On the Design of Price Caps as Sanctions

by

David E. M. Sappington* and Douglas C. Turner**

Abstract

A ceiling has been imposed on the price at which Russian producers can sell oil. The price cap is intended to reduce Russian government tax revenue without increasing the world price of oil excessively. We show that such price caps can have counterintuitive effects. A price cap can induce sanctioned producers to increase their output, thereby increasing their revenue. A price cap can also reduce the world price of the homogenous product supplied by sanctioned and non-sanctioned producers. We also show that a welfare-maximizing price cap, which is often well below the unrestricted world price, can increase welfare substantially.

Keywords: price caps; sanctions; limiting tax revenue

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* Department of Economics, University of Florida
PO Box 117140, Gainesville, Florida 32611 USA (sapping@ufl.edu).

** Department of Data Science and Business Analytics, Florida Polytechnic University
4700 Research Way, Lakeland, Florida 33805 USA (dturner@floridapoly.edu).

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1 Introduction.

In response to Russia's military operations in Ukraine, G7 countries and other allies ("the Alliance") have imposed a cap on the price at which Russian firms can sell the oil they supply using key Alliance inputs (e.g., shipping and insurance).[1] The price cap is intended to reduce the (tax) revenue that Russia has available to finance its operations in Ukraine[2] without causing the sharp increase in the world price of oil that would likely arise if the Alliance were to withhold its inputs from Russian oil suppliers altogether.[3]

Using price caps to reduce the (tax) revenue that accrues to a sanctioned nation is a relatively novel undertaking,[4] and so has received little formal analysis to date. The primary analysis of this issue has (appropriately) examined the effects of price caps on non-renewable resources. Johnson, Rachel, and Wolfram et al. (2023a) (hereinafter JRW) demonstrate that an exogenous price reduction often encourages a producer to increase its supply of a non-renewable resource. A lower price reduces the value of the remaining reserves, thereby enhancing incentives for current extraction and sale of the resource.[5] It follows that the imposition of a binding cap on the price at which a supplier can sell a non-renewable product can induce the firm to increase its current supply of the product.

The present research is intended to complement JRW's important work by examining the effects of imposing a price ceiling on a product supplied by a "rogue" supplier (R), even if the product is not a non-renewable resource. Historically, restrictions have been imposed on many different types of exports. For example, the U.S. has restricted the flows of a broad spectrum of goods and services to and from many countries, including Cuba, Iran, Libya,

[1] See Wolfram et al. (2022), Baumeister (2023), Horwich (2023), and Johnson et al. (2023a) for details.

[2] Johnson et al. (2023a, p. 3) observe that "The price cap has two main goals. First, it is an integral part of a broader sanctions package designed to reduce Russia's foreign exchange revenues and reduce its capacity to wage war in Ukraine. ... The second goal of the price cap was to make it possible for Russian oil to stay on the world market."

[3] "Without the price cap policy, many analysts predicted that the EU embargo and services ban would prevent Russia from exporting 1-2 mbpd of oil, potentially increasing oil prices significantly and, in turn, adding to global inflationary pressures" (Wolfram et al., 2022, pp. 4-5). "[I]f Russian oil doesn't get to the market somewhere, then there's a global shortfall that would have significant ramifications for the price" (Horwich, 2023, p. 1).

[4] Neil Mehrotra, one of the architects of the cap on the price at which Russian oil can be sold, observes that "The price cap is an entirely novel effort. Typically, U.S. sanctions have been just outright prohibitions on certain types of business with certain entities. The price cap is novel in that we are trying to facilitate trade, but only under certain terms. ... I think this is definitely a new front in the tools of economic statecraft" (Horwich, 2023, p. 5). Johnson et al. (2023a, p. 16) observe that "The price cap on Russian oil reflects a novel approach to sanctions and the world is just beginning to understand its impacts on Russian oil revenues, geopolitical alignments, and oil trade."

[5] Also see Johnson et al. (2023b).

North Korea, and South Africa.[6] In principle, corresponding future restrictions might take the form of price restrictions rather than quantity restrictions. Therefore, it is important to understand the likely effects of price restrictions on a wide variety of products.

If all prices were exogenous in our static model, a binding ceiling on the price at which R can sell the product it supplies using an Alliance input would induce R to reduce its supply of this product.[7] Thus, there is no natural tendency for a binding price cap to induce expanded supply in our model, in contrast to JRW's model. Nevertheless, when prices are endogenous in our model,[8] the imposition of a price cap can induce R to increase its supply of the product, and thereby reduce the (unrestricted, endogenous) world price of the product.[9] These potentially counterintuitive findings arise because, in the presence of a binding price cap, an increase in R's output no longer reduces the price at which some of R's output is sold. This reduced exposure to the key deterrent to output expansion induces R to increase its output.[10]

Even as R's increased output reduces the world price, it can increase R's revenue. Consequently, a price cap can have two effects that differ from the effects typically recognized by policymakers. First, a price cap can reduce, not increase, the world price of the product in question. Second, a price cap on a portion of a sanctioned supplier's output can increase, not reduce, the supplier's revenue. These findings imply that the optimal design of a price cap entails important subtleties even in the absence of the intertemporal considerations in JRW's analysis.

We show that the subtle qualitative effects we identify can be economically significant under arguably plausible conditions. Specifically, modest reductions in the price cap below the prevailing world price of the product can cause R's revenue to increase substantially. Consequently, relatively stringent price caps can be required to reduce R's revenue. Furthermore, even stringent price caps can cause the world price of the product to decline.

We also characterize the price cap ($\overline{p}^*$) that maximizes the difference between consumer

[6] See U.S. Senate and House of Representatives (1986) and U.S. Government Accountability Office (1987, 1988, 2007, 2010, 2015).

[7] See Sappington and Turner (2023) for a formal proof of this conclusion.

[8] The uncapped equilibrium price of the product is affected by the strategic output decisions of industry suppliers in our model.

[9] The unrestricted world price is the price at which suppliers other than R sell the product. It is also the price at which R sells the output that it produces without using an Alliance input.

[10] In this respect, a price cap functions much like forward contracting (i.e., arranging to deliver future output at a fixed price that does not vary with the (spot) price that ultimately prevails). Allaz and Vila (1993) demonstrate that forward contracting can enhance incentives for output expansion by Cournot competitors. It can be shown that a corresponding effect arises in our model even if R is a monopolist.

surplus and a multiple ($d > 0$) of R's revenue. We demonstrate that the welfare-maximizing price cap often is well below the uncapped price of the product. We also demonstrate that the optimal price cap can increase welfare substantially under arguably plausible conditions. In addition, we show that welfare under $\overline{p}^*$ is higher than when the Alliance refuses to supply its input to R if: (i) d is sufficiently small; or (ii) access to the Alliance input reduces R's marginal cost sufficiently. In contrast, such a refusal can maximize welfare if conditions (i) and (ii) do not hold.

Our analysis and JRW's analysis are related to Sturm (2022a)'s analysis of the design of tariffs and taxes that maximize the welfare of a home country for any level of welfare reduction imposed on a sanctioned country.[11] However, our work differs substantially from Sturm's analysis in part because the suppliers in Sturm's model are price takers.[12] Consequently, the key considerations that underlie our primary findings do not arise in Sturm's model.[13]

The analysis proceeds as follows. Section 2 describes our model. Section 3 identifies conditions under which a binding price cap increases R's revenue and reduces the uncapped price of the sanctioned product. Section 4 examines the welfare-maximizing choice of a price cap. Section 5 summarizes our key findings and suggests directions for future research. The Appendix provides the proofs of all formal conclusions in the text.

[11] We share JRW's focus on the effects of a price cap rather than the effects of tariffs and taxes. However, we abstract from the stochastic prices, risk aversion, and degree of intertemporal elasticity of substitution that underlie JRW's key findings. We focus on the strategic interaction between the sanctioned supplier and a non-sanctioned supplier, both of which have market power. JRW explain that they "do not model the strategic interaction between Russia and other global producers, [although their] model features parameters that reflect the responsiveness of other producers, such as OPEC, to shocks originating from Russia or elsewhere" (p. 4) In contrast to JRW, we also examine the design of a welfare-maximizing price cap.

[12] Wachtmeister et al. (2022) also abstract from strategic oligopolistic interactions among suppliers. The authors compare the effects of price restrictions and quantity restrictions after estimating prevailing demand and supply functions. They find that price discounts often are better able than quantity restrictions to reduce the profits of Russian oil producers without reducing unduly the surplus secured by oil consumers. Ehrhart and Schlecht (2022) also do not model formally the strategic interactions among industry suppliers. The authors identify conditions under which a sanctioned supplier will accept the price cap imposed by buyers of its product.

[13] Furthermore, we examine the effects of a price cap on some of R's output, rather than a tax on all of R's output. Sturm (2022b) extends the analysis in Sturm (2022a) in part to examine the design of tariffs that maximize the difference between the welfare of the home country and a multiple of the welfare of the sanctioned country. Sturm (2022b) also considers retaliatory tariffs by the sanctioned country. Sturm (2023) extends his earlier work to focus on how the presence of non-sanctioning countries that can either purchase the sanctioned product or supply substitute products affects the optimal design of sanctions.

2 The Model.

We consider a setting in which R and a rival producer supply a homogeneous product. Aggregate (inverse) demand for the product is $P(Q) = a - b\,Q$, where $a > 0$ and $b > 0$ are parameters, Q is aggregate output, and $P(\cdot)$ denotes price.

The rival's cost of producing q units of output is $C(q) = c\,q + \frac{k}{2}\,q^2$, where $c > 0$ and $k > 0$ are parameters. R produces $q_A \geq 0$ units of output using an input (e.g., shipping and/or insurance) supplied by an (Alliance) input owner ("A"). R also produces $q_N \geq 0$ units of output without employing this input.[14] R's corresponding total cost is

$$C^R(q_A, q_N) \;=\; c_A\; q_A + \frac{k_A}{2}\,[\,q_A\,]^2 + c_N\; q_N + \frac{k_N}{2}\,[\,q_N\,]^2 + \frac{k^R}{2}\,[\,q_A + q_N\,]^2\,. \tag{1}$$

The parameter $k^R > 0$ scales the nonlinear component of R's "manufacturing" costs, i.e., costs that do not vary with the presence or absence of A's input. k_A and k_N ($\geq k_A$) are parameters that scale the nonlinear component of R's "transactions" costs, i.e., costs that can vary according to whether R's output is supplied using A's input. c_A and c_N ($\geq c_A$) scale the linear component of R's costs that can vary according to whether R employs A's input.[15]

To avoid relatively uninteresting outcomes in which some equilibrium output is 0 in the absence of a stringent price cap, we assume that market demand is sufficiently pronounced relative to cost, i.e., $a > \max\{\,c, c_N\,\}$. We also assume that costs are sufficiently nonlinear, i.e.,

$$D \;\equiv\; [\,2\,b + k\,]\,\left[\,k_N\left(k_A + k^R\right) + k_A\,k^R\,\right] + b\,k_A\,[\,3\,b + 2\,k\,] - b^2\,[\,b + k\,] \;>\; 0\,, \text{ and} \tag{2}$$

$$k_A\,[\,(a - c_N)\,(2\,b + k) - b\,(a - c)\,] \;>\; [\,c_N - c_A\,]\left[\,3\,b^2 + 2\,b\left(k + k^R\right) + k\,k^R\,\right]. \tag{3}$$

The activity in our static model proceeds as follows. First, A specifies the maximum price, $\overline{p}$, at which R can sell the output it produces using A's input. Then R chooses q_A and q_N, and the rival chooses q (simultaneously and noncooperatively). The resulting output, $Q = q_A + q_N + q$, gives rise to a market-clearing equilibrium price, $P(Q)$. Finally, R sells q_N and the rival sells q at price $P(Q)$. R also sells q_A at this price if $\overline{p} > P(Q)$. Otherwise, R sells q_A at price $\overline{p}$.

R's formal problem is:

$$\underset{q_A \geq 0,\; q_N \geq 0}{\text{Maximize}} \quad P_A(\,q_A + q_N + q\,)\,q_A + [\,a - b\,(\,q_A + q_N + q\,)\,]\,q_N - C^R(q_A, q_N)$$

[14] R can procure a substitute, but potentially more costly, input from a supplier other than A.

[15] For expositional ease, we abstract from any fixed costs of production.

$$\text{where} \quad P_A(Q) \;=\; \begin{cases} \bar{p} & \text{if } P(Q) \,\geq\, \bar{p} \\ P(Q) & \text{if } P(Q) \,<\, \bar{p}\,. \end{cases} \tag{4}$$

The rival's problem is:

$$\underset{q \geq 0}{\text{Maximize}} \quad [\, a - b\,(\, q_A + q_N + q\,)\,]\, q \,-\, C(q)\,. \tag{5}$$

3 A Price Cap Can Reduce $P(Q)$ and Increase R's Revenue.

Proposition 1 examines the relationships among the level of the price cap ($\bar{p}$), the equilibrium unrestricted price of the product ($P(Q)$), and R's output using A's input (q_A).

Proposition 1. *There exist values of the price cap, $0 < \bar{p}_1 < \bar{p}_2 < \bar{p}_3$, such that, in equilibrium, $q_A = 0$ if and only if $\bar{p} \leq \bar{p}_1$. Furthermore: (i) $\bar{p} < P(Q)$ if $\bar{p} \leq \bar{p}_2$; (ii) $\bar{p} = P(Q)$ if $\bar{p} \in (\bar{p}_2, \bar{p}_3\,]$; and (iii) $\bar{p} > P(Q)$ if $\bar{p} > \bar{p}_3$.*[16]

Proposition 1 reports that for the highest values of $\bar{p}$ (i.e., for $\bar{p} > \bar{p}_3$), the cap does not bind, so it has no impact on equilibrium outcomes. As $\bar{p}$ declines below $\bar{p}_3$,[17] the price cap binds and the equilibrium uncapped price declines at the same rate that $\bar{p}$ declines. Consequently, $P(Q) = \bar{p}$ over an entire range of values, $\bar{p} \in (\bar{p}_2, \bar{p}_3\,]$. As $\bar{p}$ declines further (i.e., for $\bar{p} \in (\bar{p}_1, \bar{p}_2\,]$), $\bar{p}$ falls below $P(Q)$, but R continues to supply $q_A > 0$. For the lowest values of $\bar{p}$ (i.e., for $\bar{p} \leq \bar{p}_1$), the price cap remains below $P(Q)$, and the particularly stringent price cap induces R to set $q_A = 0$.

To explain the presence of an entire range of price caps for which the capped and uncapped prices coincide, it is helpful to determine how equilibrium outputs change as the price cap declines below the level at which it first binds.

Proposition 2. *In equilibrium, for $\bar{p} \in (\bar{p}_2, \bar{p}_3)$, $\frac{dq_A}{d\bar{p}} < 0$, $\frac{dq_N}{d\bar{p}} < 0$, $\frac{dq}{d\bar{p}} > 0$, $\frac{dQ}{d\bar{p}} < 0$, and $\frac{dP(Q)}{d\bar{p}} = 1$.*

Proposition 2 reports that q_A and q_N both increase as $\bar{p}$ declines in $[\,\bar{p}_2, \bar{p}_3\,]$, causing $P(Q)$ to decline at the same rate that $\bar{p}$ declines. This finding reflects the net impact of two countervailing effects of a binding price cap. A reduction in $\bar{p}$ reduces the unit compensation that R derives from selling q_A. The reduced unit compensation induces R to reduce q_A, *ceteris paribus*. We call this the *compensation reduction effect* of a binding price cap. A

[16] The values of $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_3$ are specified in the Appendix. We assume that $\bar{p}_2 > c$ to help ensure that $q > 0$ in equilibrium.

[17] $\bar{p}_3$ is the equilibrium price, $P(Q)$, in the absence of a price cap.

countervailing *output enhancement effect* of a binding price cap also arises. The cap shields a portion of R's total output ($Q^R = q_A + q_N$) from the key drawback to an increase in Q^R, namely the associated reduction in $P(Q)$. The price cap thereby enhances R's incentive to increase its output.

