler chicken and wholesale fuel industries.[6] An example of an index-linked contract is presented in Technical Appendix C.1. Index-linked contracts have a number of benefits. First, they reduce transaction and negotiation costs for repeat buyers of a product (Johnson, 2017; Levenstein and Suslow, 2023). Second, by signing a long-term contract, the buyer eliminates the risk of failing to secure a supplier and losing access to the input (Garrido et al., 2022*b*). Price indexes can also improve matching efficiency and increase participation by less-informed agents (Duffie and Stein, 2015).

There are over 100 PRAs, but the five largest are S&P Global Platts, Argus Media, Oil Price Information Service (OPIS), Independent Commodity Intelligence Services (ICIS), and Fastmarkets. Thousands of price indexes are published by PRAs every day across a wide variety of commodities markets. The price indexes PRAs publish influence billions of dollars in trade (Johnson, 2017). At its simplest, the calculation of an index involves collecting invoices voluntarily submitted by market participants (buyers and sellers) and calculating the average price across these transactions. In practice, index methodologies vary considerably

[6]See Nasta (2016) (natural gas), European Commission (2022*b*) (styrene), European Commission (2020) (ethylene), Tita (2025) (aluminum), Garrido et al. (2022*a*) (cattle), London Metal Exchange (2025*a*) (steel), In re Broiler Chicken (2016) (broiler chicken), and Bundeskartellamt (2025) (wholesale fuel).

across market and PRA, ranging from highly informal and subjective to entirely automated, from extensively verified to virtually no verification or auditing of any kind, and from simple arithmetic averages to complex trimmed and weighted means/medians. Submitting information to a PRA is entirely voluntary (European Commission, 2020; Johnson, 2017; Reed Smith, 2025).

Academics, regulators, and antitrust authorities have expressed concern regarding the risk of market participants manipulating price indexes to their benefit.[7] In response, policy-makers have introduced a number of measures encouraging PRAs to design their indexes so as to reduce the risk of manipulation, including the IOSCO Principles, the EU Benchmark Regulation, and a variety of sector specific regulations. Relevant regulations will be introduced and analyzed in subsequent sections. Index manipulation regulation is surveyed in detail in Technical Appendix C.2.

2.2 Price Index Manipulation Case Examples

In this subsection, we review a number of recent antitrust cases where firms have manipulated price indexes through practices such as price fixing, dishonest price reporting, concerted refusals to deal, or selective reporting of transactions.

Caustic Soda. IHS Markit compiles a monthly caustic soda[8] price index by consulting buyers and sellers, and more than 60% of the caustic soda sold in the United States is priced at least in part off this index. In a private antitrust suit covering 2015 to 2019, caustic soda buyers allege producers agreed to manipulate the index by selectively reporting prices to IHS Markit, and in some instances by submitting false prices outright (Precious Plate v. Olin, 2019). The litigation is ongoing, though several defendants have settled.

Styrene. From 2012 to 2018, a cartel of styrene buyers manipulated a monthly styrene index published by ICIS and referenced formulaically in long-term supply contracts. Cartel members colluded on the price they wished to achieve in their bilateral negotiations with sellers, coordinating the transactions that fed into the index so as to depress it. In 2022, the European Commission imposed fines on five participating firms for cartel activity (European Commission, 2022*b*).

Ethanol. Platts publishes a daily ethanol benchmark based on transactions concluded during a short "Market on Close" window. Long-term ethanol supply contracts are priced formulaically off the Platts index. Between roughly 2011 and 2014, two producers coordinated their conduct before, during, and after this window, agreeing to limit the supply of physical ethanol in the Rotterdam area that could enter the assessment

[7]The German competition authority, in its review of the German wholesale fuel industry, concluded that "the way [price indexes] are currently compiled poses considerable risks to competition" (Bundeskartellamt, 2025). The International Organization of Securities Commissions (IOSCO), the global standard-setting body for securities regulators, noted that "there may be conflicts of interests in and incentives to manipulate the determination process where the Submitters are also Market Participants with stakes in the level of the [indexes]" (IOSCO, 2013).

[8]Caustic soda (sodium hydroxide) is an industrial chemical used in alumina refining, pulp and paper production, water treatment, and chemical manufacturing.

and exchanging commercially sensitive information to sustain the scheme (European Commission, 2022*a*). By restricting the trades that reached the window, the cartel raised the benchmark and the contract prices tied to it. The European Commission imposed fines of €20 million and €47.7 million on the participants.

Broiler Chicken. The Georgia Dock, a weekly whole-bird chicken benchmark published by the Georgia Department of Agriculture until its suspension in late 2016, was widely referenced in supply contracts. Unlike comparable benchmarks such as those of Urner Barry and the USDA, which cross-checked submissions against buyers, the Georgia Dock relied on producers' reported prices without independent verification. Plaintiffs allege that producers exploited this lack of oversight by collectively submitting artificially high and nearly identical prices, inflating the index (In re Broiler Chicken, 2016).

A number of further cases follow a similar pattern, including the manipulation of price indexes for Atlantic salmon (Asmat et al., 2023), beef (Garrido et al., 2022*b*), bananas (European Commission, 2008), shrimp (European Commission, 2013), ethylene (European Commission, 2020), gasoline (Levenstein and Suslow, 2023), eggs (DOJ, 2026), and natural gas (FERC, 2016).[9] See Levenstein and Suslow (2023) for a review and discussion of prominent cases of price index manipulation.

2.3 Industry Characteristics

In this section, we review the salient characteristics of the industries where index-linked contracts are prevalent, with a particular focus on industries where index manipulation has previously occurred. These characteristics will inform the modeling choices in the subsequent section.

Transport Costs and Spatial Differentiation. Index-linked contracts are prevalent in commodity and goods markets (e.g., chemicals, oil, perishable food) where shipping is costly and suppliers are spatially differentiated (Johnson, 2017; Zhang, 2022). For example, transport costs are high in the market for fed cattle as cattle lose weight (and therefore value) during the trip (Garrido et al., 2022*b*). In the European styrene industry, where a price index was manipulated between 2012 and 2018, transport costs "form an integral part of the value of purchases of styrene" (European Commission, 2022*b*). Ethylene, an industry where European producers manipulated a price index between 2011 and 2017, is considered costly to transport (European Commission, 2022*b*, 2020). Transportation costs for natural gas, another industry where index manipulation has taken place (Johnson, 2017), are exceptionally high (Adelman and Lynch, 2004).[10] Specifically, transport costs may constitute greater than 50% of the total cost of natural gas (Molnar, 2022). Caustic soda is costly

[9] Traders at a number of major banks manipulated an interest rate benchmark known as LIBOR in the 2000s (Duffie and Stein, 2015). Among other practices, manipulation involved banks submitting dishonest judgment-based reports of their borrowing rates. Dishonest reporting of transaction prices is explored in Section 6. However, the model in this article is tailored to physical product markets involving transportation costs, rather than financial markets.

[10] Transport costs for natural gas are significant due to its low density. A shipment takes up a large amount of space (and is therefore costly to transport) relative to its value (Adelman and Lynch, 2004).

to transport as it is a heavy, corrosive chemical that requires specialized tank trucks, railcars, or barges (Denny, 2023; BMT Surveys, 2026).

Due to significant transportation costs, buyers typically purchase from nearby suppliers. In its review of the German wholesale fuel sector (where index-linked contracts are commonplace), the German competition authority found that "suppliers within a radius of up to 200 km are widely accepted as an alternative source of supply. However, more remote suppliers [...] are only considered if the difference in price is significant" (Bundeskartellamt, 2025). Similarly, beef packers typically purchase cattle from nearby producers: "[b]eef packers procured an average of 64% of their cattle within 75 miles of packing plants, 82% within 150 miles, and 92% within 250 miles" (Schroeder, 1997). Prior literature has analyzed and highlighted the importance of spatial differentiation in cattle (Moschini and Smith, 2026), broiler chicken (Vukina and Zheng, 2015), aluminum (Gilbert, 2021) and natural gas industries (Oliver, Mason and Finnoff, 2014).

Thin Spot Markets. In industries where allegations of index manipulation have arisen, a relatively large fraction of demand is procured through index-linked contracts. For example, 80% of natural gas is procured through index linked contracts (Nasta, 2016; Boyd, 2020), as is more than 60% of caustic soda sold in the US (Precious Plate v. Olin, 2019), 60-80% of US cattle purchases (Garrido et al., 2022*b*), between 80 and 95% of broiler chicken sales (In re Broiler Chicken, 2016), and over 50% of all German wholesale fuel supply (Bundeskartellamt, 2025).

Due to the prevalence of index-linked contracts in these industries, only a small fraction of supply is purchased through the spot market. As a result, the index is determined by a relatively small number of spot sales, increasing the risk of manipulation (Bundeskartellamt, 2025; Armstrong, 2024). For example, in the European styrene industry, about 5-10 deals are concluded each week (ICIS, 2015; European Commission, 2022*b*). The Dubai crude oil index is based on an exceptionally small number of actual transactions. Verstein (2014) notes that "half of trading days see literally no trading in Dubai crude in the principal venue that informs the benchmark [index] price." An index for Norwegian salmon, allegedly manipulated by large salmon producers, was based on spot market sales that constituted only 1-2.5% of Norwegian salmon production (Salmon Antitrust Litig., 2021). PRAs explicitly acknowledge that the markets they serve are often highly illiquid and transaction volume may be light or non-existent (e.g., ICIS (2019); IHS Markit (2020)). As a result, price indexes are often determined by the transactions of a small number of sellers (Bundeskartellamt, 2025; Verstein, 2014; European Commission, 2022*b*).

Privately Negotiated Prices. In the industries PRAs serve, spot prices are typically bilaterally and privately negotiated between buyer and seller (Johnson, 2017; Armstrong, 2024; London Metal Exchange, 2025*b*). Thus, the price each buyer pays is personalized and depends on, for example, its location and the location of the seller.

3 Model

Consider a market consisting of two symmetric firms.[11] Both sellers have previously signed index-linked contracts for the delivery of $\alpha \in [0,1)$ units of the good in each period at a price equaling the published price index.[12] The quantity to be delivered in each period is fixed (Rogers and Phua, 2015; Ason, 2022).[13]

Each firm is located in a "city" containing N_B spot market buyers. Each spot buyer demands $(1-\alpha)\, q_{spot}$ units of the good each period where $q_{spot} = \frac{1}{N_B}$. Spot buyers are homogeneous, each with a common per-unit valuation r for the good. Heterogeneous buyers are analyzed in Section 4 and Section 7. If each seller serves all spot buyers within its city, it supplies $1-\alpha$ units through the spot market and its total quantity supplied (across spot and contract sales) is 1. Sellers' marginal costs of production are normalized to 0. The cost of transporting a unit of the good to a neighboring city is Δ. Transport costs to local buyers within a firm's city are normalized to 0. We assume $r > \Delta\left(1 + \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{2}\right)$, which ensures buyers' valuations exceed the maximum Nash price.[14]

In each period, the PRA randomly samples $M \in \{1, 2, \ldots, 2N_B\}$ spot transactions and takes the arithmetic mean of prices across these transactions (Fastmarkets, 2025*a*). Market participants report their spot transactions truthfully to the PRA[15] and all transactions are equally likely to be sampled.

The total cost a buyer incurs to obtain the good has two components: the materials price $\bar{p}$ (the price of the underlying good) and a transport cost. Whether transport costs appear in the transaction price reported to the PRA depends on whether mill or delivered pricing prevails. Under mill pricing, the buyer collects the material at the seller's plant and arranges transport separately, so the price reported to the PRA is the materials price alone. Under delivered pricing, the seller arranges and bills transport, so the reported price includes both components. Because indexes are intended to track the cost of the material itself, not fluctuations in how far spot sales happened to travel in a given period, PRAs calculate price indexes using materials prices, not total prices. Under mill pricing, no adjustment is necessary because the reported price equals the materials price. Thus, reported prices enter the index directly. Under delivered pricing, the PRA recovers the materials price by subtracting the transport cost, which is typically itemized on the invoice or

[11]We show results are robust to $N > 2$ firms in Technical Appendix D.1.