When $\overline{p}$ is set marginally below the unrestricted equilibrium price ($\overline{p}_3$), the impact of the compensation reduction effect is relatively limited, so the marginally lower price that R secures for q_A induces a relatively small reduction in q_A, *ceteris paribus*. The predominant effect of reducing $\overline{p}$ marginally below $\overline{p}_3$ is to increase R's output, reflecting the output enhancement effect.[18] The expanded output reduces $P(Q)$, causing this price to decline at the same rate that $\overline{p}$ declines.[19]

q_A increases further as $\overline{p}$ declines farther below $\overline{p}_3$. The increase in q_A increases the magnitude of the compensation reduction effect, causing R's profit from supplying q_A to decline more rapidly as $\overline{p}$ declines. Eventually, the compensation reduction effect outweighs the output enhancement effect, inducing R to reduce q_A as $\overline{p}$ declines below $\overline{p}_2$.[20] The corresponding increase in $P(Q)$ causes $P(Q)$ to exceed $\overline{p}$ when $\overline{p} < \overline{p}_2$.[21]

Having established how a binding price cap affects equilibrium outputs and prices for $\bar{p} \in (\bar{p}_2, \bar{p}_3)$, we can determine the corresponding impact on R's revenue:

$$V(\overline{p}) \equiv \overline{p}\, q_A(\cdot) + P(Q(\cdot))\, q_N(\cdot)\,. \tag{6}$$

Proposition 3. *For $\bar{p} \in (\bar{p}_2, \bar{p}_3)$: (i) $V(\overline{p})$ is a strictly concave function of $\overline{p}$; (ii) $\frac{\partial V(\overline{p})}{\partial \overline{p}} \lesseqgtr 0 \Leftrightarrow \overline{p} \gtreqless \overline{p}_{V_3M}$ where $\overline{p}_{V_3M} \in [\,\overline{p}_2, \overline{p}_3)$; and (iii) $\overline{p}_{V_3M} = \overline{p}_2$ if $\Phi_1 \geq 0$, whereas $\overline{p}_{V_3M} > \overline{p}_2$ if $\Phi_1 < 0$, where*

$$\Phi_1 \equiv \left[k^R + \frac{b^2}{2\,b + k} \right] [\, k_A + k_N \,]\, A + 2\, b\, [\, b + k\,]\, c_A\, [\, k_N + b\,]$$
$$+ \, [\, 2\, b\, (b + k)\, c_N + A\, k_N \,]\, [\, k_A - b\,] \quad \textit{and} \quad A \equiv a\, [\, b + k\,] + b\, c\,. \tag{7}$$

Proposition 3 reports that as $\overline{p}$ declines below $\overline{p}_3$, a more stringent price cap *increases* R's revenue. Furthermore, R's revenue increases at a decreasing rate as $\bar{p}$ declines below $\bar{p}_3$, as illustrated in Figure 1.[22]

[**Figure 1 about Here**]

[18] Miller (2023) reports that Russian oil exports have increased since the Alliance imposed its price cap.

[19] R will not increase its output to a level that causes $P(Q)$ to decline below $\overline{p}$. If R did so, the entirety of its output would be exposed to any reduction in $P(Q)$, which would eliminate R's enhanced incentive to expand its output relatively aggressively.

[20] Lemma 3 (below) establishes that $\frac{dq_A}{d\,\overline{p}} > 0$ when $\bar{p} \in (\bar{p}_1, \bar{p}_2)$.

[21] Recall from Proposition 1 that $P(Q) > \overline{p}$ when $\overline{p} < \overline{p}_2$.

[22] As Figure 1 illustrates, $V_3(\bar{p})$ continues to increase as $\bar{p}$ declines below $\bar{p}_3$ to $\bar{p}_{V_3M}$, which: (i) exceeds $\bar{p}_2$

R's revenue increases as $\bar{p}$ declines marginally below $\bar{p}_3$ because the relatively pronounced output enhancement effect of a reduction in $\bar{p}$ induces R to increase q_A relatively rapidly. R continues to increase q_A as $\bar{p}$ declines further below $\bar{p}_3$. (Recall Proposition 2.) The higher level of q_A increases the impact of the compensation reduction effect, which causes R's revenue to increase more slowly as $\bar{p}$ declines in $(\bar{p}_2, \bar{p}_3)$. Consequently, as conclusion (i) in Proposition 3 reports, $V(\bar{p})$ is a concave function of $\bar{p}$ when $\bar{p} \in (\bar{p}_2, \bar{p}_3)$.

If q_A and q_N increase sufficiently rapidly as $\bar{p}$ declines in $(\bar{p}_2, \bar{p}_3)$, the compensation reduction effect can outweigh the output expansion effect, so a reduction in $\bar{p}$ can reduce R's revenue as $\bar{p}$ declines toward $\bar{p}_2$. This is the case when $\Phi_1 < 0$, as illustrated in Figure 1.[23] Alternatively, R's revenue can continue to increase as $\bar{p}$ declines for all $\bar{p} \in [\bar{p}_2, \bar{p}_3]$.[24]

Propositions 1 – 3 establish that a price cap can introduce two effects that are not commonly recognized in policy discussions. First, R's output and its revenue can increase as the cap declines below the level at which it first binds ($\bar{p}_3$). Second, the increase in R's output can cause $P(Q)$ to decline.[25]

To assess the practical importance of these potentially counterintuitive findings, it is useful to consider the following *baseline setting*. Although our analysis abstracts from the intertemporal considerations associated with non-renewable resources, the parameters in the baseline setting are chosen to reflect selected elements of Russia's activity in the oil sector, given the world's focus on the cap that is presently being imposed on the price of oil sold by Russian suppliers that employ Alliance inputs.[26]

We choose demand parameters a and b to ensure that in the absence of a price cap, the equilibrium price is 70 (dollars) and equilibrium total output is 90 million units (e.g., barrels of oil per day) when the price elasticity of demand is -0.75.[27] This elasticity, which exceeds common estimates of the price elasticity of demand for oil,[28] helps to ensure that the

if $\Phi_1 < 0$; and (ii) is equal to $\bar{p}_2$ if $\Phi_1 \geq 0$.

[23] It is apparent from (7) that if $\Phi_1 < 0$, then $k_A < b$. When k_A is relatively small, q_A is relatively large. Consequently, R's revenue from supplying q_A declines relatively rapidly as $\bar{p}$ declines (reflecting a relatively pronounced compensation reduction effect).

[24] Proposition 4 (below) establishes that $\overline{p}_3 - \overline{p}_2$ becomes smaller as c_A, k_A, or k^R increases. It is apparent from (7) that Φ_1 increases as c_A, k_A, or k^R increases. Thus, Conclusion (iii) in Proposition 3 indicates that $V(\overline{p})$ declines as $\overline{p}$ declines throughout the entire $[\bar{p}_2, \bar{p}_3]$ interval when this interval is relatively small.

[25] In contrast, $P(Q)$ would increase if R were denied all access to A's input.

[26] The Appendix considers substantial variation of the parameters in the baseline setting.

[27] In 2021 (the year prior to Russia's invasion of Ukraine), the average Brent oil price was approximately \$71 per barrel (U.S. Energy Information Administration, 2023). The average daily world production of oil in 2021 was approximately 89.9 million barrels (bp, 2022, p. 15).

[28] Caldara et al. (2016)'s review of studies of the short-run price elasticity of demand for oil reports an average elasticity of -0.22.

specified equilibrium price and output prevail in our duopoly model when arguably plausible values for cost parameters are adopted.[29] These considerations imply that $a = 163.33$ and $b = 1.03703 \times 10^{-6}$ because:

$$\frac{\partial Q}{\partial p} \frac{p}{Q} = -\frac{1}{b} \left[\frac{70}{90,000,000} \right] = -0.75 \;\; \Rightarrow \;\; b = 1.03703 \times 10^{-6} \, ; \;\; \text{and}$$

$$P(Q) = a - b\,[\,90,000,000\,] = 70 \;\; \Rightarrow \;\; a = 70 + 1.03703\,[\,90\,] \approx 163.33 \, .$$

The cost parameters in our baseline setting are chosen so that, in the absence of a price cap, R's equilibrium marginal cost when it employs A's input is approximately 25 (dollars), and R's corresponding average variable cost is approximately 15.[30] Furthermore, the rival's cost is presumed to parallel's R's cost when R employs A's input (i.e., $c = c_A$ and $k = k_A + k^R$). In addition, we assume $c_A = \beta\, c_N$ and $k_A = \beta\, k_N$, and set $\beta = 0.5$ to capture R's cost saving from employing A's input. Table 1 records the parameter values in the baseline setting.[31]

Parameter	Parameter Value		Parameter	Parameter Value
a	163.33		c_N	5
b	1.03703×10^{-6}		k_N	1×10^{-6}
c_A	2.5		c	2.5
k_A	5×10^{-7}		k	6×10^{-7}
k^R	1×10^{-7}			

Table 1. Parameters in the Baseline Setting.

Table 2 identifies key equilibrium outcomes in the baseline setting.[32] The first column of data implies that $P(Q) = \overline{p}$ as $\overline{p}$ declines from $\overline{p}_3 = 71.52$ to $\overline{p}_2 = 56.35$.[33] Thus, as indicated

[29] The identified equilibrium price and output can arise when equilibrium demand is substantially less elastic if the number of industry suppliers is sufficiently large. We consider duopoly competition for analytic ease.

[30] Horwich (2023) estimates Russia's marginal cost of supplying oil to be approximately \$20 per barrel. The Center for Research on Energy and Clean Air (2023) estimates this cost to be between \$2.70 and \$25. Hausmann (2022) suggests that Russia's average variable cost may be less than \$6 per barrel. Kennedy (2022)'s corresponding estimate is between \$20 and \$25 per barrel.

[31] These parameters ensure that in the absence of a binding price cap, $P(Q) = 71.52$, $Q = 88.535$ million, R's marginal cost ($c_A + k_A\, q_A + k^R\,[\,q_A + q_N\,]$) is 23.43, and R's average variable cost ($c_A + \frac{1}{2}\, k_A\, q_A + \frac{1}{2}\, k^R \left[\frac{(q_A + q_N)^2}{q_A} \right]$) is 13.34.

[32] It can be verified that $\Phi_1 > 0$ (so $\overline{p}_{V_3 M} = \overline{p}_2$) in the baseline setting. (Recall Proposition 3.) $\Phi_1 < 0$ (so $\overline{p}_{V_3 M} > \overline{p}_2$) if, for example, k_A is reduced by 50% (to 2.5×10^{-7}) while all other parameters remain at their values in the baseline setting.

[33] All entries in Table 2 (and subsequent tables) are rounded.

in the last column in Table 2, $P(Q)$ declines at the same rate that $\overline{p}$ declines as $\overline{p}$ declines by as much as 21% below $\overline{p}_3$. The middle columns in Table 2 report corresponding changes in R's revenue. As illustrated in Figure 2 and as summarized in the last column in Table 2, R's revenue increases by approximately 19% as $\overline{p}$ declines from $\overline{p}_3 = 71.52$ to $\overline{p}_2 = 56.35$.

Price Cap		R's Revenue		Variation
$\overline{p}_1 = 41.82$		$V(\overline{p}_1) = 2.70 \times 10^9$		$\frac{\overline{p}_3 - \overline{p}_2}{\overline{p}_3} = 0.21$
$\overline{p}_2 = 56.35$		$V(\overline{p}_2) = 3.95 \times 10^9$		$\frac{V(\overline{p}_2) - V(\overline{p}_3)}{V(\overline{p}_3)} = 0.19$
$\overline{p}_3 = 71.52$		$V(\overline{p}_3) = 3.32 \times 10^9$		

Table 2. Equilibrium Outcomes in the Baseline Setting.

[**Figure 2 about Here**]

Table 2 indicates that under arguably plausible conditions, $P(Q)$ declines at the same rate that $\overline{p}$ declines for a relatively broad range of $\overline{p}$ values. Furthermore, more stringent price caps can increase R's equilibrium revenue considerably. Table A1 in the Appendix demonstrates that values of $\frac{\overline{p}_3 - \overline{p}_2}{\overline{p}_3}$ and $\frac{V(\overline{p}_2) - V(\overline{p}_3)}{V(\overline{p}_3)}$ similar to those in Table 1 arise in equilibrium as parameter values diverge from their values in the baseline setting below $\overline{p}_3$.[34]

Proposition 4 identifies how production costs influence the extent of the range in which $P(Q)$ declines at the same rate that $\overline{p}$ declines.

Proposition 4. $\overline{p}_3 - \overline{p}_2$ *increases as: (i)* c_A, k_A, *or* k^R *declines; (ii)* c *or* c_N *increases; or (iii)* k_N *increases if* $k_A - b$ *is sufficiently small.*

Conclusion (i) in Proposition 4 holds because q_A increases as R's cost of supplying q_A declines (i.e., as c_A, k_A, or k^R declines). The higher level of q_A increases the amount of R's output that is not exposed to a reduction in $P(Q)$. A binding price cap thereby provides R with a relatively strong incentive to expand its output aggressively, which increases the range of $\overline{p}$'s for which q_A and q_N increase as $\overline{p}$ declines (so $P(Q) = \overline{p}$).

Conclusions (ii) and (iii) in Proposition 4 reflect in part the fact that q_N declines as c_N or k_N increases. The reduction in q_N leads R to increase q_A for two reasons. First, it is apparent from (1) that R's marginal cost of supplying q_A declines as q_N declines. This *marginal cost effect* of a reduction in q_N induces R to increase q_A. Second, the amount of output that R sells at price $P(Q)$ declines as q_N declines. This reduced exposure to the profit-reducing effects of a reduction in $P(Q)$ limits R's concern about the reduction in $P(Q)$ caused by an

[34] Sappington and Turner (2023) provides additional evidence to this effect.

increase in q_A. This *exposure effect* of a reduction in q_N also induces R to increase q_A.[35] The increase in q_A induced by the marginal cost effect and the exposure effect of a reduction in q_N increases the range of $\overline{p}$'s for which q_A and q_N increase as $\overline{p}$ declines, for the reasons noted immediately above.

Finally, observe that q declines and $P(Q)$ increases as c increases. The higher price and increased potential market share for R enhances R's incentive to increase output aggressively when a binding price cap eliminates the exposure of some of R's output to the corresponding reduction in $P(Q)$. Consequently, $\overline{p}_3 - \overline{p}_2$ increases as c increases.[36]

4 Welfare.

We now examine how $\bar{p}$ can be set to limit R's revenue without harming consumers unduly. To do so, we assume that welfare, $W(\cdot)$, is the difference between consumer surplus, $S(\cdot)$, and a multiple ($d > 0$) of R's revenue. Formally:

$$W(\overline{p}) \;=\; S(\overline{p}) - d\,[\,\overline{p}\;q_A(\overline{p}) + P(Q(\overline{p}))\,q_N(\overline{p})\,] \tag{8}$$

where $S(\overline{p})$ denotes equilibrium consumer surplus when the price cap is $\overline{p}$.[37] To characterize $\bar{p}^* \equiv \arg\max\{\,W(\bar{p})\,\}$, we first examine the properties of consumer surplus when $\bar{p} \in (\bar{p}_2, \bar{p}_3)$.

Lemma 1. *For $\bar{p} \in (\bar{p}_2, \bar{p}_3)$, $S(\overline{p})$ is a strictly decreasing, strictly convex function of $\overline{p}$.*

Lemma 1 establishes that consumer surplus increases at an increasing rate as $\bar{p}$ declines in $(\bar{p}_2, \bar{p}_3)$. (See Figure 1.) This is the case because reductions in $\bar{p}$ and $P(Q)$ both increase consumer surplus. (Recall that $\bar{p} = P(Q)$ for all $\bar{p} \in (\bar{p}_2, \bar{p}_3)$.) As $\bar{p}$ declines in $(\bar{p}_2, \bar{p}_3)$, equilibrium output increases, reflecting the output enhancement effect. (Recall Proposition 2.) The increased output causes consumer surplus to increase more rapidly as the prevailing price ($\bar{p} = P(Q)$) declines.

Proposition 3 and Lemma 1 imply that welfare is a strictly convex function of $\bar{p}$ for $\bar{p} \in (\bar{p}_2, \bar{p}_3\,]$. Consequently, $\bar{p}^*$ is never in $(\bar{p}_2, \bar{p}_3)$. Furthermore, a binding price cap always improves welfare, i.e., $\bar{p}^* < \bar{p}_3$. This conclusion reflects in part:

[35] The exposure effect is relatively pronounced when $P(Q)$ is relatively sensitive to changes in output, i.e., when b is relatively large (so $k_A - b$ is relatively small).

[36] Corresponding analytic conclusions about the impact of parameter values on $\frac{\overline{p}_3 - \overline{p}_2}{\overline{p}_3}$ are not available. Numerical solutions reveal that $\frac{\overline{p}_3 - \overline{p}_2}{\overline{p}_3}$ often increases as: (i) a, c_A, k_A, or k^R declines; or (ii) c_N, k_N, k, or b increases. Thus, $\frac{\overline{p}_3 - \overline{p}_2}{\overline{p}_3}$ and $\overline{p}_3 - \overline{p}_2$ tend to become relatively large as c_N and k_N increase relative to c_A and k_A, i.e., as it becomes relatively costly for R to "evade" the effects of the price cap. This is the case in the baseline setting, for example, and for substantial variation in parameters around their values in the baseline setting.

[37] We assume that efficient rationing prevails, so the marginal consumer valuation of each unit of q_A that is sold is at least $P(Q)$.

Lemma 2. *$V(\bar{p}_1) < V(\bar{p}_3)$.*

Lemma 2 provides the intuitive conclusion that R's revenue is lower when the price cap is so stringent that it induces R to set $q_A = 0$ than when no price cap is imposed. This finding implies that if d is sufficiently large, then welfare is highest when a binding price cap is imposed. A binding price cap also maximizes welfare when d is small because consumer surplus increases as $\bar{p}$ declines below $\bar{p}_3$. (Recall Lemma 1 and Figure 1.) Consequently, we have:

Proposition 5. *$\bar{p}^* \in [\bar{p}_1, \bar{p}_2]$.*

To determine how $\bar{p}^*$ varies with the prevailing economic environment, it is helpful to determine how $V(\cdot)$ and $S(\cdot)$ vary with $\bar{p}$ when $\bar{p} \in (\bar{p}_1, \bar{p}_2)$. To do so, it is helpful to first establish how equilibrium outputs change as $\bar{p}$ declines in $(\bar{p}_1, \bar{p}_2)$.

Lemma 3. *In equilibrium, for $\bar{p} \in (\bar{p}_1, \bar{p}_2)$, $\frac{dq_A}{d\bar{p}} > 0$, $\frac{dq_N}{d\bar{p}} < 0$, $\frac{dq}{d\bar{p}} < 0$, $\frac{dQ^R}{d\bar{p}} > 0$, $\frac{dQ}{d\bar{p}} > 0$, and $\frac{dP(Q)}{d\bar{p}} < 0$.*

Lemma 3 reflects standard considerations. As the price cap declines below $\bar{p}_2$, the reduced unit compensation for q_A induces R to reduce q_A. The reduction in q_A increases the price at which q_N and q are sold, which induces increases in these outputs.[38] The reduction in q_A exceeds the increase in q_N and q, so Q^R and Q decline, and $P(Q)$ increases.

Lemmas 4 and 5 establish how $V(\cdot)$ and $S(\cdot)$ vary with $\bar{p}$ when $\bar{p} \in (\bar{p}_1, \bar{p}_2)$.

Lemma 4. *For $\bar{p} \in (\bar{p}_1, \bar{p}_2)$: (i) $V(\overline{p})$ is a strictly convex function of $\overline{p}$; (ii) $\frac{\partial V(\overline{p})}{\partial \overline{p}} \lesseqgtr 0 \Leftrightarrow \overline{p} \lesseqgtr \overline{p}_{V_2 m}$ where $\overline{p}_{V_2 m} \in [\overline{p}_1, \overline{p}_2)$; and (iii) $\overline{p}_{V_2 m} > \overline{p}_1$ if $\Phi_2 \geq 0$, where*

$$\begin{aligned} \Phi_2 \equiv \; & \{ k^R [2b + k] \left[k^R (2b + k) + 2b(3b + 2k) \right] + k_N [2b + k] \left[k^R (2b + k) + b^2 \right] \\ & + b^2 \left[5b^2 + 6bk + 2k^2 \right] \} \, c_N - \left\{ b[3b + 2k] + [2b + k] \left[k_N + k^R \right] \right\}^2 c_A \\ & - b \left[b^2 - k k_N + (2b + k) k^R \right] [a(b + k) + bc]. \end{aligned} \tag{9}$$

Lemma 5. *For $\bar{p} \in (\bar{p}_1, \bar{p}_2)$: (i) $S(\overline{p})$ is a strictly concave function of $\overline{p}$; (ii) $\frac{\partial S(\overline{p})}{\partial \overline{p}} \gtreqless 0 \Leftrightarrow \overline{p} \lesseqgtr \overline{p}_{S_2 M}$ where $\overline{p}_{S_2 M} \in (\overline{p}_1, \overline{p}_2]$; and (iii) $\overline{p}_{S_2 M} > \overline{p}_{V_2 m}$.*

Lemma 4 reports that R's revenue declines as $\bar{p}$ declines below $\bar{p}_2$ toward $\overline{p}_{V_2 m}$, the $\overline{p} \in [\overline{p}_1, \overline{p}_2]$ at which $V(\overline{p})$ is minimized. (See Figure 1.) The revenue reduction reflects: (i)

[38] The reduction in q_A also reduces R's marginal cost of producing q_N, which enhances R's incentive to increase q_N.

the lower unit compensation that R receives for q_A as $\bar{p}$ declines; and (ii) the reduction in q_A that arises as $\bar{p}$ declines in $(\bar{p}_1, \bar{p}_2)$. (Recall Lemma 3.) The convexity of $V(\overline{p})$ reported in Lemma 4 implies that $V(\cdot)$ declines more slowly as $\bar{p}$ declines toward $\overline{p}_{V_2m}$ (as depicted in Figure 1). This is the case because R's supply of q_A declines as $\bar{p}$ declines in this range, which diminishes the revenue-reducing compensation reduction effect of a more stringent price cap.

Lemma 5 reports that when $\overline{p}_{S_2M}$, the $\overline{p} \in [\overline{p}_1, \overline{p}_2]$ at which $S(\overline{p})$ is maximized, is strictly below $\overline{p}_2$ (as in Figure 1), consumer surplus initially increases as $\bar{p}$ declines below $\bar{p}_2$. The increase in $S(\cdot)$ reflects in part the lower $\bar{p}$ at which q_A is sold. The concavity of $S(\cdot)$ reported in Lemma 5 implies that the rate at which consumer surplus increases as $\bar{p}$ declines diminishes as $\bar{p}$ declines toward $\overline{p}_{S_2M}$. The diminishing rate of increase in $S(\cdot)$ reflects the reduction in q_A that R implements as $\bar{p}$ declines in $(\bar{p}_1, \bar{p}_2)$. Eventually, $S(\cdot)$ declines as $\bar{p}$ declines (below $\overline{p}_{S_2M}$), reflecting the increase in $P(Q)$ induced by the reduction in q_A (and the fact that the reduction in $\bar{p}$ reduces the price at which a relatively small number of units are sold as $\bar{p}$ approaches $\bar{p}_1$). (See Figure 1.)

For emphasis, we state the following direct implication of Lemma 4.

Corollary to Lemma 4. $\left.\frac{\partial V(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_1} < 0$ *if* $\Phi_2 \geq 0$.

This corollary states that R's revenue declines as $\bar{p}$ increases above $\bar{p}_1$ when $\Phi_2 \geq 0$. Consequently, relaxing the price cap by raising $\overline{p}$ above the level at which it induces R to set $q_A = 0$ reduces R's revenue when $\Phi_2 \geq 0$. As (9) suggests, $\Phi_2 \geq 0$ if $c_N - c_A$ is sufficiently large. In this case, R reduces q_N relatively rapidly as q_A increases in response to the increase in $\overline{p}$ above $\overline{p}_1$. The reduction in q_N (sold at the relatively high price, $P(Q)$) reduces R's revenue, despite the increase in q_A (sold at the relatively low price, $\overline{p}$).[39]

Lemma 5 and the Corollary to Lemma 4 imply that when $\Phi_2 \geq 0$, an increase in $\bar{p}$ above $\bar{p}_1$ both reduces R's revenue and increases consumer surplus.[40] Consequently, the welfare-maximizing price cap generates a strictly higher level of welfare than does a refusal to supply any of the Alliance input to R (which would induce R to set $q_A = 0$). In contrast, such a refusal (or setting $\bar{p} \leq \bar{p}_1$) maximizes welfare when $\Phi_2 < 0$ (so R's revenue increases as $\bar{p}$ increases above $\bar{p}_1$) and society is primarily concerned with limiting R's revenue. Formally:

[39] $\Phi_2 < 0$ in the baseline setting, and for the variations in the baseline parameters identified in Table A1. $\Phi_2 > 0$ if, for example, c_N exceeds 21 while all other parameters remain at their values in the baseline setting.

[40] Consumer surplus increases in part because as $\bar{p}$ increases above $\bar{p}_1$, there is no first-order effect on consumer surplus associated with q_A (because $q_A \approx 0$). Furthermore, when c_A is sufficiently small relative to c_N, the increase in $\bar{p}$ induces R to increase q_A by more than q_N and q decline, so $P(Q)$ declines.

Proposition 6. *$\bar{p}^* > \bar{p}_1$ if $\Phi_2 \geq 0$. $\bar{p}^* = \bar{p}_1$ if $\Phi_2 < 0$ and d is sufficiently large.*

The properties of $V(\bar{p})$ and $S(\bar{p})$ in $(\bar{p}_1, \bar{p}_2)$ provide:

Proposition 7. *$\bar{p}^* \in [\bar{p}_{V_2m}, \bar{p}_{S_2M}]$. Furthermore: (i) $\bar{p}^* < \bar{p}_{S_2M}$ when $\bar{p}_{S_2M} < \bar{p}_2$ and $d > 0$; (ii) $\bar{p}^* > \bar{p}_{V_2m}$ when $\bar{p}_{V_2m} > \bar{p}_1$; (iii) $\bar{p}^* \to \bar{p}_{S_2M}$ as $d \to 0$; and (iv) $\bar{p}^* \to \bar{p}_{V_2m}$ as $d \to \infty$.*

Proposition 7 states that the welfare-maximizing price cap ($\bar{p}^*$) lies between the $\bar{p}$ that minimizes R's revenue ($\bar{p}_{V_2m}$) and the $\bar{p}$ that maximizes consumer surplus ($\bar{p}_{S_2M}$). This finding reflects two observations. First, $\bar{p}^*$ cannot lie in $(\bar{p}_{S_2M}, \bar{p}_2)$ when $\bar{p}_{S_2M} < \bar{p}_2$. This is the case because if $\bar{p} \in (\bar{p}_{S_2M}, \bar{p}_2)$, then a reduction in $\bar{p}$ would both reduce R's revenue and increase consumer surplus. Second, $\bar{p}^*$ cannot lie in $(\bar{p}_1, \bar{p}_{V_2m})$ when $\bar{p}_{V_2m} > \bar{p}_1$. This is the case because if $\bar{p} \in (\bar{p}_1, \bar{p}_{V_2m})$, then an increase in $\bar{p}$ would both reduce R's revenue and increase consumer surplus. (See Figure 1.)

Conclusion (i) in Proposition 7 reports that $\bar{p}^*$ lies below $\bar{p}_{S_2M}$ when $\bar{p}_{S_2M} < \bar{p}_2$ and $d > 0$.[41] This conclusion arises because a reduction in $\bar{p}$ has no first-order effect on $S(\bar{p})$ when $\bar{p} = \bar{p}_{S_2M}$. In contrast, the same reduction in $\bar{p}$ reduces R's revenue because q_A declines as $\bar{p}$ declines in $(\bar{p}_1, \bar{p}_2)$.[42] (Recall Lemma 3.) Conclusion (ii) in Proposition 7 reports that $\bar{p}^*$ exceeds $\bar{p}_{V_2m}$ when $\bar{p}_{V_2m} > \bar{p}_1$. This is the case because an increase in $\bar{p}$ has no first-order effect on $V(\bar{p})$ when $\bar{p} = \bar{p}_{V_2m}$. In contrast, the same increase in $\bar{p}$ increases consumer surplus due to the reduction in $P(Q)$ induced by the corresponding increase in q_A.

Conclusion (iii) in Proposition 7 provides the intuitive conclusion that $\bar{p}^*$ approaches the level of $\bar{p}$ that maximizes $S(\bar{p})$ as the social concern with consumer surplus becomes particularly pronounced. Similarly, as conclusion (iv) in Proposition 7 reports, $\bar{p}^*$ approaches the level of $\bar{p}$ that minimizes $V(\bar{p})$ as the social concern with limiting R's revenue becomes particularly pronounced.

Figure 3 illustrates how welfare varies with $\bar{p}$ in the baseline setting when $d = \frac{1}{2}$, so $W(\bar{p}) = S(\bar{p}) - \frac{1}{2}V(\bar{p})$. As $\bar{p}$ declines from $\bar{p}_3 = 71.52$ to $\bar{p}^* = 54.31$, welfare increases by nearly 50%, from 2.40 (million dollars) to 3.58. Welfare then declines to 2.06 as $\bar{p}$ declines from $\bar{p}^*$ to $\bar{p}_1 = 41.86$.[43] Table A1 in the Appendix reports that the welfare-maximizing price

[41] $\bar{p}^* = \bar{p}_{S_2M}$ when $d = 0$.

[42] More precisely, $\left.\frac{\partial S(\bar{p})}{\partial \bar{p}}\right|_{\bar{p}=\bar{p}_{S_2M}} = 0 < \left.\frac{\partial V(\bar{p})}{\partial \bar{p}}\right|_{\bar{p}=\bar{p}_{S_2M}}$ when $\bar{p}_{S_2M} < \bar{p}_2$.

[43] Numerical solutions reveal that $W(\bar{p}^*)$ often increases as: (i) c_A, k_A, k^R, c, or k declines; or (ii) a, c_N, k_N, or b increases. This is the case, for example, as parameters vary (substantially) around their values in the baseline setting. These findings indicate in part that higher levels of welfare can often be achieved when it is more costly for R to diminish the impact of a binding price cap by producing more of its output

cap generates corresponding increases in welfare as parameter values diverge substantially from the levels in the baseline setting. Table A2 in the Appendix reports how $\bar{p}^*$, $\frac{\bar{p}^*}{\bar{p}_3}$, $W(\bar{p}^*)$, and $\frac{W(\bar{p}^*) - W(\bar{p}_3)}{|\, W(\bar{p}_3)\,|}$ vary as d varies in the baseline setting.[44]

[**Figure 3 about Here**]

To understand how $\bar{p}^*$ changes as industry costs change, it is helpful to first determine how equilibrium outputs change as costs change.

Lemma 6. *When $\bar{p} \in (\bar{p}_1, \bar{p}_2)$:*

$$\text{(i)}\quad \frac{dq_A}{dc} < 0\,,\ \frac{dq_N}{dc} > 0\,,\ \frac{dQ^R}{dc} \gtreqless 0 \ \Leftrightarrow\ k_A \gtreqless b\,,$$

$$\frac{dq}{dc} < 0 \text{ if } b \text{ is sufficiently small}\,, \text{ and } \frac{dQ}{dc} < 0\,;$$

$$\text{(ii)}\quad \frac{dq_A}{dc_A} < 0\,,\ \frac{dq_N}{dc_A} > 0\,,\ \frac{dQ^R}{dc_A} < 0\,,\ \frac{dq}{dc_A} > 0\,, \text{ and } \frac{dQ}{dc_A} < 0\,;$$

$$\text{(iii)}\quad \frac{dq_A}{dc_N} > 0\,,\ \frac{dq_N}{dc_N} < 0\,,\ \frac{dQ^R}{dc_N} \lesseqgtr 0 \ \Leftrightarrow\ k_A \gtreqless b\,,$$

$$\frac{dq}{dc_N} \gtreqless 0 \ \Leftrightarrow\ k_A \gtreqless b\,, \text{ and } \frac{dQ}{dc_N} \lesseqgtr 0 \ \Leftrightarrow\ k_A \gtreqless b\,.$$

To understand the conclusions in Lemma 6, recall that q_A declines as q_N increases due to the marginal cost effect and the exposure effect of an increase in q_N.[45] When b is large, $P(Q)$ is relatively sensitive to changes in q_A. Consequently, when b is large, an increase in q_N induces R to reduce q_A relatively rapidly (reflecting the relatively pronounced exposure effect). Therefore, when b is sufficiently large, Q^R declines as q_N increases because q_A declines more rapidly than q_N increases. These considerations help to explain the findings in Lemma 6 as follows.