[12]In practice, index-linked contracts may include negotiated differentials from the index (e.g., the price in an index-linked oil supply contract may equal the index price plus or minus \$0.05 per barrel). Including these differentials essentially increases or decreases sellers' contract profits by a fixed constant and therefore does not impact equilibrium outcomes.

[13]For example, see the index-linked contract in Technical Appendix C.1, wherein the quantity of marine diesel fuel to be delivered (36,000 barrels per month) is fixed explicitly in the contract.

[14]Absent this condition, the largest possible Nash equilibrium price would be the minimum of $\Delta\left(1 + \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{2}\right)$ and r. If $r \leq \Delta\left(1 + \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{2}\right)$, index manipulation would not be necessary as firms could charge monopoly prices in the Nash equilibrium.

[15]This assumption is likely to hold when the PRA double verifies reported prices with both buyers and sellers, or when firms face harsh penalties for dishonestly reporting spot market prices. See Section 6 for an analysis of outcomes when sellers can lie to the PRA.

collected directly from submitters (Fastmarkets, 2024). For example, OPIS derives its naphtha index by taking an observed delivered price and "deduct[ing] freight differentials," that is, it subtracts the shipping cost to recover the materials price (OPIS, 2018). The materials price is used in the PRA's calculation of the price index. Transport cost adjustments of this kind are standard practice across PRAs and industries (Platts, 2025; OPIS, 2018; Fastmarkets, 2025*a*,*b*; Argus Media, 2025*a*,*b*, 2026) and are documented/evidenced in detail in Technical Appendix C.3.

The expected value of the index is[16]

$$p_{index} = \frac{1}{2N_B} \sum_{k=1}^{N_B} \sum_{i=1}^{2} p_i^{(k)}$$

where $p_i^{(k)}$ is the materials price paid by buyer k in city i.

The timing of the game is as follows. First, sellers make private take-it-or-leave-it offers to each buyer. Let $p_{ij}^{(k)}$ denote the offer of seller i to buyer $k \in \{1, \dots N_B\}$ located in city j. We do not consider offers involving negative materials prices.[17] Next, each buyer selects one seller and procures their spot demand of $\frac{1-\alpha}{N_B}$ from that seller. When indifferent between two sellers' offers, we assume buyers choose to purchase from the closest seller.[18] Third, the PRA samples transactions, calculates the materials price in each transaction, averages the collected materials prices, and publishes the index. Fourth, contract buyers make the payments stipulated in their index-linked contracts. An equilibrium is a vector of spot price offers from each seller to each buyer. We restrict attention to pure strategy sub-game perfect Nash equilibria throughout. We state offers as total (delivered) prices. Under mill pricing a seller instead names the materials price $\bar{p}_{ij}^{(k)} = p_{ij}^{(k)} - t_{ij}$ where t_{ij} is the transport cost between city i and city j. As t_{ij} is a route-specific constant, this is a bijective relabeling of the strategy space that leaves equilibrium outcomes unchanged. All equilibrium offers below therefore translate to mill offers by subtracting transport costs.

In equilibrium, sellers only serve buyers in their respective cities. Firm i's profit is therefore $\Pi_i = q_{spot}(1-\alpha)\sum_{k=1}^{N_B} p_i^{(k)} + \alpha p_{index}$ where $p_i^{(k)}$ is the price paid by buyer k in city i (to the local seller). As demand is perfectly inelastic, buyer surplus is $BS = R - 2\,\Pi$ in a symmetric equilibrium, where R is the sum of all contract and spot buyers' valuations for the good.

Remark 1. Buyer surplus BS is declining in per-firm profit in any symmetric equilibrium where all sellers serve their local buyers.[19]

[16]To see this, note that the price index involves randomly sampling a subset of $M \leq 2N_B$ spot transactions without replacement. Each transaction is equally likely to be sampled and included in the index.

[17]See Technical Appendix D.2 for a discussion of how permitting negative materials price offers would impact results.

[18]This tie-breaking convention avoids the familiar open-set problem in Bertrand-style pricing games.

[19]For certain equilibria in Section 6, Remark 1 does not hold due to the presence of penalties impacting firm profit but not buyer surplus. Buyer surplus results will be established without the use of Remark 1 in these cases.

3.1 Nash Competition

The following proposition characterizes Nash equilibria in the base model outlined above.

Proposition 1. *The Nash equilibria are all price offer vectors such that* $p_{11}^{(k)} = p_{21}^{(k)}$, $p_{22}^{(k)} = p_{12}^{(k)}$, $p_{11}^{(k)} \in \left[\Delta, \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)\right]$, *and* $p_{22}^{(k)} \in \left[\Delta, \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)\right]$, *for all* $k \in \{1, 2, \ldots N_B\}$.

In the Nash equilibrium, sellers match each other's price offers and all buyers purchase from their local seller. Proposition 1 implies that, absent index-linked contracts (i.e., $\alpha = 0$), the unique Nash equilibrium spot price is Δ, in both cities and for all buyers. Index-linked contracts create additional equilibria involving higher spot prices. As α increases, the set of possible Nash equilibrium prices expands and the highest possible Nash price increases. To understand this result, note that undercutting a rival's price offer and stealing a spot buyer carries an important drawback when a seller has signed index-linked contracts. Stealing a buyer in another city requires offering the buyer a reduced materials (mill) price. Transport costs force sellers to shade their materials price offer to win more distant customers.[20] By stealing a distant buyer, the seller generates a transaction with a relatively low materials price, which reduces the expected value of the price index and reduces its profit from long-term contracts. The presence of index-linked contracts therefore softens price competition between sellers and permits higher Nash equilibrium prices.

Proposition 1 is an oligopoly counterpart of an established mechanism, identified in the oligopsony setting by Xia and Sexton (2004); Garrido et al. (2022*b*) and Moschini and Smith (2026), in which buyers with index-linked procurement contracts bid less aggressively in the spot market because higher spot bids raise the price they pay through their procurement contracts. Elevated Nash equilibrium prices in Proposition 1 occur not due to any agreement between firms, dishonest reporting of prices, or fraudulent attempts to manipulate the index, but simply due to relaxed competition induced by the presence of index-linked contracts.

3.2 Collusion

We next consider outcomes in a repeated game where sellers may collude in the spot market to inflate the index. Let $\delta \in (0, 1)$ denote firms' common discount factor. During collusion, all firms make a common price offer of p_C to every buyer, and all buyers purchase from their local seller. If any seller deviates, then firms revert to Nash competition for all subsequent periods (i.e., grim trigger strategies). As established in Subsection 3.1, multiple Nash equilibria exist under index-linked contracts. For concreteness, we assume firms revert to the harshest Nash equilibrium (i.e., a Nash price of Δ) in the main text. We show that results are robust to alternative equilibria in the punishment phase in Technical Appendix D.3. Firms choose p_C

[20]Miller and Osborne (2014), in their study of pricing in the US cement industry, find evidence of this pattern. They find that cement plants charge higher mill prices to nearby customers and that "mill prices and margins of these plants decline quickly with distance to the customer."

(the optimal collusive price) to maximize the discounted present value of profit subject to the constraint that no seller wishes to deviate.

For tractability, we model index-linked contracts as infinitely lived as these contracts are long-term and span many spot-pricing periods in practice.[21] Let $\delta^*(r)$ denote the smallest discount factor at which collusion on buyers' reservation price r (i.e., the monopoly price) is sustainable.

Proposition 2. *The optimal collusive price is*

$$p_C = \begin{cases} r & \text{if } \delta \geq \delta^*(r) \\ \Delta + \frac{\Delta}{2}\left(\frac{\alpha}{1-\alpha}\right)\frac{1-\delta}{1-\delta\left(\frac{\alpha}{1-\alpha}\right)-2\delta} & \text{if } \delta < \delta^*(r) \end{cases}$$

where $\delta^*(r) = \frac{(1-\alpha)(r-\Delta)-\alpha\frac{\Delta}{2}}{(r-\Delta)(2-\alpha)-\alpha\frac{\Delta}{2}}$.

Figure 1 plots the collusive price as a function of δ. Two regimes arise. When $\delta \geq \delta^*(r)$, the incentive compatibility constraint does not bind and sellers collude by charging each buyer its reservation price. When $\delta < \delta^*(r)$, the constraint binds and the collusive price lies between the highest Nash equilibrium price and r. In this second regime, the collusive price is increasing in δ and converges to the highest Nash equilibrium price $\Delta + \frac{\Delta}{2}\left(\frac{\alpha}{1-\alpha}\right)$ as $\delta \to 0$.

Index-linked contracts relax sellers' incentive compatibility constraints. Absent such contracts (i.e., $\alpha = 0$), Proposition 2 reduces to the familiar case of collusion under homogeneous product Bertrand competition with marginal cost Δ. Sellers collude on r when $\delta \geq \frac{1}{2}$ and set a price of Δ (the competitive price) otherwise. When $\alpha > 0$, the critical discount factor $\delta^*(r)$ is strictly below $\frac{1}{2}$. $\delta^*(r)$ is also declining in α, so collusion on the reservation price is sustainable for a wider range of discount factors as α increases. When $\delta < \delta^*(r)$, the optimal collusive price under index-linked contracts ($\alpha > 0$) exceeds the price absent these contracts ($\alpha = 0$). Collusive prices are increasing in α. As price offers are symmetric and each seller earns p_C on both its spot and its contract sales, per-firm profit equals p_C. A larger share of index-linked contracts therefore raises collusive profit and, by Remark 1, reduces buyer surplus.

Two forces drive these results. First, a seller that undercuts its rival must ship to the rival's city and therefore transacts at a reduced materials price. The deviation lowers the expected value of the index and the revenue the seller earns on its contract sales, which reduces the current period gain from deviating. This force also operates under Nash competition. Second, a deviating seller raises its spot profit in the

[21] For example, natural gas is supplied under contracts of up to 15 years (Duke Energy, 2000) while its price index is assessed daily (Argus Media, 2025*c*). Thus, a single contract spans thousands of spot-pricing periods. Our focus is the manipulation of indexes given that index-linked contracts prevail, rather than the signing and negotiation of index-linked contracts. We therefore do not model the endogenous process by which contracts are offered and signed. See Xia and Sexton (2004) and Hatfield and Lowery (2023) for a detailed treatment of this topic.

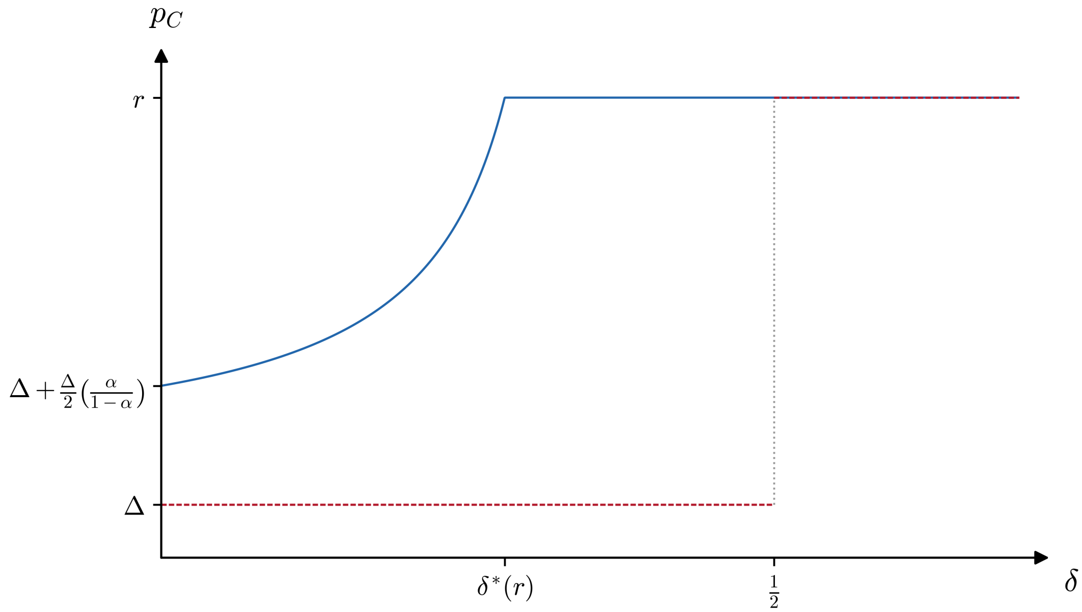

Figure 1: Optimal Collusive Prices p_C in the Baseline Model when $\alpha > 0$ (blue, solid) and $\alpha = 0$ (red, dashed)

current period, but the subsequent reversion to competition reduces both its spot and contract profit. The punishment falls on the seller's entire output while the gain from deviation accrues only on spot sales. Both forces relax the incentive compatibility constraint and permit colluding sellers to sustain higher prices when index-linked contracts are prevalent.