As c increases: (i) q_N increases, reflecting in part the weaker competitive position of R's rival; (ii) q_A declines in response, reflecting the marginal cost effect and the exposure effect; (iii) when b is sufficiently large ($b > k_A$), q_A declines more rapidly than q_N increases, causing

without employing the Alliance input.

[44] The absolute value sign in the denominator of the proportionate increase in welfare reflects the fact that welfare as defined in (8) can be negative if d is suffiicently large.

[45] Recall that the marginal cost effect arises because R's marginal cost of supplying q_A increases as q_N increases. The exposure effect arises because the amount of output that R sells at price $P(Q)$ increases as q_N increases.

Q^R to decline; (iv) total output declines, reflecting in part the higher industry costs; and (v) q declines if b is sufficiently small, in which case Q^R increases.[46]

As c_A increases: (i) q_A declines, reflecting R's higher cost; (ii) q_N increases because the reduction in q_A increases $P(Q)$ and reduces R's marginal cost of supplying q_N; (iii) Q^R declines in part because the increase in q_N further enhances R's incentive to reduce q_A (due to the marginal cost effect and the exposure effect); (iv) q increases because q and Q^R are strategic substitutes; and (v) Q declines, reflecting in part the higher industry production cost.

As c_N increases: (i) q_N declines, reflecting R's higher cost; (ii) q_A increases, reflecting the marginal cost effect and the exposure effect; (iii) Q^R declines if b is sufficiently small, reflecting the relatively limited exposure effect; (iv) q increases if b is sufficiently small, because q and Q^R are strategic substitutes; and (v) Q declines if b is sufficiently small, reflecting the relatively large decline in Q^R in this case (due to the relatively limited exposure effect).

These considerations help to explain how $\overline{p}^*$ changes as industry costs change.

Proposition 8. *When* $\bar{p}^* \in (\bar{p}_1, \bar{p}_2)$*: (i)* $\frac{d\,\overline{p}^*}{dc_A} > 0$*; (ii)* $\frac{d\,\overline{p}^*}{dk_A} > 0$*; (iii)* $\frac{d\,\overline{p}^*}{dc} > 0$*; and (iv)* $\frac{d\,\overline{p}^*}{dc_N} < 0$*.*

To understand the conclusions in Proposition 8, first observe that when $\overline{p}^* \in (\bar{p}_{V_2m}, \bar{p}_{S_2M})$, a more stringent price cap reduces R's revenue and also reduces consumer surplus (by increasing $P(Q)$). (See Figure 1.) Both effects reflect in part the reduction in q_A (which exceeds the increase in q_N and q) induced by a reduction in $\overline{p}$.

Conclusions (i) and (ii) in Proposition 8 reflect the fact that q_A declines as c_A or k_A increases. The reduction in q_A diminishes the potential welfare gain from reducing $\overline{p}$ for two reasons. First, when q_A is small, the surplus of consumers that purchase q_A increases relatively slowly as $\bar{p}$ declines. Second, when q_A is small, R's revenue from selling q_A declines relatively slowly as $\overline{p}$ declines. Both sources of diminished benefit from reducing $\overline{p}$ toward $\bar{p}_{V_2m}$ imply that $\overline{p}^*$ increases (i.e., $\frac{d\,\overline{p}^*}{dc_A} > 0$).

Conclusion (iii) in Proposition 8 holds because q_N increases as c increases, reflecting the weakened competitive position of the rival. The increase in q_N induces R to reduce q_A, reflecting the marginal cost effect and the exposure effect. The reduction in q_A implies that R's revenue from selling q_A declines relatively slowly as $\overline{p}$ declines. This diminished welfare gain from reducing $\overline{p}$ implies that $\overline{p}^*$ increases toward $\bar{p}_{S_2M}$ (i.e., $\frac{d\,\overline{p}^*}{dc} > 0$).

[46] q and Q^R are strategic substitutes.

Conclusion (iv) in Proposition 8 arises because q_N declines as c_N increases. The reduction in q_N induces R to increase q_A, reflecting both the marginal cost effect and the exposure effect. The increase in q_A implies that R's revenue from selling q_A declines relatively rapidly as $\bar{p}$ declines. This increased welfare gain from reducing $\bar{p}$ implies that $\bar{p}^*$ declines toward $\bar{p}_{V2m}$ (i.e., $\frac{d\bar{p}^*}{dc_N} < 0$).[47]

Proposition 8 implies that the welfare-maximizing price cap becomes more stringent as access to A's input reduces R's marginal cost more substantially (i.e., $\bar{p}^*$ declines as $c_N - c_A$ increases). Intuitively, the welfare-maximizing price cap becomes more stringent as the value of A's input increases.

5 Conclusions.

We have examined the design of price caps as an instrument to reduce the (tax) revenue available to a sanctioned nation without causing the world price of a key product to increase excessively. We have shown that a price cap on a portion of a supplier's output can have potentially counterintuitive effects. Specifically, the price cap can increase, not reduce, the supplier's revenue by inducing the supplier to increase its output. Furthermore, the sanctioned supplier's increased output can cause the world price of the product to decline, not increase.

We have also shown that the welfare-maximizing price cap often is well below the prevailing market price of the product, and that a price cap can enhance welfare considerably under arguably plausible conditions. In addition, we have shown that raising a price cap above the level that would eliminate sales at the capped price often can both increase consumer surplus and reduce the revenue of the sanctioned producer. Thus, moderately stringent price caps often outperform relatively lenient or particularly severe price caps.

Our streamlined duopoly model was designed to illustrate simply and clearly potentially subtle effects of price caps as sanctions. Future research should consider more general demand and cost functions, differentiated products, more than two suppliers, and alternative market interactions (e.g., bargaining among industry suppliers and large buyers). Future research should also consider alternative (e.g., nonlinear) welfare functions and allow the sanctioned supplier to act to reduce the cost it incurs when it operates without access to key (Alliance) inputs. Future research might also consider the coordination (and enforcement) problems that arise when the nations that impose the price cap differ in their valuations of the sanctioned product.

[47] Numerical solutions reveal that $\bar{p}^*$ also often increases as: (i) a, k, or k^R increases; or (ii) k_N or b declines. This is the case, for example, in the baseline setting and for substantial variation in parameters around their values in the baseline setting.

These model extensions likely will alter the extent to which a more stringent price cap increases the revenue of a sanctioned supplier, the magnitude of the welfare-maximizing pice cap, and the potential welfare gains from a price cap. However, the model extensions seem unlikely to alter the conclusion that strategic, oligopolistic considerations merit careful consideration in any comprehensive analysis of the use of price caps as sanctions.

Appendix

Part A of this Appendix illustrates how equilibrium outcomes change as parameter values diverge from their levels in the baseline setting. Part B presents the proofs of the formal conclusions in the text.

A. Outcomes in Settings Other Than the Baseline Setting.

Parameter Variation	$\frac{\overline{p}_3 - \overline{p}_2}{\overline{p}_3}$	$\frac{V(\overline{p}_2) - V(\overline{p}_3)}{V(\overline{p}_3)}$	$\overline{p}^*$	$\frac{\overline{p}^*}{\overline{p}_3}$	$\frac{W(\overline{p}^*) - W(\overline{p}_3)}{W(\overline{p}_3)}$
$1.50\, a$	0.21	0.19	81.08	0.76	0.48
$0.50\, a$	0.21	0.21	27.55	0.75	0.52
$1.50\, b$	0.26	0.19	49.57	0.74	0.51
$0.50\, b$	0.13	0.19	59.56	0.72	0.65
$1.50\, c_A$	0.20	0.20	55.24	0.77	0.46
$0.50\, c_A$	0.22	0.19	53.39	0.75	0.51
$1.50\, k_A$	0.15	0.18	57.86	0.79	0.34
$0.5\, k_A$	0.35	0.15	45.36	0.65	0.87
$1.50\, k^R$	0.20	0.20	55.47	0.77	0.46
$0.50\, k^R$	0.23	0.19	53.06	0.75	0.52
$1.50\, c_N$	0.22	0.20	53.58	0.75	0.52
$0.50\, c_N$	0.20	0.19	55.05	0.77	0.46
$1.50\, k_N$	0.24	0.21	51.03	0.71	0.60
$0.50\, k_N$	0.17	0.15	58.81	0.83	0.34
$1.50\, c$	0.21	0.19	54.51	0.76	0.50
$0.50\, c$	0.21	0.19	54.12	0.76	0.48
$1.50\, k$	0.22	0.17	56.22	0.75	0.63
$0.50\, k$	0.20	0.23	51.78	0.77	0.36

Table A1. The Effects of Changing Baseline Parameters.

The first column in Table A1 identifies the single parameter that is changed in the baseline setting and the amount by which it is changed. All other parameters remain at their levels in the baseline setting.[48] The remaining columns in Table A1 identify the outcomes that arise in equilibrium. The welfare calculations in the last column assume $d = \frac{1}{2}$.

[48] For example, the first row of data in Table A1 records the outcomes that arise in equilibrium when a is increased by 50% above its level in the baseline setting, holding all other parameters at their values in the baseline setting.

d	$\overline{p}^{*}$	$\frac{\overline{p}^{*}}{\overline{p}_3}$	$W(\overline{p}^{*})$	$\frac{\mid W(\overline{p}^{*}) - W(\overline{p}_3) \mid}{\mid W(\overline{p}_3) \mid}$
0.0	56.35	0.79	5.518×10^9	0.36
0.25	56.26	0.79	4.529×10^9	0.40
0.50	54.31	0.76	3.579×10^9	0.49
0.75	52.79	0.74	2.688×10^9	0.70
1.0	51.56	0.72	1.837×10^9	1.46
2.0	48.36	0.68	-1.325×10^9	0.48
10.0	42.70	0.60	-23.873×10^9	0.18

Table A2. The Effects of Changing d in the Baseline Setting.

The first column in Table A2 identifies the value of d in the welfare function $W(\cdot) = S(\cdot) - d\,V(\cdot)$. The remaining columns report the corresponding welfare-maximizing price cap, the ratio of this price cap to the unrestricted equilibrium price ($\overline{p}_3$), the maximized level of welfare, and the proportionate maximum welfare gain, respectively.[49]

B. Proofs of Formal Conclusions in the Text[50]

Proof of Proposition 1. The proof follows directly from Lemmas A1 – A6 (below), which refer to the following definitions.[51]

$$\bar{p}_1 \equiv c_A + \frac{[\,a - c_N\,]\,[\,2\,b + k\,] - b\,[\,a - c\,]}{[\,2\,b + k_N + k^R\,]\,[\,2\,b + k\,] - b^2}\,\left[\,b + k^R\,\right]. \tag{10}$$

$$\bar{p}_2 \equiv \frac{1}{D_2}\,\{\,[\,a\,(b+k) + b\,c\,]\,\left[\,\left(b + k^R\right)(k_N + k_A) + k_N\,k_A - b\,k_N\,\right] + \; b\,[\,b+k\,]\,[\,k_A - b\,]\,c_N + b\,[\,k_N + b\,]\,[\,b+k\,]\,c_A\,\}$$

$$\text{where } \; D_2 \equiv b\,[\,b+k\,]\,[\,k_N + k_A\,] + k_N\,[\,k_A - b\,]\,[\,2\,b + k\,] + \; [\,k_N + k_A\,]\,[\,2\,b+k\,]\,\left[\,b + k^R\,\right]. \tag{11}$$

$$\bar{p}_3 \equiv \frac{1}{D_3}\,\{\,[\,a\,(b+k) + b\,c\,]\,\left[\,\left(b + k^R\right)(k_N + k_A) + k_N\,k_A\,\right] + \; b\,c_N\,[\,b+k\,]\,k_A + b\,k_N\,[\,b+k\,]\,c_A\,\}$$

$$\text{where } \; D_3 \equiv b\,[\,b+k\,]\,[\,k_N + k_A\,] + k_N\,k_A\,[\,2\,b + k\,]$$

[49] The relatively large welfare gain that arises when $d = 1$ arises in part because $W(\overline{p}_3)$ is relatively close to 0 in the baseline setting when $d = 1$.

[50] Part B of this Appendix sketches the proofs of the formal conclusions in the text. Detailed proofs are available in Sappington and Turner (2023).

[51] The proofs of Lemmas A1, A2, and A4 – A6 employ relatively standard techniques, and so are omitted. Detailed proofs of these lemmas are available in Sappington and Turner (2023).

$$+ \; [\, k_N + k_A \,] \, [\, 2\, b + k \,] \left[\, b + k^R \,\right] \; = \; D_2 + b\, k_N \, [\, 2\, b + k \,] \, . \tag{12}$$

Lemma A1. Suppose $\bar{p} \le \bar{p}_1$. Then in equilibrium:

$$q_A \; = \; 0 \, , \quad q_N \; = \; \frac{[\, a - c_N \,] \, [\, 2\, b + k \,] - b \, [\, a - c \,]}{[\, 2\, b + k_N + k^R \,] \, [\, 2\, b + k \,] - b^2} \, ,$$

$$q \; = \; \frac{[\, a - c \,] \left[\, 2\, b + k_N + k^R \,\right] - b \, [\, a - c_N \,]}{[\, 2\, b + k_N + k^R \,] \, [\, 2\, b + k \,] - b^2} \, , \text{ and}$$

$$Q \; = \; q_A + q_N + q \; = \; \frac{[\, a - c \,] \left[\, b + k_N + k^R \,\right] + [\, a - c_N \,] \, [\, b + k \,]}{[\, 2\, b + k_N + k^R \,] \, [\, 2\, b + k \,] - b^2} \, . \tag{13}$$

Lemma A2. *Suppose* $\bar{p} \in (\, \bar{p}_1, \bar{p}_2 \,]$. *Then in equilibrium*:

$$\begin{aligned} q_A \; = \; & \frac{1}{D} \{ \left[\, 3\, b^2 + 2\, b \left(k + k_N + k^R \right) + k \left(k_N + k^R \right) \right] [\, \overline{p} - c_A \,] \\ & + \; b \left[\, b + k^R \,\right] [\, a - c \,] - [\, 2\, b + k \,] \left[\, b + k^R \,\right] [\, a - c_N \,] \, \} \, ; \end{aligned} \tag{14}$$

$$\begin{aligned} q_N \; = \; & \frac{1}{D} \, \{ \, [\, 2\, b + k \,] \left[\, k_A + k^R \,\right] [\, a - c_N \,] - b \left[\, k_A + k^R \,\right] [\, a - c \,] \\ & - \; \left[\, b \left(b + 2\, k^R \right) + k \left(b + k^R \right) \right] [\, \overline{p} - c_A \,] \, \} \, ; \end{aligned} \tag{15}$$

$$\begin{aligned} Q^R \; \equiv \; q_A + q_N \; = \; & \frac{1}{D} \{ \, [\, 2\, b + k \,] \, [\, b + k_N \,] \, [\, \overline{p} - c_A \,] + [\, 2\, b + k \,] \, [\, k_A - b \,] \, [\, a - c_N \,] \\ & - \; b \, [\, k_A - b \,] \, [\, a - c \,] \, \} \, ; \end{aligned} \tag{16}$$

$$\begin{aligned} q \; = \; & \frac{1}{D} \, \{ \left[\, k_N \left(k_A + k^R \right) + k_A \, k^R + 2\, b\, k_A - b^2 \,\right] [\, a - c \,] \\ & - \; b \, [\, k_A - b \,] \, [\, a - c_N \,] - b \, [\, b + k_N \,] \, [\, \overline{p} - c_A \,] \, \} \, ; \text{ and} \end{aligned} \tag{17}$$

$$\begin{aligned} Q \; = \; q + q_A + q_N \; = \; & \frac{1}{D} \, \{ \, [\, b + k \,] \, [\, b + k_N \,] \, [\, \overline{p} - c_A \,] + [\, b + k \,] \, [\, k_A - b \,] \, [\, a - c_N \,] \\ & + \; \left[\, k^R \, (\, k_A + k_N \,) + k_A \, (\, b + k_N \,) \right] [\, a - c \,] \, \} \, . \end{aligned} \tag{18}$$