4 Heterogeneous Buyer Valuations

In the baseline model of Section 3, all spot buyers share the same reservation price. In this section, we consider a variant of the baseline model wherein spot buyers are heterogeneous in their valuations. Differences in valuations arise naturally when buyers put the good to different end uses. In the caustic soda industry discussed in Subsection 2.2, alumina refiners and many chemical producers have no practical substitute for caustic soda in their production processes. In contrast, water treatment facilities, pulp mills, and other general-purpose users can use an alternative (soda ash) when caustic soda prices are high (U.S. Geological Survey, 2025). Buyers that cannot employ a substitute are willing to pay more for the good than buyers that can switch to an alternative. A buyer's end use, and therefore its valuation, is readily observable to producers (Olin Corporation, 2024).

Suppose there are two spot buyers in each city ($N_B = 2$) that differ in their valuations: one buyer has a low reservation price r_L and the other buyer has a high reservation price $r_H > r_L$. Buyers are otherwise identical, with each demanding $\frac{1-\alpha}{2}$ units of the good. As in the baseline model, we assume $r_H > \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)$, but place a weaker restriction on $r_L : r_L > \Delta$.[22] As in Section 3, sellers make matching price offers to each buyer in all equilibria. We restrict attention to symmetric equilibria in the sense that the price paid by the low (high) valuation buyer in city 1 equals the price paid by the low (high) valuation buyer in city 2. Let $p^{(L)}$ denote the price paid by low-valuation buyers and let $p^{(H)}$ be the price paid by high-valuation buyers. Sellers may refuse to deal with a buyer, either by declining to make the buyer an offer or by making an offer sufficiently high that the buyer will not accept it. Let a price offer of R_D denote a refusal to deal.

We first characterize Nash equilibria in which no refusals to deal occur and all buyers are served.

Lemma 1. (No Refusal Equilibria) *A strategy profile where each seller offers $p^{(H)} \neq R_D$ to high-valuation buyers and $p^{(L)} \neq R_D$ to low-valuation buyers is a Nash equilibrium if and only if $p^{(H)} \in \left[\Delta, \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)\right]$ and $p^{(L)} \in \left[\Delta, \min\left\{\Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right), r_L\right\}\right]$.*

The equilibria identified in Lemma 1 coincide with the Nash equilibria of the baseline model (Proposition 1), with the exception that r_L may be sufficiently low that it constrains the price charged to low-valuation buyers. When $r_L < \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)$, the highest price the low-valuation buyer pays in a Nash equilibrium is its reservation price r_L. Sellers may instead refuse to deal with the low-valuation buyer. The following lemma characterizes Nash equilibria involving refusals to deal.

Lemma 2. (Refusal Equilibria) *A strategy profile where each seller offers $p^{(H)} \neq R_D$ to high-valuation buyers and $p^{(L)} = R_D$ to low-valuation buyers is a Nash equilibrium if and only if $\alpha > \frac{3}{5}$ and*

$$\max\left\{r_L\left(\frac{3-\alpha}{2\alpha}\right), \frac{(r_L - \Delta)(3-\alpha)}{(5\alpha - 3)}\right\} \leq p^{(H)} \leq \min\left\{\Delta + \frac{\alpha}{1-\alpha}\Delta, r_H\right\}.$$

In the equilibria of Lemma 2, both sellers refuse to deal with low-valuation buyers while charging high-valuation buyers a price that can exceed the maximum Nash equilibrium price when all buyers are served (Lemma 1 or Proposition 1). Sellers have two incentives to refuse to deal with the low-valuation buyer. First, refusing to supply the low-valuation buyer removes a relatively low-priced transaction from the spot market, raising the index and, as a result, sellers' contract profits. Second, by reducing the number of spot transactions, refusals to deal increase the weight of each remaining transaction in the index and, as a result, the sensitivity of the index to deviations. The index therefore declines by a larger amount when a seller undercuts its rival, which weakens incentives to deviate and supports higher prices for the high-valuation buyer. Indeed, in some equilibria, sellers charge the high-valuation buyer its reservation price r_H. Equilibria

[22]See Technical Appendix B.2 for results when $r_L \leq \Delta$. $r_L \leq \Delta$ implies that a supplier cannot profitably serve the low-valuation buyer in the other city. Thus, suppliers essentially do not compete for low-valuation buyers. As results are qualitatively similar when $r_L \leq \Delta$, we present the simpler case of $r_L > \Delta$ in the main text for brevity.

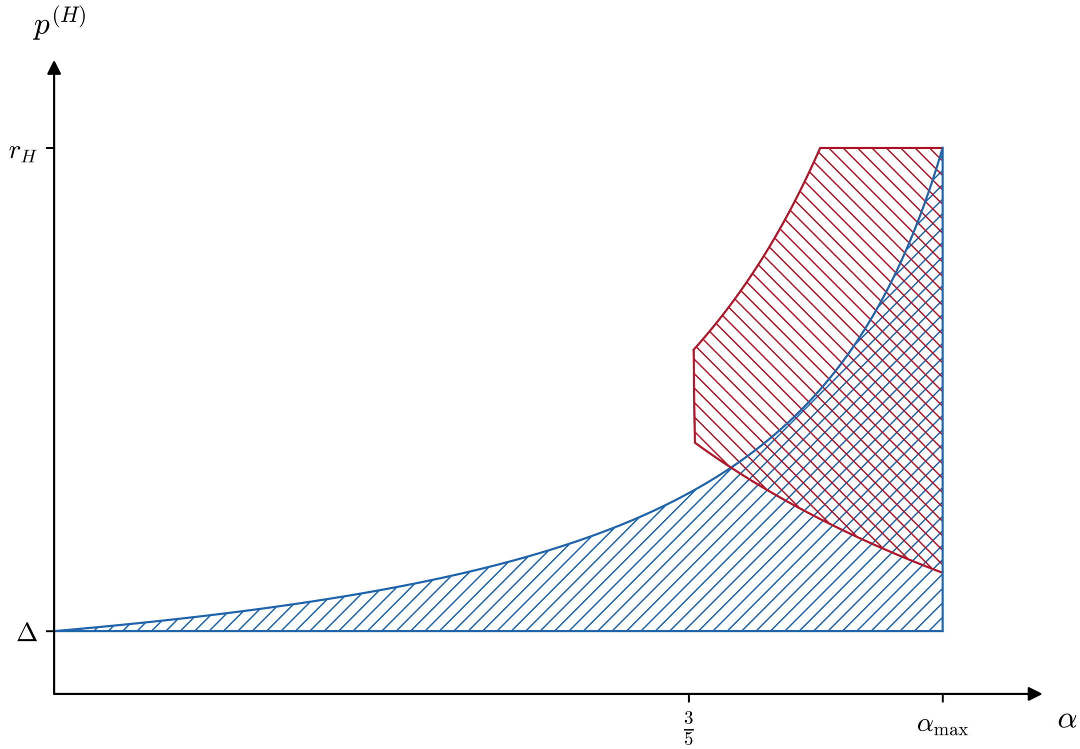

Figure 2: Refusal (red) and No Refusal (blue) Equilibria when Buyers Differ in their Valuations for the Good

involving refusals to deal arise only when the share of index-linked contracts is large ($\alpha > \frac{3}{5}$). Otherwise, the gains from inflating the index do not compensate sellers for the loss in spot profits from refusing to serve the low-valuation buyer. Additionally, equilibria involving refusals to deal exist when the asymmetry in buyers' valuations is sufficiently large (i.e., the second condition in Lemma 2 holds when r_L is small relative to r_H).

The bounds on $p^{(H)}$ in Lemma 2 reflect two constraints. If $p^{(H)}$ is too high, a seller prefers to deviate and steal the rival's high-valuation buyer. If $p^{(H)}$ is too low, the index is relatively low. In this case, a seller prefers to deviate by serving its own low-valuation buyer, as the additional transaction only modestly reduces an already low index. Figure 2 depicts the set of Nash equilibria as α varies.[23]

Consistent with these results, suppliers in a number of recent antitrust and market manipulation cases allegedly refused to sell to certain buyers in order to inflate a price index. Caustic soda purchasers allege that producers refused to supply certain customers as part of a scheme to increase the level of a prominent caustic soda price index (Precious Plate v. Olin, 2019; In re Caustic Soda, 2019). In February 2004, BP acquired a dominant position in physical propane at the Mont Belvieu trading hub (the main US trading hub for propane) and, according to the government's indictment, "withheld February 2004 TET propane from the market" (United States v. Radley et al., 2007). By refusing to supply certain buyers, BP inflated an

[23] $\alpha_{max} = \frac{2(r_H - \Delta)}{2r_H - \Delta}$ denotes the largest α such that the maintained assumption of $r_H > \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)$ holds.

OPIS propane price index referenced in propane supply contracts and, as a result, increased its profits from contract-based sales (CFTC v. BP Products, 2006). In the early 2010s, the owners of warehouses storing physical aluminum allegedly restricted the outflow of metal from storage, creating long delivery delays that inflated the Platts Midwest Premium, a benchmark referenced in index-linked aluminum supply contracts (In re Aluminum Warehousing, 2013).

The results of this section suggest that sellers may refuse to deal with low-valuation buyers when index-linked contracts are prevalent. Refusals to deal are particularly harmful from a buyer surplus perspective as the refused buyer does not obtain the good and loses the associated surplus entirely, creating a deadweight loss.

5 Concealment of Transactions

In the baseline model of Section 3, all transaction prices can potentially be sampled/collected by the PRA and included in the index. However, a number of recent antitrust cases have involved suppliers selectively reporting transactions to the PRA. To illustrate, suppose a supplier concludes two transactions during the relevant reporting window: one at \$75 and one at \$100. When contacted by a PRA and asked the prices at which it transacted, the firm reports the \$100 transaction but does not report the \$75 transaction. This strategy artificially distorts the index upwards and increases a supplier's contract profits. Between early 2015 and late 2016, energy companies Vitol and SK Energy allegedly selectively reported spot gasoline transactions to OPIS (a PRA) in order to inflate OPIS's California gasoline price index, which served as the reference price for their index-linked gasoline supply contracts (Cal. AG, 2020). Recent antitrust litigation alleges caustic soda producers "reached an understanding or agreement to manipulate the IHS Markit price index by selectively providing information to IHS Markit" (Precious Plate v. Olin, 2019). Regulators have repeatedly expressed concern about the risk of selective reporting.[24]

To explore the possibility of sellers manipulating the index through selective reporting, we consider an extension of the baseline model where suppliers can choose to conceal transactions from the PRA and prevent their inclusion in the index. For tractability, suppose there are two buyers in each city ($N_B = 2$). The timing of the game is as follows. First, sellers make private take-it-or-leave-it offers to each buyer. Next, each buyer selects one seller and procures its spot demand of $\frac{1-\alpha}{N_B}$ from that seller. Third, for each offer that has been accepted by a buyer, the seller decides whether to conceal the transaction from the PRA. Fourth, the PRA samples transactions that have not been concealed, averages the collected prices, and publishes the index. If

[24] The German competition authority notes that "there is a risk that individual market players may manipulate price [indexes] to their advantage, for example by selectively reporting data" (Bundeskartellamt, 2025). Similarly, IOSCO has expressed concerns about "opportunities for manipulative conduct created by the possibility of voluntary and/or selective submissions" (IOSCO, 2013).

all transactions are concealed from the PRA, we assume the PRA exerts additional effort to uncover hidden transactions[25] (e.g., contacting buyers rather than sellers, or exerting additional pressure on their contacts to reveal transactions) and, as a result, all transactions become visible.[26] Fifth, contract buyers make the payments stipulated in their index-linked contracts.