Lemma A3. *Suppose* $\bar{p} \in (\bar{p}_2, \bar{p}_3\,]$, *where* $\bar{p}_2 < \bar{p}_3$. *Then in equilibrium,* $P(Q) = \bar{p}$. *Furthermore*:

$$q_A \; = \; \frac{b \, [\, b + k \,] \, [\, c_N - c_A \,] + k_N \, [\, a - \bar{p} \,] \, [\, b + k \,] - b\, k_N \, [\, \bar{p} - c \,]}{b \, [\, b + k \,] \, [\, k_N + k_A \,]} \, ;$$

$$q_N \; = \; \frac{k_A \, [\, b + k \,] \, [\, a - \bar{p} \,] - b\, k_A \, [\, \bar{p} - c \,] - b \, [\, b + k \,] \, [\, c_N - c_A \,]}{b \, [\, b + k \,] \, [\, k_N + k_A \,]} \, ; \quad q \; = \; \frac{\bar{p} - c}{b + k} \, ;$$

$$Q^R \equiv q_A + q_N = \frac{[\,b+k\,]\,[\,a - \bar{p}\,] - b\,[\,\bar{p} - c\,]}{b\,[\,b+k\,]}\,; \text{ and } Q \equiv \frac{a - \bar{p}}{b}\,. \tag{19}$$

Proof. (4) implies that R's problem can be written as:

$$\underset{q_A,\, Q^R}{\text{Maximize}} \;\; \Pi_R \equiv \left[\, P_A(q + Q^R) - c_A \,\right] q_A + \left[\, P(Q^R + q) - c_N \,\right] \left[\, Q^R - q_A \,\right]$$
$$- \frac{k_A}{2}\,[\,q_A\,]^2 - \frac{k_N}{2}\left[\, Q^R - q_A \,\right]^2 - \frac{k^R}{2}\left[\, Q^R \,\right]^2$$

$$\text{where } \; P_A(q + Q^R) = \begin{cases} \bar{p} & \text{if } P(\,q + Q^R\,) \geq \bar{p} \\ P(\,q + Q^R\,) & \text{if } \bar{p} > P(\,q + Q^R\,). \end{cases} \tag{20}$$

(20) implies that the necessary conditions for a solution to R's problem are:

$$\frac{\partial \Pi_R}{\partial q_A} = P_A\left(q + Q^R\right) - c_A - k_A\, q_A - \left[\, P\left(q + Q^R\right) - c_N \,\right] + k_N \left[\, Q^R - q_A \,\right] = 0 \tag{21}$$

$$\text{and} \quad \frac{\partial^+ \Pi_R}{\partial Q^R} \leq 0 < \frac{\partial^- \Pi_R}{\partial Q^R}\,, \tag{22}$$

where $\frac{\partial^- \Pi_R}{\partial Q^R}$ denotes the left-sided derivative of Π_R with respect to Q^R, which is relevant when $P_A(\cdot) = \bar{p}$, and $\frac{\partial^+ \Pi_R}{\partial Q^R}$ denotes the right-sided derivative of Π_R with respect to Q^R, which is relevant when $P_A(\cdot) = P(Q)$.

(5) implies that the rival's choice of q is determined by:

$$\bar{p} - b\,q - c - k\,q = 0 \quad \Leftrightarrow \quad q = \frac{\bar{p} - c}{b + k}\,. \tag{23}$$

Because $\bar{p} = a - b\left[\, q + Q^R \,\right]$, (23) implies:

$$\bar{p} = a - b\left[\frac{\bar{p} - c}{b + k} + Q^R\right] \quad \Leftrightarrow \quad Q^R = \frac{[\,a - \bar{p}\,]\,[\,b+k\,] - b\,[\,\bar{p} - c\,]}{b\,[\,b+k\,]}\,. \tag{24}$$

Because $\bar{p} = P_A(q + Q^R)$ in equilibrium, (21) holds if:

$$\bar{p} - c_A - k_A\, q_A - [\,\bar{p} - c_N\,] + k_N \left[\, Q^R - q_A \,\right] = 0$$
$$\Leftrightarrow \quad c_N - c_A - k_A\, q_A + k_N\, Q^R - k_N\, q_A = 0\,. \tag{25}$$

(24) implies that (25) holds if:

$$q_A = \frac{b\,[\,b+k\,]\,[\,c_N - c_A\,] + k_N\,[\,a - \bar{p}\,]\,[\,b+k\,] - b\,k_N\,[\,\bar{p} - c\,]}{b\,[\,b+k\,]\,[\,k_N + k_A\,]}\,. \tag{26}$$

(24) and (26) imply:

$$q_N = Q^R - q_A = \frac{k_A\,[\,b+k\,]\,[\,a - \bar{p}\,] - b\,k_A\,[\,\bar{p} - c\,] - b\,[\,b+k\,]\,[\,c_N - c_A\,]}{b\,[\,b+k\,]\,[\,k_N + k_A\,]}\,. \tag{27}$$

(26) and (27) imply:

$$Q^R \;\equiv\; q_A \,+\, q_N \;=\; \frac{[\,b+k\,]\,[\,a-\,\bar{p}\,]-b\,[\,\bar{p}-c\,]}{b\,[\,b+k\,]}\,. \tag{28}$$

(23) and (28) imply:

$$Q \;\equiv\; Q^R+q \;=\; \frac{[\,b+k\,]\,[\,a-\,\bar{p}\,]-b\,[\,\bar{p}-c\,]}{b\,[\,b+k\,]}+\frac{b\,[\,\bar{p}-c\,]}{b\,[\,b+k\,]} \;=\; \frac{a-\,\bar{p}}{b}\,.$$

(20) implies:

$$\begin{aligned}\frac{\partial^+\,\Pi_R}{\partial Q^R} \;&=\; a-2\,b\,Q^R-b\,q-c_N-k_N\left[\,Q^R-q_A\,\right]-k^R\,Q^R\\ &=\; \bar{p}-b\,Q^R-c_N-k_N\,q_N-k^R\,Q^R \;=\; \bar{p}-\left[\,b+k^R\,\right]Q^R-c_N-k_N\,q_N\,;\end{aligned} \tag{29}$$

$$\begin{aligned}\frac{\partial^-\,\Pi_R}{\partial Q^R} \;&=\; a-2\,b\,Q^R-b\,q-c_N+b\,q_A-k_N\left[\,Q^R-q_A\,\right]-k^R\,Q^R\\ &=\; \bar{p}-\left[\,b+k^R\,\right]Q^R-c_N+b\,q_A-k_N\,q_N\,.\end{aligned} \tag{30}$$

(29) and (30) imply that (22) can be written as:

$$\left[\,b+k^R\,\right]Q^R+c_N+k_N\,q_N-b\,q_A \;<\; \bar{p} \;\leq\; \left[\,b+k^R\,\right]Q^R+c_N+k_N\,q_N\,. \tag{31}$$

(12), (24), and (27) imply:

$$\begin{aligned}&\bar{p} \;\leq\; \left[b+k^R\right]Q^R+c_N+k_N\,q_N\\ \Leftrightarrow\quad &\left[\,b+k^R\,\right][\,a\,(b+k)+b\,c\,]\,[\,k_N+k_A\,]+c_N\,b\,[\,b+k\,]\,[\,k_N+k_A\,]\\ &\qquad +\;k_N\,[\,k_A\,(b\,+\,k)\,a+b\,k_A\,c-b\,(b+k)\,(c_N-c_A)\,]\\ &\geq\;\bar{p}\,[\,b\,(b+k)\,(k_N+k_A)+k_N\,k_A\,(2\,b+k)\\ &\qquad +\,(k_N+k_A)\,(2\,b+k)\,\big(b+k^R\big)\,] \;=\; \bar{p}\,D_3.\end{aligned} \tag{32}$$

(32) implies:

$$\bar{p} \;\leq\; \left[\,b+k^R\,\right]Q^R+c_N+k_N\,q_N \quad\Leftrightarrow\quad \bar{p} \;\leq\; \bar{p}_3\,. \tag{33}$$

(11), (24), (26), and (27) imply:

$$\begin{aligned}&\left[\,b+k^R\,\right]Q^R+c_N+k_N\,q_N-b\,q_A \;<\; \bar{p}\\ \Leftrightarrow\quad &\left[\,b+k^R\,\right][\,a\,(b+k)+b\,c\,]\,[\,k_N+k_A\,]+c_N\,b\,[\,b+k\,]\,[\,k_N+k_A\,]\\ &\quad +\,k_N\,[\,k_A\,(b+k)\,a+b\,k_A\,c-b\,(b+k)\,(c_N-c_A)\,]\\ &\quad -\,b\,[\,b\,(b+k)\,(c_N-c_A)+k_N\,a\,(b+k)+b\,k_N\,c\,]\end{aligned}$$

$$< \ \bar{p} \left[b \, (b+k) \, (k_N + k_A) + k_N \, (k_A - b) \, (2 \, b + k) \right.$$

$$\left. + \ (k_N + k_A) \, (2 \, b + k) \left(b + k^R \right) \right] \ = \ \bar{p} \, D_2 \, . \tag{34}$$

(34) implies:

$$\left[b + k^R \right] Q^R + c_N + k_N \, q_N - b \, q_A \ < \ \bar{p}$$

$$\Leftrightarrow \ \bar{p} \ > \ \frac{1}{D_2} \left\{ \left[a \, (b+k) + b \, c \right] \left[\left(b + k^R \right) (k_N + k_A) + k_N \, k_A - b \, k_N \right] \right.$$

$$\left. + \ b \, [\, b + k \,] \, [\, k_A - b \,] \, c_N + b \, [\, k_N + b \,] \, [\, b + k \,] \, c_A \right\} \ \equiv \ \bar{p}_2 \, . \tag{35}$$

(12), (29), (30), (33), and (35) imply:

$$\bar{p}_2 \ = \ \left[b + k^R \right] Q^R + c_N + k_N \, q_N - b \, q_A \quad \text{and}$$

$$\bar{p}_3 \ = \ \left[b + k^R \right] Q^R + c_N + k_N \, q_N \, . \tag{36}$$

(36) implies that $\bar{p}_2 < \bar{p}_3$ because $q_A > 0$ when $\bar{p} > \bar{p}_1$. $\square$

Lemma A4. *Suppose* $\bar{p} > \bar{p}_3$*. Then in equilibrium*:

$$q_A \ = \ \frac{1}{D_3} \left\{ [\, a - c_A \,] \left[2 \, b \, k + 2 \, b \, k_N + 2 \, b \, k^R + k \, k_N + k \, k^R + 3 \, b^2 \right] \right.$$

$$\left. - \ [\, a - c_N \,] \left[2 \, b \, k + 2 \, b \, k^R + k \, k^R + 3 \, b^2 \right] - b \, k_N \, [\, a - c \,] \right\} ; \tag{37}$$

$$q_N \ = \ \frac{1}{D_3} \left\{ [\, a - c_N \,] \left[2 \, b \, k + 2 \, b \, k_A + 2 \, b \, k^R + k \, k_A + k \, k^R + 3 \, b^2 \right] \right.$$

$$\left. - \ [\, a - c_A \,] \left[2 \, b \, k + 2 \, b \, k^R + k \, k^R + 3 \, b^2 \right] - b \, k_A \, [\, a - c \,] \right\} ; \tag{38}$$

$$q \ = \ \frac{1}{D_3} \left\{ [\, a - c \,] \left[2 \, b \, k_A + 2 \, b \, k_N + k_A \, k_N + k_A \, k^R + k_N \, k^R \right] \right.$$

$$\left. - \ b \, k_A \, [\, a - c_N \,] - b \, k_N \, [\, a - c_A \,] \right\} ; \quad \text{and} \tag{39}$$

$$Q^R \ \equiv \ q_A + q_N \ = \ \frac{1}{D_3} \left\{ [\, a - c_A \,] \, k_N \, [\, 2 \, b + k \,] + [\, a - c_N \,] \, k_A \, [\, 2 \, b + k \,] \right.$$

$$\left. - \ b \, [\, k_A + k_N \,] \, [\, a - c \,] \right\} \tag{40}$$

where D_3 *is as specified in (12).*

<u>Definitions</u>

$q_{A1}(\bar{p}_1)$, $q_{N1}(\bar{p}_1)$, and $q_1(\bar{p}_1)$, respectively, denote the values of $q_A(\cdot)$, $q_N(\cdot)$, and $q(\cdot)$ specified in Lemma A1, where $\bar{p} \leq \bar{p}_1$.

$q_{A2}(\bar{p}_1)$, $q_{N2}(\bar{p}_1)$, and $q_2(\bar{p}_1)$, respectively, denote the values of $q_A(\cdot)$, $q_N(\cdot)$, and $q(\cdot)$ specified in Lemma A2, where $\bar{p} \in (\, \bar{p}_1, \bar{p}_2 \,]$.

Lemma A5. $\lim_{\bar{p} \to \bar{p}_1} q_{A2}(\bar{p}) = q_{A1}(\bar{p}_1)$, $\lim_{\bar{p} \to \bar{p}_1} q_{N2}(\bar{p}) = q_{N1}(\bar{p}_1)$, *and* $\lim_{\bar{p} \to \bar{p}_1} q_2(\bar{p}) = q_1(\bar{p}_1)$.

Lemma A6. $0 < \bar{p}_1 < \bar{p}_2 < \bar{p}_3$. ■

Proof of Proposition 2. The conclusions in the proposition follow directly from Lemma A3. ■

Proof of Proposition 3. (24) implies that for $\bar{p} \in (\bar{p}_2, \bar{p}_3)$, R's revenue is:

$$V(\bar{p}) = \bar{p} \left[\frac{a\,(b+k) + b\,c - \bar{p}\,(2\,b+k)}{b\,[\,b+k\,]} \right] = \frac{[\,a\,(b+k) + b\,c\,]\,\bar{p} - [\,2\,b+k\,]\,\bar{p}^2}{b\,[\,b+k\,]}\,. \tag{41}$$

The value of $\bar{p}$ at which $V(\bar{p})$ in (41) is maximized is determined by:

$$a\,[\,b+k\,] + b\,c - 2\,[\,2\,b+k\,]\,\bar{p} = 0 \;\Rightarrow\; \bar{p} = \frac{a\,[\,b+k\,] + b\,c}{2\,[\,2\,b+k\,]} \equiv \bar{p}_{V_3M}\,. \tag{42}$$

(12) and (42) imply that $\bar{p}_{V_3M} < \bar{p}_3$ if:

$$\frac{a\,[\,b+k\,] + b\,c}{2\,[\,2\,b+k\,]} < \frac{\left[\,(\,b+k^R\,)\,(\,k_N + k_A\,) + k_N\,k_A\,\right] \frac{a\,[\,b+k\,]+b\,c}{b\,[\,b+k\,]} + c_N\,k_A + k_N\,c_A}{k_N + k_A + [\,(\,b+k^R\,)\,(k_N + k_A\,) + k_N\,k_A\,]\,\frac{2\,b+k}{b\,[\,b+k\,]}}$$

$$\Leftrightarrow \quad \frac{[\,2\,b+k\,]\,\left[\,b+k^R\,\right] - b\,[\,b+k\,]}{b\,[\,b+k\,]}\,[\,k_N + k_A\,] + k_N\,k_A \left[\frac{2\,b+k}{b\,(b+k)} \right] > 0\,. \tag{43}$$

It is readily verified that the inequality in (43) always holds, so $\bar{p}_{V_3M} < \bar{p}_3$.

(41) and (42) imply that for $\bar{p} \in (\bar{p}_2, \bar{p}_3)$, $V(\bar{p})$ is a strictly concave function that attains its maximum at $\bar{p}_{V_3M}$. Therefore, $\frac{\partial V(\bar{p})}{\partial \bar{p}} < 0$ for $\bar{p} \in (\bar{p}_{V_3M}, \bar{p}_3)$.