Let $V_j^{(k)} = 1$ (for visible) if the relevant seller does not conceal its transaction to buyer $k \in \{1, 2\}$ in city j and let $V_j^{(k)} = 0$ if the seller conceals this transaction. When indifferent between concealing and not concealing a transaction, suppose the seller does not conceal. This assumption could, for example, reflect a small effort or reputational cost of concealing. Let N_V be the number of visible (i.e., non-concealed) transactions. We restrict attention to pure strategy sub-game perfect equilibria. The expected value of the index is

$$p_{index} = \begin{cases} \frac{1}{N_V} \sum_{j=1}^{2} \sum_{k=1}^{N_B} p_j^{(k)} V_j^{(k)} & \text{if } N_V > 0 \\ \frac{1}{2N_B} \sum_{j=1}^{2} \sum_{k=1}^{N_B} p_j^{(k)} & \text{if } N_V = 0 \end{cases}.$$

Sellers face no formal penalties for concealing transactions. Firms are under no legal obligation to report their transactions to a PRA (IEA et al., 2011; Johnson, 2017). Selective reporting has attracted attention (and fines) from antitrust authorities only when it constituted part of a conspiracy/agreement, such as a violation of Section 1 of the Sherman Act (Precious Plate v. Olin, 2019). Similarly, legal precedent suggests unilateral concealment, without an explicit agreement/conspiracy, would likely not be considered fraud or a violation of anti-market manipulation laws either (Verstein, 2014).[27] Let $p_{max} \equiv \max_{m,k}\{p_m^{(k)}\}$ denote the largest transaction price across all buyers.

Lemma 3. *In any sub-game perfect Nash equilibrium,* $V_j^{(k)} = 1$ *when* $p_j^{(k)} = p_{max}$ *and* $V_j^{(k)} = 0$ *when* $p_j^{(k)} < p_{max}$*.*

Lemma 3 establishes that firms reveal only the highest priced transactions and conceal all other transactions.

Proposition 3. *In the unique sub-game perfect Nash equilibrium,* $p_1^{(1)} = p_2^{(1)} = p_1^{(2)} = p_2^{(2)} = \Delta$*.*

In the baseline model without concealment, the Nash equilibrium price can be as high as $\Delta \left(1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{1}{2}\right)$. Perhaps surprisingly, the ability to conceal transactions reduces Nash equilibrium prices, resulting in a lower index price, reduced seller profits and higher buyer surplus (see Remark 1). This is the case as deviating

[25] PRAs have strong incentives to uncover enough transactions to publish an index in every period. Failing to publish an index would likely result in significant reputational damage for the PRA and could cause market participants to switch to an alternative price index.

[26] This assumption is made for concreteness. All results hold under a more general assumption that, when all transactions are concealed, the PRA publishes an index price that is less than or equal to the maximum transaction price.

[27] A joint report by leading energy organizations and securities regulators found that firms have "no contractual, legal or regulatory requirement to report [...] transactions to PRAs" and that "[i]t is therefore open to companies to report only those deals that are in their own best interests for the rest of the market to see" (IEA et al., 2011).

sellers can conceal the relatively low prices they must offer a buyer to steal it from a rival. Therefore, the reduced price does not affect the index and does not reduce a seller's contract profits. This effect eliminates the key downside of undercutting a rival, thereby intensifying competition.

We next consider outcomes under collusion. Under collusion, sellers price discriminate between the two buyers. This is the case as sellers can raise the price charged to one buyer, and report that transaction to the PRA, while concealing a lower price charged to the other buyer. Let $p^{(k)}$ denote the collusive price charged to buyer $k \in \{1, 2\}$ in each city. Sellers choose prices and concealment decisions to maximize profits subject to the constraint that no firm wishes to defect from the collusive agreement.

Proposition 4. *Let* $\delta_0 = \frac{1}{2+2\left(\frac{\alpha}{1-\alpha}\right)}$ *and* $\delta_C^*(r) = \frac{1}{2+\left(\frac{\alpha}{1-\alpha}\right)}$. *The optimal collusive strategy involves*[28]

i) $V_j^{(1)} = 1$ *and* $V_j^{(2)} = 0$ *for* $j \in \{1, 2\}$, *and*

ii) collusive prices of

$$(p^{(1)}, p^{(2)}) = \begin{cases} (\Delta, \Delta) & \text{if } \delta < \delta_0 \\ \left(r, p^{(2)}(\delta)\right) & \text{if } \delta_0 \leq \delta < \delta_C^*(r) \\ (r, r) & \text{if } \delta \geq \delta_C^*(r) \end{cases}$$

where $p^{(2)}(\delta) = r - (r - \Delta)\,\frac{1-2\delta-\delta\left(\frac{\alpha}{1-\alpha}\right)}{\frac{1}{2}-\delta}$.

Proposition 4 reports that firms set Nash equilibrium prices when $\delta < \delta_0$. For intermediate discount factors, the optimal collusive strategy involves firms price discriminating across buyers. Firms charge a higher price to one buyer (WLOG, buyer 1) and a lower price to the other buyer. The lower priced transaction is concealed from the PRA while the higher priced sale serves as an index-anchoring price and is revealed to the PRA. Monopoly collusion (i.e., $p_1 = p_2 = r$) is achievable when δ is sufficiently large.

Let $\Pi^{NC}(\delta)$ denote collusive profit when firms cannot conceal transactions from the PRA and let $\Pi^{C}(\delta)$ denote collusive profit when firms can conceal transactions from the PRA.

Proposition 5. *i)* $\Pi^{NC}(\delta) > \Pi^{C}(\delta)$ *if* $\delta < \delta_0$.

ii) $\Pi^{NC}(\delta) > \Pi^{C}(\delta)$ *if* $\delta_0 \leq \delta < \delta^*(r)$ *and* $2\frac{\frac{r}{\Delta}-1}{1+\left(\frac{\alpha}{1-\alpha}\right)} < \frac{1-2\delta}{1-\delta\left(\frac{\alpha}{1-\alpha}\right)-2\delta}$.

iii) $\Pi^{NC}(\delta) < \Pi^{C}(\delta)$ *if* $\delta_0 \leq \delta < \delta^*(r)$ *and* $2\frac{\frac{r}{\Delta}-1}{1+\left(\frac{\alpha}{1-\alpha}\right)} > \frac{1-2\delta}{1-\delta\left(\frac{\alpha}{1-\alpha}\right)-2\delta}$.

iv) $\Pi^{NC}(\delta) > \Pi^{C}(\delta)$ *if* $\delta^*(r) \leq \delta < \delta_C^*(r)$.

v) $\Pi^{NC}(\delta) = \Pi^{C}(\delta)$ *if* $\delta_C^*(r) \leq \delta$.

Proposition 5 establishes that the level of collusive profits when firms cannot conceal transactions may exceed the level of collusive profits when firms are able to conceal transactions. Concealing generates higher

[28]Without loss of generality, we suppose firms offer the higher price to buyer 1 and the lower price to buyer 2. As buyers are symmetric, outcomes are identical under the opposite assumption.

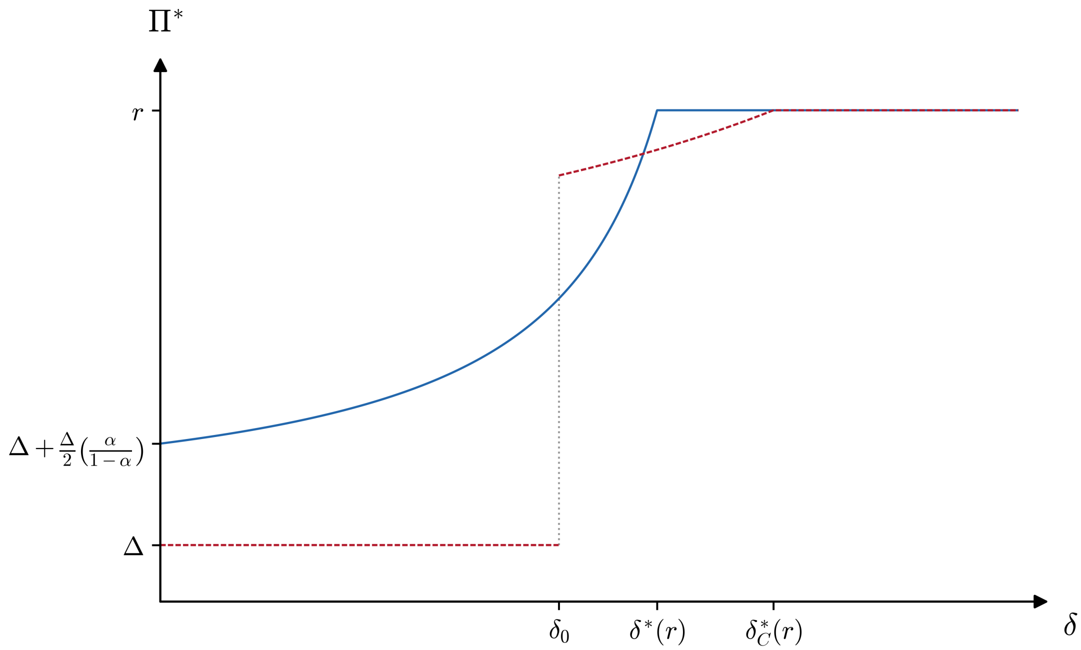

Figure 3: Collusive Prices without Concealment (solid, blue) and with Concealment (dashed, red)

profits only for a range of intermediate discount factors (see Figure 3). This finding reflects two opposing effects of concealment. First, all else equal, concealment allows sellers to increase the index by inflating the price paid by one buyer and revealing only that price to the PRA. Thus, even when firms are too impatient to sustain high prices for both buyers, they can inflate the index by raising the price of only one buyer. However, concealment also enhances incentives to deviate. A deviating seller can undercut its rival and hide those transactions from the PRA, preventing any reduction in the index. This effect destabilizes collusion and results in lower prices and profits. Concealment raises profits only when the first effect dominates.[29] By Remark 1, Proposition 5 implies that forbidding concealment can reduce buyer surplus.

Recent regulations, such as the IOSCO principles (IOSCO, 2012) and the European Union's Benchmark Regulation (EU BMR, 2016*a*), have encouraged PRAs to adopt policies to prevent concealment and to exert effort to uncover concealed transactions.[30] In response, many PRAs have instituted such policies (Johnson, 2017). For example, "Platts encourages entities that submit input data for consideration in its assessment processes to submit all market data which is relevant" (Platts, 2025). Fastmarkets includes a clause in its contractual agreements with data submitters that "require[s] all relevant data to be provided" and has adopted a general policy explicitly forbidding selective reporting: "[u]nder no circumstances should selective

[29]In Technical Appendix B.3.1, we establish that $\Pi^{NC}(\delta) < \Pi^{C}(\delta)$ occurs (i.e., case (iii) in Proposition 5) if and only if α is sufficiently small. Thus, the ability to conceal increases profits only when the prevalence of index-linked contracts is low.

[30]In U.S. wholesale natural gas and electricity markets, the Federal Energy Regulatory Commission forbids sellers from "omit[ting] material information" when reporting transactions to price index publishers (FERC, 2003, 2007).

data be submitted to attempt to influence the price assessment process" (Fastmarkets, 2024). Additionally, PRAs may use alternative sources (e.g., buyers) to verify that a submitting seller has not concealed any unreported sales (Johnson, 2017). The results of this section suggest that the ability to conceal has subtle strategic implications that can reduce Nash equilibrium prices, hinder collusion, and benefit buyers. Thus, regulations and policies intended to prevent concealment can have countervailing effects.