(11) and (42) imply that $\bar{p}_2 \geq \bar{p}_{V_3M}$ if and only if:

$$\frac{1}{b\,[\,b+k\,]\,[\,k_N + k_A\,] + [\,k_A\,k_N - k_N\,b\,]\,[\,2\,b+k\,] + [\,k_N + k_A\,]\,[\,2\,b+k\,]\,[\,b+k^R\,]}$$

$$\cdot \left\{ [\,(b+k)\,a + b\,c\,] \left[\left(b+k^R\right)(k_N + k_A) + k_N\,k_A - b\,k_N \right] \right.$$

$$\left. +\; b\,[\,b+k\,]\,[\,k_A - b\,]\,c_N + b\,[\,k_N + b\,]\,[\,b+k\,]\,c_A \right\}$$

$$\geq \frac{a\,[\,b+k\,] + b\,c}{2\,[\,b+k\,]}$$

$$\Leftrightarrow \left[\frac{(b+k)\,a + b\,c}{2\,b+k} \right] \left[\left(b^2 + k^R\,[\,2\,b+k\,]\right)(k_N + k_A) + k_N\,k_A\,(2\,b+k) - b\,k_N\,(2\,b+k) \right]$$

$$+\; 2\,b\,[\,b+k\,]\,[\,k_A\,c_N + k_N\,c_A - b\,(c_N - c_A)\,] \geq 0\,. \tag{44}$$

It is readily verified that:

$$\left[\, b^2 + k^R\,(2\,b+k)\,\right][\,k_N + k_A\,] + k_N\,k_A\,[\,2\,b+k\,] - b\,k_N\,[\,2\,b+k\,] \;=\; -\,2\,b\,[\,b+k\,]\,[\,k_N+k_A\,] + D_2 .$$

Therefore, (44) implies that $\bar{p}_2 \geq \bar{p}_{V_3M} \;\Leftrightarrow\; \widetilde{\Phi}_1 \geq 0$, where:

$$\widetilde{\Phi}_1 \;\equiv\; \left[\frac{(b+k)\,a + b\,c}{2\,b+k}\right]\{\,D_2 - 2\,b\,[\,b+k\,]\,[\,k_N+k_A\,]\,\}$$
$$+\;\; 2\,b\,[\,b+k\,]\,[\,k_A\,c_N + k_N\,c_A - b\,(c_N - c_A)\,] .$$

It is readily verified that $\widetilde{\Phi}_1 = \Phi_1$. ■

Proof of Proposition 4. Let $q_A(\bar{p})$ denote R's equilibrium output using A's input when the price cap is $\bar{p} \in [\,\bar{p}_2, \bar{p}_3\,]$. Let $q_N(\bar{p})$ denote R's corresponding output when R does not employ A's input. Also let $Q^R(\bar{p}) = q_A(\bar{p}) + q_N(\bar{p})$.

To prove that $\frac{\partial(\,\bar{p}_3 - \bar{p}_2)}{\partial k^R} < 0$, observe that (36) implies:

$$\bar{p}_3 \;=\; \left[\, b + k^R\,\right] Q^R(\bar{p}_3) + c_N + k_N\,q_N(\bar{p}_3)$$

where, from (19):

$$q_N(\bar{p}_3) \;=\; \frac{k_A\,[\,b+k\,]\,[\,a-\,\bar{p}_3\,] - b\,k_A\,[\,\bar{p}_3 - c\,] - b\,[\,b+k\,]\,[\,c_N - c_A\,]}{b\,[\,b+k\,]\,[\,k_N+k_A\,]} \quad \text{and}$$

$$Q^R(\bar{p}_3) \;=\; \frac{[\,b+k\,]\,[\,a-\,\bar{p}_3\,] - b\,[\,\bar{p}_3 - c\,]}{b\,[\,b+k\,]} . \tag{45}$$

(45) implies that $q_N(\bar{p}_3)$ and $Q^R(\bar{p}_3)$ vary with k^R only through $\bar{p}_3$. Therefore:

$$\frac{\partial \bar{p}_3}{\partial k^R} \;=\; Q^R(\bar{p}_3) + \left[\, b + k^R\,\right] \frac{\partial Q^R(\bar{p}_3)}{\partial \bar{p}_3}\,\frac{\partial \bar{p}_3}{\partial k^R} + k_N\,\frac{\partial q_N(\bar{p}_3)}{\partial \bar{p}_3}\,\frac{\partial \bar{p}_3}{\partial k^R} ,$$

$$\frac{\partial q_N(\bar{p}_3)}{\partial \bar{p}_3} \;=\; -\,\frac{k_A\,[\,b+k\,] + b\,k_A}{b\,[\,b+k\,]\,[\,k_N+k_A\,]} \;\equiv\; D_N \;<\; 0\,, \text{ and } \frac{\partial Q^R(\bar{p}_3)}{\partial \bar{p}_3} \;=\; -\,\frac{2\,b+k}{b\,[\,b+k\,]} \;\equiv\; D_R \;<\; 0$$

$$\Rightarrow\; \frac{\partial \bar{p}_3}{\partial k^R} \;=\; \frac{Q^R(\bar{p}_3)}{1 - [\,b+k^R\,]\,D_R - k_N\,D_N} \;>\; 0\,. \tag{46}$$

(19) and (36) imply:

$$\bar{p}_2 \;=\; \left[\, b + k^R\,\right] Q^R(\bar{p}_2) + c_N + k_N\,q_N(\bar{p}_2) - b\,q_A(\bar{p}_2)$$

$$\text{where} \;\; q_A(\bar{p}_2) \;=\; \frac{b\,[\,b+k\,]\,[\,c_N - c_A\,] + k_N\,[\,a - \bar{p}\,]\,[\,b+k\,] - b\,k_N\,[\,\bar{p} - c\,]}{b\,[\,b+k\,]\,[\,k_N+k_A\,]}\,;$$

$$q_N(\bar{p}_2) \;=\; \frac{k_A\,[\,b+k\,]\,[\,a-\,\bar{p}_2\,] - b\,k_A\,[\,\bar{p}_2 - c\,] - b\,[\,b+k\,]\,[\,c_N - c_A\,]}{b\,[\,b+k\,]\,[\,k_N+k_A\,]}\,; \;\text{ and}$$

$$Q^R(\bar{p}_2) \;=\; \frac{[\,b+k\,]\,[\,a-\,\bar{p}_2\,] - b\,[\,\bar{p}_2 - c\,]}{b\,[\,b+k\,]} . \tag{47}$$

(47) implies that $q_A(\bar{p}_2)$, $q_N(\bar{p}_2)$, and $Q^R(\bar{p}_2)$ vary with k^R only through $\bar{p}_2$. Therefore:

$$\frac{\partial \bar{p}_2}{\partial k^R} \;=\; Q^R(\bar{p}_2) + \left[\, b + k^R \,\right] \frac{\partial Q^R(\bar{p}_2)}{\partial \bar{p}_2} \frac{\partial \bar{p}_2}{\partial k^R} + k_N \frac{\partial q_N(\bar{p}_2)}{\partial \bar{p}_2} \frac{\partial \bar{p}_2}{\partial k^R} - b \frac{\partial q_A(\bar{p}_2)}{\partial \bar{p}_2} \frac{\partial \bar{p}_2}{\partial k^R} \,;$$

$$\frac{\partial q_A(\bar{p}_2)}{\partial \bar{p}_2} \;=\; -\frac{k_N\,[\,b+k\,] + b\,k_N}{b\,[\,b+k\,]\,[\,k_N + k_A\,]} \;\equiv\; D_A \;<\; 0\,;$$

$$\frac{\partial q_N(\bar{p}_2)}{\partial \bar{p}_2} \;=\; -\frac{k_A\,[\,b+k\,] + b\,k_A}{b\,[\,b+k\,]\,[\,k_N + k_A\,]} \;\equiv\; D_N < 0\,; \quad \frac{\partial Q^R(\bar{p}_2)}{\partial \bar{p}_2} \;=\; -\frac{2\,b+k}{b\,[\,b+k\,]} \;\equiv\; D_R < 0\,. \quad (48)$$

(48) implies:

$$\frac{\partial \bar{p}_2}{\partial k^R} \;=\; \frac{Q^R(\bar{p}_2)}{1 - [\,b+k^R\,]\,D_R - k_N\,D_N + b\,D_A}\,, \ \text{ and} \quad (49)$$

$$-\,b\,D_R + b\,D_A \;=\; b\left[\frac{2\,b+k}{b\,(b+k)}\right]\left[1 - \frac{k_N}{k_N+k_A}\right] \;>\; 0\,. \quad (50)$$

Because $D_N \,<\, 0$ and $D_R \,<\, 0$ from (48), (50) implies:

$$1 - \left[\, b + k^R \,\right] D_R - k_N\,D_N + b\,D_A \;>\; 1 - k^R\,D_R - k_N\,D_N \;>\; 0\,. \quad (51)$$

Because $D_A < 0$ from (48), (51) implies:

$$1 - \left[\, b + k^R \,\right] D_R - k_N\,D_N \;>\; 0\,. \quad (52)$$

(49) and (51) imply:

$$\frac{\partial \bar{p}_2}{\partial k^R} \;=\; \frac{Q^R(\bar{p}_2)}{1 - [\,b+k^R\,]\,D_R - k_N\,D_N + b\,D_A} \;>\; 0\,. \quad (53)$$

(46) and (51) – (53) imply:

$$\frac{\partial \bar{p}_3}{\partial k^R} - \frac{\partial \bar{p}_2}{\partial k^R} \;<\; 0 \quad\Leftrightarrow\quad \frac{Q^R(\bar{p}_3)}{Q^R(\bar{p}_2)} \;<\; \frac{1 - \left[\, b+k^R \,\right] D_R - k_N\,D_N}{1 - [\,b+k^R\,]\,D_R - k_N\,D_N + b\,D_A}\,. \quad (54)$$

(24) implies that $\frac{Q^R(\bar{p}_3)}{Q^R(\bar{p}_2)} < 1$. Furthermore, because $1 - \left[\, b + k^R \,\right] D_R - k_N D_N + b D_A \,>\, 0$ from (51):

$$\frac{1 - \left[\, b+k^R \,\right] D_R - k_N\,D_N}{1 - [\,b+k^R\,]\,D_R - k_N\,D_N + b\,D_A} \;>\; 1 \quad\Leftrightarrow\quad D_A \;<\; 0\,. \quad (55)$$

(48) implies that the last inequality in (55) holds. Therefore, (54) holds. Consequently, because $\bar{p}_3 > \bar{p}_2 > 0$ from Proposition 1, (46) and (54) imply that $\frac{\partial(\bar{p}_3 - \bar{p}_2)}{\partial k^R} < 0\,.$

To prove that $\frac{\partial(\bar{p}_3 - \bar{p}_2)}{\partial c_N} \,>\, 0$, observe that (11) and (12) imply:

$$\frac{\partial \bar{p}_3}{\partial c_N} - \frac{\partial \bar{p}_2}{\partial c_N} \;=\; \frac{b\,[\,b+k\,]\,k_A}{D_2 + b\,k_N\,[\,2\,b+k\,]} - \frac{b\,[\,b+k\,]\,[\,k_A - b\,]}{D_2} \;>\; 0$$

$$\Leftrightarrow\quad D_2 - k_N\,[\,2\,b+k\,]\,[\,k_A - b\,] \;>\; 0\,. \quad (56)$$

It is readily verified that the inequality in (56) holds.

To prove that $\frac{\partial(\bar{p}_3 - \bar{p}_2)}{\partial c_A} < 0$, observe that (11) and (12) imply:

$$\frac{\partial \bar{p}_3}{\partial c_A} - \frac{\partial \bar{p}_2}{\partial c_A} = \frac{b\,[\,b+k\,]\,k_N}{D_2 + b\,k_N\,[\,2\,b+k\,]} - \frac{b\,[\,b+k\,]\,[\,k_N+b\,]}{D_2} < 0$$

$$\Leftrightarrow \quad D_2 + k_N\,[\,2\,b+k\,]\,[\,k_N+b\,] > 0\,. \tag{57}$$

It is readily verified that $D_2 > 0$, so the inequality in (57) holds.

To prove that $\frac{\partial(\bar{p}_3 - \bar{p}_2)}{\partial c} > 0$, observe that (11) and (12) imply:

$$\frac{\partial\,(\,\bar{p}_3\, - \,\bar{p}_2)}{\partial c} \stackrel{s}{=} \frac{k_A\left[\,b+k^R\,\right] + k_N\left[\,k_A + k^R + b\,\right]}{D_3} - \frac{k_A\left[\,b+k^R\,\right] + k_N\left[\,k_A + k^R\,\right]}{D_2} > 0$$

$$\Leftrightarrow \quad b\,[\,b+k\,]\,[\,k_N+k_A\,] + k_N\,[\,k_A - b\,]\,[\,2\,b+k\,] + [\,k_N+k_A\,]\,[\,2\,b+k\,]\left[\,b+k^R\,\right]$$

$$> \left[\,\left(b+k^R\right)(k_A+k_N) + k_N\,(k_A - b)\,\right][\,2\,b+k\,] \quad \Leftrightarrow \quad b\,[\,b+k\,]\,[\,k_N+k_A\,] > 0\,.$$

The proofs of the remaining conclusions are similar, but more tedious. See Sappington and Turner (2023) for details. ■

Recall that welfare is:

$$W(\,\bar{p}\,) \equiv S(\,\bar{p}\,) - d\,[\,\bar{p}\,q_A + (\,a - b\,[\,q_A + q_N + q\,]\,)\,q_N\,] = S(\,\bar{p}\,) - d\,V(\,\bar{p}\,) \tag{58}$$

where $d > 0$ is a parameter and $S(\cdot)$ denotes consumer surplus. The gross value that consumers derive from Q units of output is:

$$\frac{1}{2}\,[\,a - P(Q)\,]\,Q + P(Q)\,Q = \frac{1}{2}\,[\,a + P(Q)\,]\,Q = \frac{1}{2}\,[\,a + a - b\,Q\,]\,Q = a\,Q - \frac{b}{2}\,Q^2.$$

Therefore, consumer surplus when the price cap is $\bar{p}$ is:

$$S(\,\bar{p}\,) = a\,Q - \frac{b}{2}\,Q^2 - \bar{p}\,q_A - P(Q)\,[\,q_N + q\,]\,. \tag{59}$$

Proof of Lemma 1. (19) implies that when $\bar{p} \in (\bar{p}_2, \bar{p}_3)$ (so $P(Q) = \bar{p}$), $Q = \frac{a-\bar{p}}{b} \Rightarrow \frac{\partial Q}{\partial \bar{p}} = -\frac{1}{b}$. Therefore, (59) implies:

$$\frac{\partial S(\,\bar{p}\,)}{\partial \bar{p}} = -\,\frac{a-\bar{p}}{b} < 0 \quad \Rightarrow \quad \frac{\partial^2 S(\,\bar{p}\,)}{\partial\,(\bar{p})^2} = \frac{1}{b} > 0\,. \;\blacksquare \tag{60}$$

Proof of Lemma 2. Lemmas A1 and A3 imply that because $q_A(\bar{p}_1) = 0$ and $P(Q(\bar{p}_3)) = \bar{p}_3$:

$$V(\bar{p}_1) = \bar{p}_1\,q_N(\bar{p}_1) = \bar{p}_1\,\frac{[\,a - c_N\,]\,[\,2\,b+k\,] - b\,[\,a-c\,]}{[\,2\,b + k_N + k^R\,]\,[\,2\,b+k\,] - b^2}\,;$$

$$V(\bar{p}_3) = \bar{p}_3\,Q^R(\bar{p}_3) = \bar{p}_3\,\frac{[\,b+k\,]\,[\,a-\bar{p}_3\,] - b\,[\,\bar{p}_3 - c\,]}{b\,[\,b+k\,]}\,. \tag{61}$$

Definition. $D_N \equiv \left[\,2\,b + k_N + k^R\,\right][\,2\,b+k\,] - b^2\,.$ (62)