One might expect that the ability to conceal transactions would benefit suppliers and harm buyers in a setting with heterogeneous buyer valuations (i.e., the extension in Section 4). Specifically, sellers can conceal transactions with low-valuation buyers to ensure the index reflects only relatively high-priced sales to high-valuation buyers. However, to the contrary, we find that the ability to conceal can also increase buyer surplus in a setting with heterogeneous buyer valuations. See Technical Appendix B.3.2 for a formal analysis of concealment in a model of heterogeneous buyer valuations. This is the case as sellers no longer engage in refusals to deal in equilibrium when they can conceal transactions. Instead, producers serve low-valuation buyers and simply conceal those (relatively low-priced) transactions from the PRA, preventing the transaction from reducing the index. Regulations prohibiting the concealment of transactions can cause producers to refuse low-valuation buyers to avoid depressing the index (as shown in Lemma 2), creating a deadweight loss.

6 Dishonest Price Reporting

In the baseline model introduced in Section 3, we assumed that firms truthfully reported their transactions to the PRA. However, a number of recent antitrust cases have involved firms manipulating the index through reporting fake or dishonest transaction prices to the PRA. For example, broiler chicken producers allegedly submitted false prices to inflate the Georgia Dock price index (In re Broiler Chicken, 2016). From 2002 to 2003, natural gas companies submitted knowingly inaccurate prices to PRAs to inflate natural gas price indexes (Johnson, 2017). In its complaint against one of the firms, the Commodity Futures Trading Commission (CFTC) concluded that about 2,800 of 3,600 natural gas trades the firm reported to Platts were inaccurate (CFTC, 2003). Plaintiffs in ongoing litigation in the caustic soda industry allege producers submitted fake information to IHS Markit (a PRA) to inflate an index (Precious Plate v. Olin, 2019). To explore the possibility of dishonest reporting, we next consider an extension of the baseline model wherein sellers can lie to the PRA and report a price that differs from their actual transaction price.

Let $x_i \in [0, r] \cup \{No\,Sales\}$ denote a firm's report of its average transaction price across its spot sales. A firm's price report is dishonest if the reported average transaction price x_i differs from the firm's actual average transaction price (or if a firm reports $No\,Sales$ when it conducted a sale or reports a price when no sale occurred). The PRA detects any dishonesty with probability $\phi \in [0,1]$.[31] ϕ reflects the auditing and verification practices of PRA staff and their compliance departments intended to uncover dishonest price reports (ICIS, 2025; Johnson, 2017).[32] If a dishonest report is detected, the PRA exerts additional effort to uncover actual transaction prices (e.g., requesting copies of invoices or contacting buyers)[33] and the expected index equals the average of actual transaction prices. If both firms report $No\,Sales$, then the PRA detects dishonesty with probability 1. This assumption reflects the fact that a sale always occurs in equilibrium. Thus, a rational PRA would recognize dishonesty whenever it received reports of $No\,Sales$ from both firms.

A dishonest firm incurs a penalty $F \geq 0$. Lying to a PRA is an offense in most jurisdictions (Johnson, 2017) and is a violation of either antitrust laws (especially when part of an agreement/conspiracy), anti-market manipulation statutes (EU REMIT, 2011), commodities/derivatives regulations (CFTC, 2011), or fraud laws (U.S. Congress, 1952).[34]

We restrict attention to symmetric equilibria in the sense that both firms offer the same price (denoted p_N) and make the same price reports in equilibrium.[35] The timing of the game is as follows. First, sellers make private take-it-or-leave-it offers to each buyer. Next, each buyer selects one seller and procures its spot demand from that seller. Third, sellers make price reports to the PRA, which may or may not be truthful. Fourth, the PRA detects dishonesty with probability ϕ and publishes the price index. Dishonest firms receive a penalty F if their dishonesty was detected. Fifth, contract buyers make the payments stipulated in their index-linked contracts.

There are three types of equilibria: truthful, semi-truthful and lying. In lying equilibria, both firms dishonestly report a transaction price of r to the PRA in equilibrium. Semi-truthful equilibria involve a strategy profile where sellers lie (and report r) in sub-games following a deviation in the spot market, but do not lie in equilibrium. Truthful equilibria involve firms reporting honestly both along the equilibrium path and following a deviation in the spot market.

Lemma 4. *There exists a truthful equilibrium with spot price p_N if and only if $\frac{\phi}{1-\phi}F \geq \alpha\left(r - p_N + \frac{\Delta}{2}\right)$ and*

[31] In the main text, the probability of detection is constant. Technical Appendix D.4 explores the robustness of results to a variety of alternative detection technologies. Specifically, we allow the probability of detection to depend on 1) the difference between a firm's report and its actual average spot price, 2) the difference between the two firms' reports, or 3) the number of lying firms.

[32] For example, the Georgia Department of Agriculture randomly audited prices submitted by producers, for inclusion in its Georgia Premium Poultry Price Index, by confirming the price of the transaction with the buyer (GDA, 2017).

[33] For example, Fastmarkets states that "suspected motivation to unfairly influence the price discovery process would typically be grounds for removal of data, [...] and efforts will be made to identify the counterparty to cross-verify information. [...] Fastmarkets reserves the right to see contracts and signed paperwork before inclusion of the data in the assessment" (Fastmarkets, 2025*a*).

[34] Duffie and Dworczak (2021) include a similar penalty in their model, capturing "the risk of detection of manipulation by the authorities, and resulting penalties or loss of reputation."

[35] This restriction is primarily expositional. Absent this assumption, duplicative lying equilibria exist where one firm reports r and another firm reports $No\,Sales$. These equilibria yield identical payoffs and prices as equilibria where both firms report r (i.e., $x_1 = x_2 = r$). Moreover, these equilibria exist for exactly the same parameter values. See Technical Appendix B.4 for details.

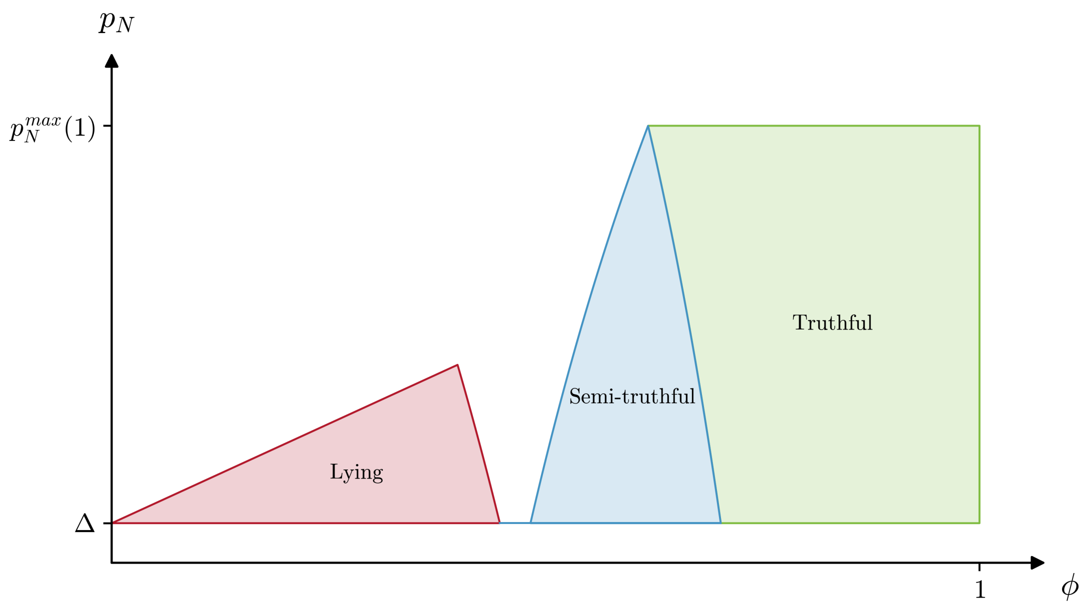

Figure 4: Spot Prices under Truthful (green), Semi-truthful (blue) and Lying (red) Equilibria when Suppliers can Dishonestly Report Transactions

$p_N \in \left[\Delta, \Delta + \frac{\alpha}{(1-\alpha)}\left(\frac{\Delta}{2}\right)\right]$.

In a truthful equilibrium, expected penalties are large enough to deter dishonest price reports both on and off the equilibrium path.

Lemma 5. *There exists a semi-truthful equilibrium with spot price p_N if and only if*

i) $\alpha \frac{r-p_N}{2} \leq \frac{\phi}{1-\phi} F \leq \alpha\left(r - p_N + \frac{\Delta}{2}\right)$, *and*

ii) either $p_N = \Delta$, *or*

$$p_N\left(\alpha\left(1-\phi\right)-\left(1-\alpha\right)\right) \geq \alpha\left(1-\phi\right)\left(r\right)-\left(1-\alpha\right)\Delta-\phi\left(\alpha\frac{\Delta}{2}+F\right)$$

and $p_N > \Delta$.

Semi-truthful equilibria arise when expected penalties for dishonest reporting are moderate. In a semi-truthful equilibrium, the price must be sufficiently high that firms do not wish to lie on the equilibrium path. Recall that lying, if undetected, raises the index from p_N to r. When p_N is relatively high, lying does not substantially increase the index and is therefore not worthwhile. Additionally, the price must be sufficiently low that a deviating firm wishes to lie. Lying after a deviation (if undetected) prevents the index from declining to $p_N - \frac{\Delta}{2}$. When $p_N - \frac{\Delta}{2}$ is sufficiently low, lying to the PRA, in an attempt to prevent a relatively low index price of $p_N - \frac{\Delta}{2}$, is worth the risk of incurring a penalty.

Lemma 6. *There exists a lying equilibrium with spot price* p_N *if and only if* $\frac{\phi}{1-\phi}F \leq \alpha\frac{r-p_N}{2}$ *and* $p_N \in \left[\Delta, \Delta + \phi\frac{\alpha}{1-\alpha}\frac{\Delta}{2}\right]$.

Lying equilibria arise when 1) the penalty for dishonest reporting and/or the likelihood of detection is relatively low, and 2) the Nash price is sufficiently low. When Nash equilibrium prices are high, firms earn relatively large contract profits by reporting these prices truthfully and therefore have weaker incentives to lie to the PRA. Figure 4 depicts Nash equilibria as ϕ varies.

Let $p_N^{max}(\phi)$ and $\Pi_N^{max}(\phi)$ denote the maximum Nash equilibrium spot price and profit (across the equilibria characterized in Lemmas 4-6) when the probability of detection is ϕ. The following proposition contrasts prices and profits when firms can lie to the PRA with outcomes when lying is not feasible (i.e., $\phi = 1$).[36]

Proposition 6. *i)* $p_N^{max}(\phi) \leq p_N^{max}(1)$ *for all* ϕ *and* $p_N^{max}(\phi) < p_N^{max}(1)$ *when* $\phi < \frac{\alpha\left(r-\frac{\Delta}{2(1-\alpha)}\right)}{F+\alpha\left(r-\frac{\Delta}{2(1-\alpha)}\right)}$.

ii) There exist ϕ_1 *and* ϕ_2 *such that* $\phi_1 < \phi_2$ *and* $\Pi_N^{max}(\phi) < \Pi_N^{max}(1)$ *when* $\phi \in (\phi_1, \phi_2)$.

Part (i) of Proposition 6 establishes that the ability to lie to the PRA does not raise the maximum Nash price and can actually reduce the maximum Nash price. The ability to lie reduces spot prices when ϕ is sufficiently low that a truthful equilibrium does not exist and, instead, a lying or semi-truthful equilibrium prevails (see Figure 4). This result reflects two considerations. First, firms that inflate the index through dishonest reporting recognize that their spot prices do not impact the index, reducing the incentive to raise these prices. Second, when deviating and undercutting rivals in the spot market, firms can lie to the PRA and prevent their deviation from reducing the price index, enhancing incentives to compete aggressively for spot buyers.[37]

Nash equilibrium profit is shaped by two countervailing effects of dishonest reporting. The ability to report dishonestly permits firms to obtain higher contract profits, by inflating the index, than when lying is not feasible. On the other hand, the ability to dishonestly report reduces spot prices (part (i) of Proposition 6).