Because $\bar{p}_1 < \bar{p}_3$, (61) and (62) imply that $V(\bar{p}_1) < V(\bar{p}_3)$ if:

$$q_N(\bar{p}_1) = \frac{[\,a - c_N\,]\,[\,2\,b + k\,] - b\,[\,a - c\,]}{D_N} < \frac{[\,b + k\,]\,[\,a - \bar{p}_3\,] - b\,[\,\bar{p}_3 - c\,]}{b\,[\,b + k\,]} = Q^R(\bar{p}_3)$$

$$\Leftrightarrow \frac{a\,[\,b + k\,] + b\,c - c_N\,[\,2\,b + k\,]}{D_N} < \frac{[\,b + k\,]\,a + b\,c - [\,2\,b + k\,]\,\bar{p}_3}{b\,[\,b + k\,]}$$

$$\Leftrightarrow \frac{[\,a\,(b + k) + b\,c\,]\left[\,b + k_N + k^R\,\right] + c_N\,b\,[\,b + k\,]}{D_N} > \bar{p}_3\,. \tag{63}$$

(12) implies:

$$\bar{p}_3 = \frac{[\,a\,(b + k) + b\,c\,]\left[\,\left(b + k^R\right)(k_N + k_A) + k_N\,k_A\,\right] + b\,c_N\,[\,b + k\,]\,k_A + b\,k_N\,[\,b + k\,]\,c_A}{b\,[\,b + k\,]\,[\,k_N + k_A\,] + k_N\,k_A\,[\,2\,b + k\,] + [\,k_N + k_A\,]\,[\,2\,b + k\,]\,[\,b + k^R\,]}\,. \tag{64}$$

As established in the proof of Proposition 4, $\bar{p}_3$ is increasing in k_A. Therefore, (64) implies that because $k_A \leq k_N$ by assumption:

$$\bar{p}_3 \leq \frac{[\,a\,(b + k) + b\,c\,]\left[\,2\,k_N\left(b + k^R\right) + (k_N)^2\,\right] + b\,c_N\,[\,b + k\,]\,k_N + b\,k_N\,[\,b + k\,]\,c_A}{2\,b\,[\,b + k\,]\,k_N + (k_N)^2\,[\,2\,b + k\,] + 2\,k_N\,[\,2\,b + k\,]\,[\,b + k^R\,]}\,. \tag{65}$$

(12) implies that $\bar{p}_3$ is increasing in c_A. Therefore, because $c_A \leq c_N$ by assumption, (65) implies:

$$\begin{aligned}\bar{p}_3 &\leq \frac{[\,a\,(b + k) + b\,c\,]\left[\,2\,k_N\left(b + k^R\right) + (k_N)^2\,\right] + 2\,b\,c_N\,[\,b + k\,]\,k_N}{2\,b\,[\,b + k\,]\,k_N + (k_N)^2\,[\,2\,b + k\,] + 2\,k_N\,[\,2\,b + k\,]\,[\,b + k^R\,]} \\ &= \frac{[\,a\,(b + k) + b\,c\,]\left[\,b + k^R + \frac{k_N}{2}\,\right] + b\,c_N\,[\,b + k\,]}{[\,2\,b + k\,]\left[\,2\,b + k^R + \frac{k_N}{2}\,\right] - b^2}\,.\end{aligned} \tag{66}$$

(62), (63), and (66) imply that the Lemma holds if:

$$\frac{[\,a\,(b + k) + b\,c\,]\left[\,b + k^R + \frac{k_N}{2}\,\right] + b\,c_N\,[\,b + k\,]}{[\,2\,b + k\,]\left[\,2\,b + k^R + \frac{k_N}{2}\,\right] - b^2}$$

$$< \frac{[\,a\,(b + k) + b\,c\,]\left[\,b + k^R + k_N\,\right] + b\,c_N\,[\,b + k\,]}{[\,2\,b + k\,]\,[\,2\,b + k^R + k_N\,] - b^2}\,.$$

It can be verified that this inequality holds. ■

Proof of Proposition 5. Proposition 3 and Lemma 1 imply that $W(\cdot)$ is a strictly convex function of $\bar{p}$ for $\bar{p} \in (\bar{p}_2, \bar{p}_3)$. Therefore, $\bar{p}^* \notin (\bar{p}_2, \bar{p}_3)$. Lemma A1 implies that $W(\bar{p}) = W(\bar{p}_1)$ for all $\bar{p} < \bar{p}_1$. Lemma A4 implies that $W(\bar{p}) = W(\bar{p}_3)$ for all $\bar{p} > \bar{p}_3$. Therefore, $\bar{p}^* \in [\,\bar{p}_1, \bar{p}_2\,] \bigcup \bar{p}_3$.

It remains to show that $\bar{p}^* \neq \bar{p}_3$. The proof of Lemma 2 establishes that:

$$Q^R(\bar{p}_1) \; < \; Q^R(\bar{p}_3)\,. \tag{67}$$

Lemma A6 and Proposition 2 imply:

$$Q^R(\bar{p}_3) \; < \; Q^R(\bar{p}_2)\,. \tag{68}$$

(67) and (68) imply that $Q^R(\bar{p}_1) < Q^R(\bar{p}_3) < Q^R(\bar{p}_2)$. $Q^R(\bar{p})$ is continuous and monotonically increasing in $\bar{p}$ for $\bar{p} \in (\bar{p}_1, \bar{p}_2)$ (from Lemma A2). Therefore, the intermediate value theorem implies that there exists a $\bar{p}_E \in (\bar{p}_1, \bar{p}_2)$ such that:

$$Q^R(\bar{p}_E) \; = \; Q^R(\bar{p}_3)\,. \tag{69}$$

(5) implies that the rival's output q is determined by:

$$a - b\left[\, Q^R(\bar{p}) + q(\bar{p})\,\right] - c - b\, q(\bar{p}) - k\, q(\bar{p}) \; = \; 0\,. \tag{70}$$

(69) and (70) imply:

$$q(\bar{p}_E) \; = \; q(\bar{p}_3)\,. \tag{71}$$

(69) and (71) imply:

$$Q(\bar{p}_E) \; = \; Q(\bar{p}_3) \;\text{ and }\; P(Q(\bar{p}_E)\,) \; = \; P(Q(\bar{p}_3)\,)\,. \tag{72}$$

R's revenue is:

$$\begin{aligned} V_2(\bar{p}_E) \; &= \; \bar{p}_E\, q_A(\bar{p}_E) + P(Q(\bar{p}_E)\,)\, q_N(\bar{p}_E) \\ &< \; P(Q(\bar{p}_E)\,)\, q_A(\bar{p}_E) + P(Q(\bar{p}_E)\,)\, q_N(\bar{p}_E) \\ &= \; P(Q(\bar{p}_E)\,)\, Q^R(\bar{p}_E) \; = \; P(Q(\bar{p}_3)\,)\, Q^R(\bar{p}_3) \; = \; V_3(\bar{p}_3)\,. \end{aligned} \tag{73}$$

The inequality in (73) holds because $\bar{p}_E < P(Q(\bar{p}_E)\,)$, since $\bar{p}_E \in (\bar{p}_1, \bar{p}_2)$. The penultimate equality in (73) reflects (72). The last equality in (73) holds because $P(Q(\bar{p}_3)\,) = \bar{p}_3$.

(59) and (72) imply:

$$\begin{aligned} S(\bar{p}_E) \; &= \; a\, Q(\bar{p}_E) - \frac{b}{2}\, Q(\bar{p}_E)^2 - P(Q(\bar{p}_E)\,)\, \left[\, q(\bar{p}_E) + q_N(\bar{p}_E)\,\right] - \bar{p}_E\, q_A(\bar{p}_E) \\ &> \; a\, Q(\bar{p}_E) - \frac{b}{2}\, Q(\bar{p}_E)^2 - P(Q(\bar{p}_E)\,)\, \left[\, q(\bar{p}_E) + q_N(\bar{p}_E) + q_A(\bar{p}_E)\,\right] \\ &= \; a\, Q(\bar{p}_E) - \frac{b}{2}\, Q(\bar{p}_3)^2 - P\left(Q(\bar{p}_3)\,\right)\, Q(\bar{p}_3) \; = \; S(\bar{p}_3)\,. \end{aligned} \tag{74}$$

The inequality in (74) holds because $\bar{p}_E < P\left(Q(\bar{p}_E)\,\right)$, since $\bar{p}_E \in (\bar{p}_1, \bar{p}_2)$. (73) and (74) imply that consumer surplus is higher and R's revenue is lower when $\bar{p} = \bar{p}_E$ than when $\bar{p} = \bar{p}_3$. Therefore, $W(\bar{p}_E) > W(\bar{p}_3)$, so $\bar{p}^* \neq \bar{p}_3$. ■

<u>Proof of Lemma 3</u>. The conclusions in the lemma follow directly from Lemma A2. ■

Proof of Lemma 4 and its Corollary.

$$\text{Define} \quad \widetilde{V}_2(\bar{p}) \;\equiv\; q_{A2}(\bar{p})\,\bar{p} + q_{N2}(\bar{p})\,P(Q_2(\bar{p})) \tag{75}$$

where $q_{A2}(\bar{p})$ and $q_{N2}(\bar{p})$ are as defined in (14) and (15), respectively. Observe that $\widetilde{V}_2(\bar{p}) = V(\bar{p})$ for $\bar{p} \in [\,\bar{p}_1, \bar{p}_2\,]$. Because $P(Q_2) = a - b\,Q_2$, (75) implies:

$$\frac{\partial \widetilde{V}_2(\bar{p})}{\partial \bar{p}} \;=\; q_{A2} + \bar{p}\,\frac{\partial q_{A2}}{\partial \bar{p}} + P(Q_2)\,\frac{\partial q_{N2}}{\partial \bar{p}} - b\,q_{N2}\,\frac{\partial Q_2}{\partial \bar{p}}\,. \tag{76}$$

(2) and Lemma A2 imply that $\frac{\partial^2 q_{A2}}{\partial(\bar{p})^2} = \frac{\partial^2 q_{N2}}{\partial(\bar{p})^2} = \frac{\partial^2 q_2}{\partial(\bar{p})^2} = \frac{\partial^2 Q_2}{\partial(\bar{p})^2} = 0$. Therefore, (76) implies:

$$\frac{\partial^2 \widetilde{V}_2(\bar{p})}{\partial\,(\bar{p})^2} \;=\; 2\,\frac{\partial q_{A2}}{\partial \bar{p}} - 2\,b\,\frac{\partial Q_2}{\partial \bar{p}}\,\frac{\partial q_{N2}}{\partial \bar{p}} \;>\; 0\,. \tag{77}$$

The inequality in (77) holds because $D > 0$ by assumption, so $\frac{\partial q_{A2}}{\partial \bar{p}} > 0$ from (14), $\frac{\partial Q_2}{\partial \bar{p}} > 0$ from (18), and $\frac{\partial q_{N2}}{\partial \bar{p}} < 0$ from (15).

$\bar{p}_{V_2 m} \equiv \arg\min\limits_{\bar{p}} \{\,\widetilde{V}_2(\bar{p})\,\}$ is unique and is determined by:

$$\frac{\partial \widetilde{V}_2(\bar{p}_{V_2 m})}{\partial \bar{p}} \;\equiv\; \left.\frac{\partial \widetilde{V}_2(\bar{p})}{\partial \bar{p}}\right|_{\bar{p} = \bar{p}_{V_2 m}} \;=\; 0. \tag{78}$$

This is the case because (2), (14) – (18), and (76) imply that $\frac{\partial \widetilde{V}_2(\bar{p})}{\partial \bar{p}}$ is a linear function of $\bar{p}$. Therefore, $\widetilde{V}_2(\bar{p})$ is a quadratic function of $\bar{p}$. Consequently, (77) implies that $\widetilde{V}_2(\bar{p})$ has a unique minimum that is determined by (78).

To prove the Corollary to Lemma 4 and thereby establish that $\bar{p}_{V_2 m} > \bar{p}_1$ when $\Phi_2 \geq 0$, observe that R's revenue is:

$$V(\bar{p}) \;=\; \bar{p}\,q_A + P(Q)\,q_N \;=\; \bar{p}\,q_A + [\,a - b\,Q\,]\,q_N\,. \tag{79}$$

(79) implies that the Corollary to Lemma 4 holds if:

$$\frac{\partial^+ V(\bar{p}_1)}{\partial \bar{p}} \;=\; q_A + \bar{p}_1\,\frac{\partial q_A}{\partial \bar{p}} - b\,\frac{\partial Q}{\partial \bar{p}}\,q_N + P(Q)\,\frac{\partial q_N}{\partial \bar{p}} \;<\; 0\,, \tag{80}$$

where: (i) $\frac{\partial^+ V(\bar{p}_1)}{\partial \bar{p}} = \left.\frac{\partial^+ V(\bar{p})}{\partial \bar{p}}\right|_{\bar{p} = \bar{p}_1}$ denotes the right-sided derivative of $V(\cdot)$; (ii) $\frac{\partial q_A}{\partial \bar{p}}$, $\frac{\partial q_N}{\partial \bar{p}}$, and $\frac{\partial Q}{\partial \bar{p}}$ pertain to the quantities identified in Lemma A2; and (iii) q_A, q_N, and Q are as defined in Lemma A1.

Lemma A2 implies that when $\bar{p} \in (\bar{p}_1, \bar{p}_2)$:

$$\frac{\partial q_N}{\partial \bar{p}} \;=\; -\,\frac{b\,k + 2\,b\,k^R + k\,k^R + b^2}{D}\,; \qquad \frac{\partial q_A}{\partial \bar{p}} \;=\; \frac{E}{D}\,; \text{ and } \quad \frac{\partial Q}{\partial \bar{p}} \;=\; \frac{[\,b + k\,]\,[\,b + k_N\,]}{D}$$

$$\text{where } \; E \;\equiv\; b\,[\,3\,b + 2\,k\,] + [\,2\,b + k\,]\left[\,k_N + k^R\,\right]. \tag{81}$$

Lemma A1 implies that when $\bar{p} \,\leq\, \bar{p}_1$:

$$q_N \;=\; \frac{[\,a-c_N\,]\,[\,2\,b+k\,]-b\,[\,a-c\,]}{E}\,,\;\; q \;=\; \frac{[\,a-c\,]\,\left[\,2\,b+k_N+k^R\,\right]-b\,[\,a-c_N\,]}{E}\,,$$

$$\text{and}\;\; P(Q) \;=\; \frac{a\,E-b\,[\,a-c_N\,]\,[\,b+k\,]-b\,\left[\,b+k_N+k^R\,\right]\,[\,a-c\,]}{E}\,. \tag{82}$$

(80) – (82) imply that because $q_A = 0$ when $\bar{p} = \bar{p}_1$ (from Lemma A1):

$$\begin{aligned}\frac{\partial^+ V(\bar{p}_1)}{\partial \bar{p}} \;=\; & \frac{1}{D\,E}\,\{\,\bar{p}_1\,E^2 - b\,[\,b+k\,]\,[\,b+k_N\,]\,[\,(a-c_N)\,(2\,b+k)-b\,(a-c\,)\,] \\ & - \left[\,a\,E-b\,(a-c_N)\,(b+k)-b\,\left(b+k_N+k^R\right)(a-c\,)\,\right] \\ & \cdot\left[\,b\,k+2\,b\,k^R+k\,k^R+b^2\,\right]\,\}\,.\end{aligned} \tag{83}$$

Tedious calculations reveal that the expression in (83) is strictly negative when $\Phi_2 \geq 0$.