Part (ii) of Proposition 6 shows that the ability to lie can reduce the maximum Nash profit level. This is the case when the loss in spot profits (part (i) of Proposition 6) from the ability to lie dominates any increase in contract profits from dishonestly reporting. This result can occur under both semi-truthful equilibria and under lying equilibria (see Figure 5). Under semi-truthful equilibria, the likelihood of detection is sufficiently large that firms do not lie in equilibrium. Therefore, the ability to lie does not inflate the index or enhance

[36] The following proposition analyzes the maximum profit obtained in a Nash equilibrium (Π_N^{max}) which measures the greatest extent to which dishonest reporting can elevate profits. See Technical Appendix B.4 for an analysis of the minimum profit across Nash equilibria.

[37] The ability to lie can also reduce the maximum value of the index (see Technical Appendix B.4 for a proof).

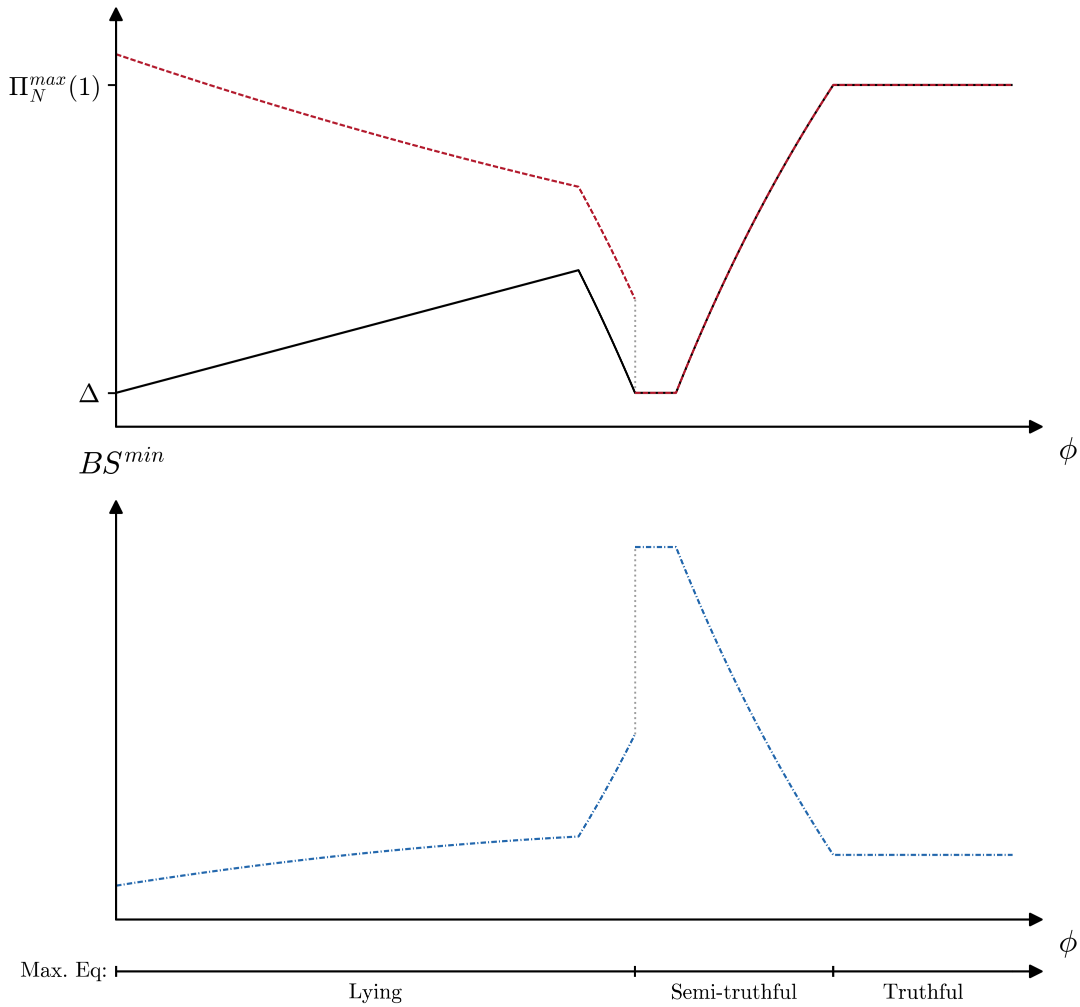

Figure 5: Top Panel: Maximum Nash Equilibrium Spot Prices (black, solid) and Maximum Nash Profits (red, dashed). Bottom Panel: Minimum Buyer Surplus (blue, dot-dash).

contract profits. However, after deviating, firms lie to the PRA, preventing the index from declining as a result of the deviation. Thus, the ability to lie reduces the loss in contract profits caused by a deviation and increases incentives to undercut rivals. This effect intensifies competition in the spot market, resulting in lower profits. Under lying equilibria, firms submit untruthful reports, artificially inflate the index, and increase their contract profits.[38] However, the ability to lie intensifies spot market competition and reduces spot profits. Reduced spot profits from the ability to lie can outweigh increased contract profits from dishonestly reporting, causing a reduction in firm profit. Due to these considerations, the ability to lie can also enhance buyer surplus and the prohibition of dishonest reporting can harm buyers. See Technical

[38]Firms' ability to inflate the expected value of the index through lying is limited by reduced spot market prices (part (i) of Proposition 6). To see this, note that the index equals the actual/honest average spot price when lying is detected. Thus, with a probability of ϕ, the index reflects reduced spot prices caused by the ability to lie. While lying still increases contract profits under lying equilibria, this effect limits the extent that sellers can enhance their contract profits through lying.

Appendix B.4 for a formal proof.[39]

Recent regulation has strongly encouraged or required PRAs to develop stronger procedures to detect or prevent the dishonest reporting of transactions. First, recent Benchmark Regulation in the European Union (EU BMR, 2016*a*) requires PRAs to strengthen their efforts to monitor and audit submitted data to detect false reports, and, when suspicious activity is detected, report that activity to the relevant authority. Second, PRAs are recommended or required to conduct extensive double verification of submitted prices (e.g., requesting copies of contracts/invoices or confirming transactions with buyers) (IOSCO, 2012; Johnson, 2017). Consistent with this recommendation, ICIS's published index methodology states that "[c]onfirmed deals, verified by both buyer and seller, provide the foundation of our price assessments" (ICIS, 2025). Due to suspected manipulation by broiler chicken producers submitting false price reports, the USDA required publishers of the Georgia Dock price index to collect invoices to verify transactions (In re Broiler Chicken, 2016; GDA, 2017).

Each of these regulations is intended to increase the probability of false price reports (ϕ) being detected. However, Proposition 6 reveals that increasing the likelihood of detecting false reports (or entirely preventing false reporting) can cause higher spot prices, higher firm profits, and reduced buyer surplus. The results of this section suggest that regulations increasing the likelihood of detecting a dishonest report can have counterintuitive effects on market outcomes.

7 Asymmetric Buyers

The preceding analysis has assumed that all spot buyers demand an identical quantity of the good. In practice, spot buyers may be heterogeneous in demand (Johnson, 2017).[40] Smaller transactions are easier for suppliers to manipulate (Duffie and Dworczak, 2021). Consider a seller who undercuts a rival to capture a small spot buyer. The gain in spot profits is modest, since the transaction is small. However, when all transactions are weighted equally in the index, the reduced transaction price needed to win that demand lowers the value of the index considerably. Thus, suppliers have relatively weak incentives to compete aggressively for small buyers.

Due to the risk of sellers inflating the price of small transactions to raise the index, PRAs, in some cases, impose minimum size constraints on the transactions included in the index. Any transaction for a quantity below the size threshold is excluded entirely. For example, in ICIS's styrene assessments for the

[39] Due to similar considerations, marginal increases in ϕ can raise the maximum Nash equilibrium spot price, raise the maximum value of the index, increase firm profit, and reduce buyer surplus (see Figure 5 and, for a formal proof, Technical Appendix B.4).

[40] For example, Duke Energy Field Services sells natural gas liquids to "a variety of customers ranging from large, multinational petrochemical and refining companies to small regional retail propane distributors" (Duke Energy, 2000).

US, "spot prices are assessed for waterborne volumes of at least 1,000 tonnes" (ICIS, 2026). In Argus's chlor-alkali indexes, "minimum volumes are typically established which may apply to each transaction accepted" (Argus Media, 2025*b*). However, other indexes do not exclude small transactions or impose any minimum size constraints.[41] Alternatively, some PRAs weight transactions by volume/quantity when calculating the index. For example, Platts' Gas Daily natural gas index "is the volume-weighted average of all transactions submitted to Platts" (FERC, 2016). However, volume-weighted indexes are not the norm (Johnson, 2017).[42] To explore the competitive implications of excluding small transactions and weighting transactions by volume, we consider a variation of the baseline model where spot buyers are heterogeneous in their demand for the good.

Suppose there are two spot buyers in each city ($N_B = 2$): a large buyer with spot demand $q_l(1-\alpha)$ and a small buyer with spot demand $q_s(1-\alpha)$, where $q_s + q_l = 1$ and $q_s < q_l$. Sellers price discriminate, potentially charging small buyers a different price than large buyers. Let $p_j^{(k)}$ denote the price paid by buyer $k \in \{s, l\}$ in city j. The PRA calculates the index as

$$p_{index} = w_s p_1^{(s)} + w_s p_2^{(s)} + w_l p_1^{(l)} + w_l p_2^{(l)},$$

where w_s denotes the weight placed on small transactions, w_l denotes the weight placed on large transactions, and $2w_s + 2w_l = 1$. Excluding small transactions corresponds to $w_s = 0$ and $w_l = \frac{1}{2}$. $w_s = \frac{q_s}{2}$ and $w_l = \frac{q_l}{2}$ yields a volume-weighted index. Equal weighting, as in the baseline model, corresponds to $w_s = w_l = \frac{1}{4}$. Mirroring the baseline model, we assume the reservation price is sufficiently large that $r > \Delta + \frac{\alpha w_l \Delta}{q_l(1-\alpha)}$ and $r > \Delta + \frac{\alpha w_s \Delta}{q_s(1-\alpha)}$ to ensure the reservation price exceeds the maximum Nash equilibrium price.

We restrict attention to Nash equilibria that are symmetric across cities (i.e., $p_1^{(k)} = p_2^{(k)}$ for $k \in \{s, l\}$). Thus, the price paid by the small (large) buyer in city 1 is the same as the price paid by the small (large) buyer in city 2. A Nash equilibrium consists of a small-buyer price p_s and a large-buyer price p_l.

Proposition 7. *The Nash equilibria are any p_s and p_l such that $p_l \in \left[\Delta, p_l^N\right]$ where $p_l^N = \Delta + \frac{\alpha w_l \Delta}{q_l(1-\alpha)}$, and $p_s \in \left[\Delta, p_s^N\right]$ where $p_s^N = \Delta + \frac{\alpha w_s \Delta}{q_s(1-\alpha)}$.*

The maximum Nash price (for both types of buyers) increases as the size of the transaction declines. Sellers have weaker incentives to undercut a rival and steal a smaller transaction as doing so reduces the index but only yields a modest increase in spot profits. The maximum Nash price is increasing in a transaction's

[41] For example, OPIS does not apply minimum volume requirements in several of its spot gas liquids assessments (OPIS, 2025).

[42] For example, Fastmarkets' scrap metal indexes place equal weight on all price quotations: "[t]he simple monthly averages are calculated by dividing the sum of the price quotations by the number of quotations published during the calendar month" (Fastmarkets, 2025*a*). Johnson (2017) notes that "the vast majority of PRA commodity benchmarks are based on other methodologies" than the volume-weighted average.

weight (w_s or w_l) which reflects the fact that sellers are hesitant to undercut rivals and reduce the price of transactions weighted heavily by the PRA. Under equal weighting (i.e., $w_s = w_l = \frac{1}{4}$), $p_l^N < p_s^N$, which captures the standard intuition that the prices of smaller transactions are easier to inflate.

Let $\Pi_N^{max}(w_s)$ denote the maximum Nash equilibrium profit when the weight placed on small transactions is w_s (and, thus, $w_l = \frac{1}{2} - w_s$).[43]

Proposition 8. *i)* $\Pi_N^{max}(w_s) > \Pi_N^{max}(\frac{q_s}{2})$ *for all* $w_s \neq \frac{q_s}{2}$.

ii) $\Pi_N^{max}(0) > \Pi_N^{max}(\frac{1}{4})$ *when* $q_s > \frac{1}{4}$.

Part (i) of Proposition 8 establishes that weighting transactions by volume yields the lowest maximum Nash equilibrium profit level and, as a result, the highest buyer surplus (recall Remark 1).[44] In particular, excluding small transactions is never optimal in the sense that it does not yield the lowest prices and highest buyer surplus. Part (ii) shows that excluding small transactions results in higher profits (and therefore lower buyer surplus) than equal weighting when the asymmetry in buyer size is modest. These results reflect a drawback of excluding small transactions. Excluding small transactions reduces the prices paid by small buyers. Indeed, $w_s = 0$ implies that small buyers pay a price of Δ. However, excluding small transactions places increased weight on large transactions, which raises the price paid by large buyers (note that p_l^N is increasing in w_l in Proposition 7). Due to the larger demand of these buyers, this effect can create greater buyer harm and larger producer profits. Additionally, when small transactions are excluded, sellers need only raise the price of two transactions, rather than four, to inflate the index.

We next consider outcomes under collusion. $\delta^*(r)$ denotes the smallest discount factor such that charging both buyers their reservation price of r during collusion is sustainable. Let $\Pi^*(w_s)$ denote the maximum sustainable collusive profit when the weight placed on small transactions is w_s.

Proposition 9. *i) Suppose* $\delta < \delta^*(r)$. *Then,* $\Pi^*(w_s) > \Pi^*(\frac{q_s}{2})$ *for all* $w_s \neq \frac{q_s}{2}$.

ii) Suppose $\delta < \delta^*(r)$. *Then,* $\Pi^*(0) > \Pi^*(\frac{1}{4})$ *when* $q_l - q_s$ *is small.*

When $\delta \geq \delta^*(r)$, collusion at the monopoly prices (i.e., $p_s = p_l = r$) is sustainable for any weights. Thus, changes in the weights do not affect profits or buyer surplus. When $\delta < \delta^*(r)$, results parallel those under Nash competition. Part (i) of Proposition 9 establishes that volume weighting yields the lowest collusive profit and the highest buyer surplus (recall Remark 1). Part (ii) shows that excluding small transactions

[43] We analyze the maximum Nash equilibrium profit level because the minimum does not vary with the index weights. By Proposition 7, $p_s = p_l = \Delta$ is a Nash equilibrium for every w_s, and it yields a profit of Δ regardless of the weights. Thus, the choice of weight impacts the upper bound on the interval of possible profit levels across Nash equilibria, not the lower bound.

[44] Recall from Remark 1 that buyer surplus is declining in per-firm profit, so the maximum profit level corresponds to the minimum level of buyer surplus. Proposition 8 therefore pertains to the minimum possible buyer surplus across the Nash equilibria in Proposition 7. As $p_s = p_l = \Delta$ is a Nash equilibrium for all w_s, the maximum buyer surplus is $R - 2\Delta$ regardless of the weights. The choice of weight w_s therefore only impacts the lower bound on the interval of possible buyer surpluses across Nash equilibria, but not the upper bound.

increases seller profit and reduces buyer surplus relative to equal weighting when the asymmetry in buyer size is small. When small transactions are excluded, sellers need only collude on large transactions to inflate the index and substantially increase their contract profits. By excluding small transactions, the weight placed on large transactions is increased and the susceptibility of these transactions to collusion is enhanced.

Prior literature in finance studying the manipulation of financial benchmarks also provides support for volume-weighted indexes (Duffie and Dworczak, 2021; Baldauf, Frei and Mollner, 2022). Duffie and Dworczak (2021) find that "smaller trades (those whose prices are most easily distorted) are optimally downweighted" and that volume-weighted indexes are often optimal. These studies employ models that differ markedly from ours and are tailored toward the manipulation of financial benchmarks by traders holding derivative positions, rather than index distortion by oligopoly producers. Despite these differences, our analysis in this section reaches a similar general conclusion that volume-weighted indexes outperform alternative index designs. The preceding analysis therefore complements these existing studies, providing further support for volume weighting in an alternative setting involving product-market competition between oligopoly producers. Additionally, our findings suggest that a policy of excluding small transactions from the index can enhance firm profits and reduce buyer surplus.

8 Conclusion

We have analyzed the possibility of suppliers manipulating a price index referenced in long-term supply contracts. Consistent with prior literature, index-linked contracts relax spot market competition and facilitate collusion in our model. Motivated by recent antitrust cases, we then examined practices that distort the index beyond what conventional competition or collusion in the spot market would produce, alongside regulations and index designs intended to prevent these practices.

Our results suggest that recent regulation intended to prevent the dishonest and selective reporting of transactions can have subtle, counterintuitive effects. The ability to conceal a transaction from the PRA can strengthen incentives to undercut rivals and compete more aggressively in the spot market. Preventing concealment can therefore raise prices and reduce buyer surplus. Our results suggest regulations prohibiting concealment are most likely to harm buyers when sellers compete in the spot market, when sellers collude and the share of index-linked contracts is high, or when buyers are heterogeneous in their valuations of the good (as sellers who can no longer conceal a low-priced sale may refuse the low-valuation buyer outright). Regulation intended to prevent sellers from lying to the PRA can benefit sellers and harm buyers for a related reason: the ability to misreport intensifies spot market competition, so raising the probability that a false report is detected, or eliminating dishonest reporting altogether, can increase spot prices and seller profit while reducing buyer surplus. These findings do not imply that the concealment or dishonest reporting of transactions should be permitted. Rather, they indicate that regulations targeting these practices involve subtle strategic considerations and can have unintended consequences.

When buyers differ in the quantity they demand, excluding small transactions from the index, a practice employed in the determination of some (but not all) indexes, may reduce buyer surplus, particularly when the asymmetry in buyer size is modest. Weighting transactions by volume yields the lowest prices and the highest buyer surplus. Finally, refusals to deal can emerge when index-linked contracts are pervasive, and are most likely when the asymmetry in buyers' valuations for the good is large.

Our analysis involves a number of simplifying assumptions. We model collusion using grim trigger strategies. Our results are likely robust to alternative punishment strategies, as the key mechanism (i.e., deviations reducing the index and, as a result, a seller's contract profits) does not depend on the form of the punishment. We also abstract from negotiation/transaction costs in the spot market. This simplification focuses the analysis on incentives to manipulate the index rather than the beneficial effects of index-linked contracts (e.g., the potential to avoid the costly and repeated negotiation of price terms).

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A Proofs

This appendix contains proofs of the results in the main text. Extended proofs, containing complete derivations and supporting lemmas are presented in Technical Appendix B.

Proof of Proposition 1. By Lemma 7, any Nash equilibrium satisfies $p_{11}^{(k)} = p_{21}^{(k)}$ and $p_{22}^{(k)} = p_{12}^{(k)}$ for all k, so every buyer purchases from its local seller. Prices below Δ cannot arise, as materials price offers are non-negative and a firm pricing below its rival could raise its offer and retain its buyer. Next, consider the deviation in which a firm undercuts to steal buyer k in the rival city. The deviation raises spot profits by $\frac{1}{N_B}(1-\alpha)\left(p^{(k)}-\Delta\right)$, where $p^{(k)}$ is the equilibrium price paid by buyer k, and lowers the index, and hence contract profits, by $\alpha\left(\frac{\Delta}{2N_B}\right)$. No such deviation is profitable if and only if

$$\frac{1}{N_B}(1-\alpha)\left(p^{(k)}-\Delta\right) \leq \alpha\left(\frac{\Delta}{2N_B}\right) \iff p^{(k)} \leq \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)$$

as stated. □

Proof of Proposition 2. Collusive profits are strictly increasing in p_C, so the optimal collusive price is the largest $p_C \in [\Delta, r]$ satisfying the incentive compatibility constraint, under which the value of colluding weakly exceeds that of undercutting both cities and triggering Nash reversion. As shown in Appendix B.1, the constraint rearranges to

$$(p_C - \Delta)\left(1 - \delta\left(\frac{\alpha}{1-\alpha}\right) - 2\delta\right) \leq \frac{\Delta}{2}\left(\frac{\alpha}{1-\alpha}\right)(1-\delta).$$

If $\delta \geq \frac{1}{2+\left(\frac{\alpha}{1-\alpha}\right)}$, the left-hand side is non-positive, the constraint holds for every p_C. Thus, $p_C = r$. Otherwise, the constraint is equivalent to $p_C \leq \Delta + \frac{\Delta}{2}\left(\frac{\alpha}{1-\alpha}\right)\frac{1-\delta}{1-\delta\left(\frac{\alpha}{1-\alpha}\right)-2\delta}$. □

Proof of Lemma 1. The binding deviations steal the rival's buyers. Stealing the rival's high-valuation buyer raises spot profits by $\left(p^{(H)}-\Delta\right)\frac{1-\alpha}{2}$ while lowering the index, and hence contract revenues, by $\alpha\frac{\Delta}{4}$, so the deviation is unprofitable if and only if

$$\left(p^{(H)}-\Delta\right)\frac{1-\alpha}{2} \leq \alpha\frac{\Delta}{4} \iff p^{(H)} \leq \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right),$$

and the analogous condition bounds $p^{(L)}$. Prices below Δ are excluded as in Proposition 1, participation of low-valuation buyers requires $p^{(L)} \leq r_L$. See Lemma 9 for detailed deviations. □

Proof of Lemma 2. Lemma 11 characterizes refusal equilibria for arbitrary r_L by checking the seven deviations available to a seller: serving its own low-valuation buyer (L1), serving the rival's low-valuation buyer

(L2), stealing the rival's high-valuation buyer (H2), and their combinations. Three of the resulting conditions bind. Deviation L1 is unprofitable if and only if

$$\alpha\frac{p^{(H)}}{3} \geq r_L\frac{1-\alpha}{2} + \alpha\frac{r_L}{3} \iff p^{(H)} \geq r_L\left(\frac{3-\alpha}{2\alpha}\right),$$

deviation H2 is unprofitable if and only if $\alpha\frac{\Delta}{2} \geq \left(p^{(H)} - \Delta\right)\frac{1-\alpha}{2}$, i.e., $p^{(H)} \leq \Delta + \frac{\alpha}{1-\alpha}\Delta$, and deviation L1-H2 is unprofitable if and only if $p^{(H)}(5\alpha - 3) \geq (r_L - \Delta)(3-\alpha)$. The conditions for the remaining deviations are implied by these three. When $r_L > \Delta$, the right-hand side of the L1-H2 condition is strictly positive, so the condition can hold only if $5\alpha - 3 > 0$. Refusal equilibria therefore require $\alpha > \frac{3}{5}$, in which case the condition rearranges to $p^{(H)} \geq \frac{(r_L-\Delta)(3-\alpha)}{5\alpha-3}$. Combining the three conditions with the participation constraint $p^{(H)} \leq r_H$ yields the statement (Lemma 12). □

Proof of Lemma 3. Disclosing a transaction at p_{max} weakly raises the index, because p_{max} is no smaller than any average of disclosed prices, and strictly raises it whenever some disclosed price is smaller; together with the assumption that indifferent sellers disclose, $V_j^{(k)} = 1$ for every transaction at p_{max}. Conversely, if a transaction below p_{max} were disclosed, the seller with the lowest disclosed price could strictly raise the index, and hence its contract revenues, by concealing it, without affecting spot profits. Thus, in any subgame perfect Nash equilibrium, exactly the transactions at p_{max} are disclosed. □

Proof of Proposition 3. By Lemma 3, the index equals the highest transaction price p_{max}. Consider a candidate equilibrium with prices $p_1^N, p_2^N \geq \Delta$ and the deviation in which a seller undercuts the prices paid by the rival's buyers and conceals the resulting transactions: the index remains p_{max} while spot profits rise. Ruling out this deviation requires $0 \geq \left(p_1^N - \Delta\right) + \left(p_2^N - \Delta\right)$, which, given $p_i^N \geq \Delta$, implies $p_1^N = p_2^N = \Delta$. Conversely, at these prices no deviation is profitable, so the profile in which all prices equal Δ is the unique subgame perfect Nash equilibrium. □

Proof of Proposition 4. Part (i) holds because disclosing only the transactions at the higher price $p^{(1)}$ sets the index to its highest attainable level. For part (ii), the binding deviation undercuts both buyers and conceals the resulting transactions, leaving the index unchanged. Let $H\left(p^{(1)}, p^{(2)}\right)$ denote the slack in the resulting incentive compatibility constraint. As shown in Appendix B.3, $\frac{\partial H}{\partial p^{(1)}} > 0 \iff \delta > \delta_0$ while $\frac{\partial H}{\partial p^{(2)}} > 0 \iff \delta > \frac{1}{2}$, and the latter never holds in the relevant region since $\delta < \delta_C^*(r) < \frac{1}{2}$. Thus, raising $p^{(2)}$ always tightens the constraint. If $\delta < \delta_0$, raising either price tightens the constraint; as the constraint holds with equality at (Δ, Δ), no higher price is sustainable. If $\delta_0 \leq \delta < \delta_C^*(r)$, raising $p^{(1)}$ relaxes the constraint while raising the objective, so $p^{(1)} = r$, and $p^{(2)}$ rises until the constraint binds; solving

$$\frac{1}{1-\delta}\left(\frac{r}{2}+\frac{p^{(2)}}{2}+\frac{\alpha}{1-\alpha}r\right) = r + p^{(2)} - \Delta + \frac{\alpha}{1-\alpha}r + \frac{\delta}{1-\delta}\left(1+\frac{\alpha}{1-\alpha}\right)\Delta$$

for $p^{(2)}$ yields $p^{(2)}(\delta)$ as stated. Finally, $\delta_C^*(r)$ is exactly the threshold at which the monopoly profile (r, r) satisfies the constraint, so $\left(p^{(1)}, p^{(2)}\right) = (r, r)$ for $\delta \geq \delta_C^*(r)$. □

Proof of Proposition 5. Let $\Pi^{NC}(\delta)$ and $\Pi^{C}(\delta)$ denote per-firm profits under the optimal collusive strategies without and with concealment (Propositions 2 and 4). Lemma 14 establishes $\delta^*(r) < \delta_C^*(r)$, which orders the thresholds below and signs the relevant denominators. For $\delta < \delta_0$, concealment sustains only Nash pricing, so $\Pi^C = \Delta < \Delta + \frac{\Delta}{2}\left(\frac{\alpha}{1-\alpha}\right)\frac{1-\delta}{1-\delta\left(\frac{\alpha}{1-\alpha}\right)-2\delta} = \Pi^{NC}$, part (i). For $\delta_0 \leq \delta < \delta^*(r)$, substituting the optimal prices and simplifying, the comparison reduces to

$$\Pi^C(\delta) \gtrless \Pi^{NC}(\delta) \iff 2\frac{\frac{r}{\Delta}-1}{1+\left(\frac{\alpha}{1-\alpha}\right)} \gtrless \frac{1-2\delta}{1-\delta\left(\frac{\alpha}{1-\alpha}\right)-2\delta},$$

yielding parts (ii) and (iii). For $\delta^*(r) \leq \delta < \delta_C^*(r)$, firms sustain the monopoly price only without concealment, and $\Pi^{NC} - \Pi^C = \frac{1-\alpha}{2}\left(r - p^{(2)}(\delta)\right) > 0$, part (iv). For $\delta \geq \delta_C^*(r)$, both regimes sustain (r, r) and profits coincide, part (v). □

The proofs of Lemmas 4, 5 and 6 share a common structure, developed in Appendix B.4. Lemma 17 shows that, in any equilibrium, a transacting firm reports either its true price or r and a non-transacting firm reports either r or no sales, as any other report is dominated. Further lemmas characterize the stage-2 reporting equilibria on the path and after an undercutting deviation, following which the deviator's transaction prices average $p_N - \frac{\Delta}{2}$. Lemmas 30 and 31 then reduce the candidate symmetric equilibria to the three types in the text. For each type, it remains to derive the conditions under which the prescribed reporting is optimal in stage 2 and no firm undercuts in stage 1.

Proof of Lemma 4. Truthful reporting must be optimal both on the equilibrium path and following an undercutting deviation. The latter constraint binds: a deviator whose transaction prices average $p_N - \frac{\Delta}{2}$ prefers reporting truthfully to reporting r in stage 2 if and only if

$$\alpha\left(p_N - \frac{\Delta}{2}\right) \geq (1-\phi)\,\alpha r + \phi\left(\alpha\left(p_N - \frac{\Delta}{2}\right) - F\right) \iff \frac{\phi}{1-\phi}F \geq \alpha\left(r - p_N + \frac{\Delta}{2}\right),$$

which implies the analogous on-path condition $\frac{\phi}{1-\phi}F \geq \alpha\frac{r-p_N}{2}$. Given truthful reporting in stage 2, undercutting raises spot profits by $(1-\alpha)(p_N - \Delta)$ and lowers contract revenues by $\alpha\frac{\Delta}{2}$, so no firm undercuts if and only if $p_N \le \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)$. See Lemma 32. $\square$

Proof of Lemma 5. A semi-truthful equilibrium differs from a truthful equilibrium only in that a deviator lies in stage 2 after undercutting. Reporting r must now be optimal for the deviator, reversing the binding inequality from the previous proof: $\frac{\phi}{1-\phi}F \le \alpha\left(r - p_N + \frac{\Delta}{2}\right)$. Truthful reporting on the equilibrium path still requires $\frac{\phi}{1-\phi}F \ge \alpha\frac{r-p_N}{2}$, giving the condition in the lemma. Because a deviator now reports r and pays the penalty only upon detection, the no-undercutting condition becomes $\alpha p_N \ge (1-\alpha)(p_N - \Delta) + \alpha(1-\phi)r + \phi\left(\alpha\left(p_N - \frac{\Delta}{2}\right) - F\right)$, which rearranges to the price condition in the statement and holds automatically at $p_N = \Delta$. See Lemma 33. $\square$

Proof of Lemma 6. In a lying equilibrium, the binding reporting constraint arises on the equilibrium path: reporting the true price p_N while the rival reports r is unprofitable if and only if

$$(1-\phi)\alpha r + \phi(\alpha p_N - F) \ge (1-\phi)\alpha\frac{r+p_N}{2} + \phi\alpha p_N \iff \frac{\phi}{1-\phi}F \le \alpha\frac{r-p_N}{2},$$

which implies the corresponding off-path condition. In stage 1, undercutting raises spot profits by $(1-\alpha)(p_N - \Delta)$ but lowers detected contract revenues by $\phi\alpha\frac{\Delta}{2}$. No firm undercuts if and only if $p_N \le \Delta + \phi\frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)$, the factor ϕ reflecting that a deviator forgoes index revenue only when the lie is detected. See Lemma 34. $\square$

Proof of Proposition 6. Appendix B.4.2 derives the maximal Nash equilibrium price sustained by each type: $p_{N,L1}^{max} = \Delta + \phi\frac{\alpha}{1-\alpha}\frac{\Delta}{2}$ and $p_{N,L2}^{max} = r - 2\frac{\phi}{\alpha(1-\phi)}F$ for lying equilibria, analogous semi-truthful expressions, and the ϕ-independent truthful maximum $p_{N,T}^{max} = \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)$. Lemmas 42 and 43 assemble $p_N^{max}(\phi)$ piecewise from these expressions, with thresholds $F_1, \ldots, F_6$ in F determining which type prevails (Lemma 35). Since $p_N^{max}(1) = p_{N,T}^{max}$, part (i) follows by showing that every prevailing price is bounded above by $p_{N,T}^{max}$ (Lemmas 40 and 41). The bound is strict exactly when a non-truthful regime prevails, which occurs when $F < F_5 \equiv \frac{\alpha(1-\phi)}{\phi}\left(r - \frac{\Delta}{2(1-\alpha)}\right)$, with $F < F_5 \iff \phi < \frac{\alpha\left(r-\frac{\Delta}{2(1-\alpha)}\right)}{F+\alpha\left(r-\frac{\Delta}{2(1-\alpha)}\right)}$, the threshold in the statement (Lemma 46). For part (ii), let ϕ_2 solve $F_5(\phi_2) = F$, so that no truthful equilibrium with $p_N > \Delta$ exists for $\phi < \phi_2$. The maximal profit of each remaining type is then shown to lie strictly below $\Pi_N^{max}(1) = \Delta + \frac{\alpha}{1-\alpha}\left(\frac{\Delta}{2}\right)$ at ϕ_2. By continuity, there exists $\phi_1 < \phi_2$ such that $\Pi_N^{max}(\phi) < \Pi_N^{max}(1)$ on (ϕ_1, ϕ_2) (Lemma 48). $\square$

Proof of Proposition 7. The binding deviations steal a single buyer in the rival city: stealing buyer $i \in \{s, l\}$ raises spot profits by $q_i(1-\alpha)(p_i - \Delta)$ and lowers the index by $w_i\Delta$, reducing contract revenues by $\alpha w_i\Delta$. No such deviation is profitable if and only if

$$q_i(1-\alpha)(p_i - \Delta) \le \alpha w_i\Delta \iff p_i \le \Delta + \frac{\alpha w_i\Delta}{q_i(1-\alpha)} = p_i^N$$

for $i \in \{s,l\}$. The condition for stealing both buyers is the sum of these two and is therefore implied (see Appendix B.5). □

Proof of Proposition 8. The maximum Nash equilibrium profit occurs at price $\left(p_s^N, p_l^N\right)$ from Proposition 7:

$$\Pi_N^{max}(w_s) = (1-\alpha)\left(q_s p_s^N + q_l p_l^N\right) + \alpha\left(2w_s p_s^N + 2w_l p_l^N\right)$$

with $w_l = \frac{1}{2} - w_s$. Differentiating with respect to w_s and simplifying,

$$\frac{\partial \Pi_N^{max}}{\partial w_s} = \frac{4\alpha^2\Delta}{1-\alpha}\left(\frac{w_s}{q_s} - \frac{w_l}{q_l}\right),$$

so Π_N^{max} attains its unique minimum at the volume weights $w_s = \frac{q_s}{2}$, part (i). For part (ii), evaluating at $w_s = 0$ and $w_s = \frac{1}{4}$ and cancelling common terms, $\Pi_N^{max}(0) > \Pi_N^{max}(\frac{1}{4})$ reduces to $\frac{4}{q_l} > \frac{1}{q_s} + \frac{1}{q_l}$, i.e., $4q_s > q_s + q_l = 1$. □

Proof of Proposition 9. By Lemma 53, the binding deviation from a collusive profile with $p_i \geq p_i^N$ steals both buyers, so the incentive compatibility constraint takes the form $\delta \geq \delta^*(p_s, p_l)$, with $\delta^*\left(p_s^N, p_l^N\right) = 0$ and $\delta^*(r,r) = \delta^*(r)$. Suppose $\delta < \delta^*(r)$. Then the constraint binds at the optimum, and Lemmas 61 and 71 locate the optimum at $\left(p_s^{I2}, r\right)$ or $\left(p_s^N, p_l^{I1}\right)$ when $w_s < \frac{q_s}{2}$, and at $\left(r, p_l^{I2}\right)$ or $\left(p_s^{I1}, p_l^N\right)$ when $w_s > \frac{q_s}{2}$ (see Appendix B.5 for expressions). By the envelope theorem, the sign of $\frac{\partial \Pi^*}{\partial w_s}$ equals the sign of $p_s^* - p_l^*$ (Lemma 78), and the boundary solutions satisfy $p_s^* < p_l^*$ when $w_s < \frac{q_s}{2}$ and $p_s^* > p_l^*$ when $w_s > \frac{q_s}{2}$ (Lemmas 79 and 80). Hence Π^* is decreasing in w_s below $\frac{q_s}{2}$ and increasing above it (Lemma 81), so $\Pi^*(w_s) > \Pi^*(\frac{q_s}{2})$ for all $w_s \neq \frac{q_s}{2}$, part (i). Part (ii) follows from part (i) at $q_s = q_l = \frac{1}{2}$, where $\frac{q_s}{2} = \frac{1}{4}$, and from continuity of Π^* when $q_l - q_s$ is small. □