It remains to prove that $\bar{p}_{V_2m} < \bar{p}_2$, which is established by demonstrating that $\left.\frac{\partial^- V(\bar{p})}{\partial\,\bar{p}}\right|_{\bar{p}=\bar{p}_2} > 0$. Define $V_2(\bar{p}) \equiv \bar{p}\,q_A(\cdot) + P(Q(\cdot))\,q_N(\cdot)$ for $\bar{p} \in (\bar{p}_1, \bar{p}_2)$. Because $P(Q) = a - b\,Q$:

$$\frac{\partial^- V_2(\bar{p}_2)}{\partial \bar{p}} \;=\; q_A + \bar{p}_2\,\frac{\partial q_A}{\partial \bar{p}} + P(Q)\,\frac{\partial q_N}{\partial \bar{p}} - b\,q_N\,\frac{\partial Q}{\partial \bar{p}} \tag{84}$$

where q_A, q_N, and Q are as specified in Lemma A2, evaluated at $\bar{p} = \bar{p}_2$. Because $\bar{p}_2 = P(Q)$, (84) implies:

$$\frac{\partial^- V_2(\bar{p}_2)}{\partial \bar{p}} \;=\; q_A + \bar{p}_2\left[\frac{\partial q_A}{\partial \bar{p}} + \frac{\partial q_N}{\partial \bar{p}}\right] - b\,q_N\,\frac{\partial Q}{\partial \bar{p}}\,. \tag{85}$$

(30) implies:

$$\bar{p}_2 \;=\; \left[\,b+k^R\,\right]Q^R + c_N + k_N\,q_N - b\,q_A \;=\; k^R\,q_A + \left[\,b+k_N+k^R\,\right]q_N + c_N\,. \tag{86}$$

(85) and (86) imply:

$$\frac{\partial^- V_2(\bar{p}_2)}{\partial \bar{p}} \;=\; q_A + \left[\,k^R\,q_A + \left(k_N+k^R\right)\,q_N + c_N\,\right]\left[\frac{\partial q_A}{\partial \bar{p}} + \frac{\partial q_N}{\partial \bar{p}}\right] - b\,q_N\,\frac{\partial q}{\partial \bar{p}} \;>\; 0\,. \tag{87}$$

The inequality holds here because $\frac{\partial q_A}{\partial \bar{p}} + \frac{\partial q_N}{\partial \bar{p}} = \frac{\partial Q^R}{\partial \bar{p}} > 0$ (from (16)) and $\frac{\partial q}{\partial \bar{p}} < 0$ (from (17)). ■

Proof of Lemma 5. As in (59), define:

$$\widetilde{S}_2(\bar{p}) \;\equiv\; a\,Q_2(\bar{p}) - \frac{b}{2}\,Q_2(\bar{p})^2 - q_{A2}(\bar{p})\,\bar{p} - [\,q_2(\bar{p}) + q_{N2}(\bar{p})\,]\,P(Q_2(\bar{p})) \tag{88}$$

where $q_{A2}(\bar{p})$, $q_{N2}(\bar{p})$, $q_2(\bar{p})$, and $Q_2(\bar{p})$ are as defined in (14), (15), (17), and (18), respectively. Observe that $\widetilde{S}_2(\bar{p}) = S(\bar{p})$ for $\bar{p} \in [\,\bar{p}_1, \bar{p}_2\,]$.

(88) implies that because $P(Q_2) = a - b\,Q_2$ and $Q_2 = q_{A2} + q_{N2} + q_2$:

$$\frac{\partial \widetilde{S}_2(\bar{p})}{\partial \bar{p}} \;=\; [\,P(Q_2) - \bar{p}\,]\,\frac{\partial q_{A2}}{\partial \bar{p}} + b\,\frac{\partial Q_2}{\partial \bar{p}}\,[\,q_{N2} + q_2\,] - q_{A2} \tag{89}$$

$$\Rightarrow \frac{\partial^2 \widetilde{S}_2(\bar{p})}{\partial (\bar{p})^2} = \left[-b \frac{\partial Q_2}{\partial \bar{p}} - 1 \right] \frac{\partial q_{A2}}{\partial \bar{p}} + b \frac{\partial Q_2}{\partial \bar{p}} \left[\frac{\partial q_{N2}}{\partial \bar{p}} + \frac{\partial q_2}{\partial \bar{p}} \right] - \frac{\partial q_{A2}}{\partial \bar{p}} < 0 . \tag{90}$$

The inequality in (90) holds because Lemma 3 implies that $\frac{\partial q_{A2}}{\partial \bar{p}} > 0$, $\frac{\partial Q_2}{\partial \bar{p}} > 0$, $\frac{\partial q_{N2}}{\partial \bar{p}} < 0$, and $\frac{\partial q_2}{\partial \bar{p}} < 0$.

$\bar{p}_{S_2 M} \equiv \arg\max_{\bar{p}} \{ \widetilde{S}_2(\bar{p}) \}$ is unique and is determined by:

$$\frac{\partial \widetilde{S}_2(\bar{p}_{S_2 M})}{\partial \bar{p}} \equiv \left. \frac{\partial \widetilde{S}_2(\bar{p})}{\partial \bar{p}} \right|_{\bar{p} = \bar{p}_{S_2 M}} = 0 . \tag{91}$$

This is the case because (2), (14) – (18), and (89) imply that $\frac{\partial \widetilde{S}_2(\bar{p})}{\partial \bar{p}}$ is a linear function of $\bar{p}$. Therefore, $\widetilde{S}_2(\bar{p})$ is a quadratic function of $\bar{p}$. Consequently, (90) implies that $\widetilde{S}_2(\bar{p})$ has a unique maximum that is determined by (91).

To prove that $\bar{p}_{S_2 M} > \bar{p}_{V_2 m}$, define $H(\bar{p}) \equiv a\, Q_2 - \frac{b}{2} Q_2^2 - [\, a - b\, Q_2 \,]\, q_2$. Observe that:

$$\frac{\partial H(\bar{p})}{\partial \bar{p}} \equiv [\, a - b\, Q_2 \,] \frac{\partial Q_2}{\partial \bar{p}} - [\, a - b\, Q_2 \,] \frac{\partial q_2}{\partial \bar{p}} + b \frac{\partial Q_2}{\partial \bar{p}}\, q_2 \tag{92}$$

$$\Rightarrow \frac{\partial^2 H(\bar{p})}{(\partial \bar{p})^2} \equiv -b \left(\frac{\partial Q_2}{\partial \bar{p}} \right)^2 + 2\, b \frac{\partial Q_2}{\partial \bar{p}} \frac{\partial q_2}{\partial \bar{p}} < 0 , \tag{93}$$

where q_2 and Q_2 are defined in (17) and (18). The inequality in (93) holds because $\frac{\partial Q_2}{\partial \bar{p}} > 0$ and $\frac{\partial q_2}{\partial \bar{p}} < 0$, from (17) and (18). (92) implies:

$$\frac{\partial H(\bar{p}_2)}{\partial \bar{p}} \equiv \left. \frac{\partial H(\bar{p})}{\partial \bar{p}} \right|_{\bar{p} = \bar{p}_2} = \bar{p}_2 \frac{\partial Q_2}{\partial \bar{p}} - \bar{p}_2 \frac{\partial q_2}{\partial \bar{p}} + b \frac{\partial Q_2}{\partial \bar{p}}\, q_2(\bar{p}_2) > 0 . \tag{94}$$

The inequality in (94) holds because $\frac{\partial Q_2}{\partial \bar{p}} > 0$ and $\frac{\partial q_2}{\partial \bar{p}} < 0$, from (17) and (18). The concavity of $H(\bar{p})$ established in (93), along with (94), imply:

$$\frac{\partial H(\bar{p})}{\partial \bar{p}} > 0 \text{ for all } \bar{p} < \bar{p}_2 \Rightarrow \frac{\partial H(\bar{p}_{V_2 m})}{\partial \bar{p}} > 0 . \tag{95}$$

The implication in (95) holds because $\bar{p}_{V_2 m} < \bar{p}_2$, from Lemma 4.

(76) and (91) imply:

$$\frac{\partial \widetilde{V}_2(\bar{p}_{V_2 m})}{\partial \bar{p}} = [\, a - b\, Q_2(\cdot) \,] \frac{\partial q_{N2}(\cdot)}{\partial \bar{p}} - b \frac{\partial Q_2(\cdot)}{\partial \bar{p}}\, q_{N2}(\cdot) + q_{A2}(\cdot) + \bar{p}_{V_2 m} \frac{\partial q_{A2}(\cdot)}{\partial \bar{p}} = 0 \tag{96}$$

where $q_{A2}(\cdot)$, $q_{N2}(\cdot)$, and $Q_2(\cdot)$ are defined in (14), (15), and (18), and evaluated at $\bar{p}_{V_2 m}$.

(89) implies:

$$\begin{aligned} \frac{\partial \widetilde{S}_2(\bar{p})}{\partial \bar{p}} = {} & [\, a - b\, Q_2 \,] \frac{\partial Q_2}{\partial \bar{p}} - [\, a - b\, Q_2 \,] \frac{\partial q_2}{\partial \bar{p}} + b \frac{\partial Q_2}{\partial \bar{p}}\, q_2 \\ & - [\, a - b\, Q_2 \,] \frac{\partial q_{N2}}{\partial \bar{p}} + b \frac{\partial Q_2}{\partial \bar{p}}\, q_{N2} - q_{A2} - \bar{p} \frac{\partial q_{A2}}{\partial \bar{p}} \end{aligned} \tag{97}$$

where q_{A2}, q_{N2}, q_2, and Q_2 are defined in (14), (15), (17), and (18). (97) implies:

$$\begin{aligned}\frac{\partial \widetilde{S}_2(\bar{p}_{V_2m})}{\partial \bar{p}} &= [\, a - b\, Q_2(\bar{p}_{V_2m})\,]\, \frac{\partial Q_2}{\partial \bar{p}} - [\, a - b\, Q_2(\bar{p}_{V_2m})\,]\, \frac{\partial q_2}{\partial \bar{p}} + b\, \frac{\partial Q_2}{\partial \bar{p}}\, q_2(\bar{p}_{V_2m}) \\ &= \frac{\partial H(\bar{p}_{V_2m})}{\partial \bar{p}} \ > \ 0\,. \end{aligned} \tag{98}$$

The last equality in (98) reflects (96). The inequality in (98) reflects (95).

(90) implies that $\widetilde{S}_2(\bar{p})$ is a strictly concave function of $\bar{p}$. Therefore, $\bar{p}_{V_2m} < \bar{p}_{S_2M}$ because: (i) $\frac{\partial \widetilde{S}_2(\bar{p}_{S_2M})}{\partial \bar{p}} = 0$ from (91); and (ii) $\frac{\partial \widetilde{S}_2(\bar{p}_{V_2m})}{\partial \bar{p}} > 0$, from (98).

To prove that $\bar{p}_{S_2M} > \bar{p}_1$, it suffices to establish that $\frac{\partial^+ S_2(\bar{p}_1)}{\partial \bar{p}} \equiv \left.\frac{\partial^+ S_2(\bar{p}_1)}{\partial \bar{p}}\right|_{\bar{p}=\bar{p}_1} > 0$. Lemma A1 implies that $q_A = 0$ when $\bar{p} = \bar{p}_1$. Therefore, (59) implies:

$$\frac{\partial^+ \widetilde{S}_2(\bar{p}_1)}{\partial \bar{p}} = [\, P(Q) - \bar{p}_1\,]\, \frac{\partial q_A}{\partial \bar{p}} + b\, [\, q_N + q\,]\, \frac{\partial Q}{\partial \bar{p}} \ > \ 0\,. \tag{99}$$

The inequality in (99) holds because $\frac{\partial q_A}{\partial \bar{p}} > 0$ and $\frac{\partial Q}{\partial \bar{p}} > 0$ from Lemma 3, and because $P(Q) > \bar{p}_1$ when $\bar{p} \in (\bar{p}_1, \bar{p}_2)$. ■

Proof of Proposition 6. The first conclusion in the Proposition holds because (58) implies that when if $\Phi_2 \geq 0$:

$$\frac{\partial^+ W_2(\bar{p}_1)}{\partial \bar{p}} \equiv \left.\frac{\partial^+ W_2(\bar{p})}{\partial \bar{p}}\right|_{\bar{p}=\bar{p}_1} = \frac{\partial^+ S_2(\bar{p}_1)}{\partial \bar{p}} - d\, \frac{\partial^+ V_2(\bar{p}_1)}{\partial \bar{p}} \ > \ 0\,. \tag{100}$$

The inequality in (100) holds because when $\Phi_2 \geq 0$: (i) $\frac{\partial^+ V_2(\bar{p}_1)}{\partial \bar{p}} < 0$ from the proof of Lemma 4 and its Corollary; and (ii) $\frac{\partial^+ S_2(\bar{p}_1)}{\partial \bar{p}} > 0$ from (99).

The second conclusion in the Proposition holds if $V(\bar{p}_1) < V(\bar{p})$ for all $\bar{p} > \bar{p}_1$ when d is sufficiently large and $\Phi_2 < 0$. The proof of Lemma 4 and its Corollary establishes that:

$$\left.\frac{\partial^+ V(\bar{p})}{\partial \bar{p}}\right|_{\bar{p}=\bar{p}_1} > \ 0 \text{ when } \Phi_2 < 0\,. \tag{101}$$

$V(\bar{p})$ is a strictly convex function of $\bar{p}$ for $\bar{p} \in (\bar{p}_1, \bar{p}_2)$, from Lemma 4. Therefore, (101) implies that $V(\bar{p})$ is a strictly increasing function of $\bar{p}$ for $\bar{p} \in [\, \bar{p}_1, \bar{p}_2\,]$ under the maintained conditions. Consequently:

$$V(\bar{p}_1) \ < \ V(\bar{p}) \text{ for all } \bar{p} \in (\bar{p}_1, \bar{p}_2\,]\,. \tag{102}$$

Lemma 2 implies that under the maintained conditions:

$$V(\bar{p}_1) \ < \ V(\bar{p}_3)\,. \tag{103}$$

(41) implies that $V(\bar{p})$ is a strictly concave function of $\bar{p}$ for $\bar{p} \in (\bar{p}_2, \bar{p}_3)$. Therefore, (102) and (103) imply:

$$V(\bar{p}) \;>\; V(\bar{p}_1) \;\text{ for all }\; \bar{p} \in \; (\bar{p}_2, \bar{p}_3\,]\,. \tag{104}$$

The conclusion follows from (102), (104), and Proposition 5. ■

Proof of Proposition 7. To prove that $\bar{p}^* \leq \bar{p}_{S_2M}$, suppose that $\bar{p}^* > \bar{p}_{S_2M}$. $\widetilde{S}_2(\bar{p})$ is a strictly concave function of $\bar{p}$, from Lemma 5. Therefore, because $\bar{p}^* > \bar{p}_{S_2M}$, (91) implies:

$$\frac{\partial \widetilde{S}_2(\bar{p}^*)}{\partial \bar{p}} \;<\; \frac{\partial \widetilde{S}_2(\bar{p}_{S_2M})}{\partial \bar{p}} \;=\; 0\,. \tag{105}$$

$\widetilde{V}_2(\bar{p})$ is a strictly convex function of $\bar{p}$, from Lemma 4. Therefore, because $\bar{p}_{V_2m} < \bar{p}_{S_2M}$ from Lemma 5 and because $\bar{p}^* > \bar{p}_{S_2M}$ by assumption, (78) implies:

$$\frac{\partial \widetilde{V}_2(\bar{p}^*)}{\partial \bar{p}} \;>\; \frac{\partial \widetilde{V}_2(\bar{p}_{S_2M})}{\partial \bar{p}} \;>\; \frac{\partial \widetilde{V}_2(\bar{p}_{V_2m})}{\partial \bar{p}} \;=\; 0\,. \tag{106}$$

(105) and (106) imply that R's revenue declines and consumer surplus increases as $\bar{p}$ declines below $\bar{p}^*$. Therefore, $\bar{p}^*$ is not the welfare-maximizing value of $\bar{p}$. Hence, by contradiction, $\bar{p}^* \leq \bar{p}_{S_2M}$.

To prove that $\bar{p}^* \geq \bar{p}_{V_2m}$, suppose that $\bar{p}^* < \bar{p}_{V_2m}$. $\widetilde{V}_2(\bar{p})$ is a strictly convex function of $\bar{p}$, from Lemma 4. Therefore, because $\bar{p}_{V_2m} < \bar{p}_{S_2M}$ from Lemma 5, (78) implies:

$$\frac{\partial \widetilde{V}_2(\bar{p}^*)}{\partial \bar{p}} \;<\; \frac{\partial \widetilde{V}_2(\bar{p}_{V_2m})}{\partial \bar{p}} \;=\; 0\,. \tag{107}$$

$\widetilde{S}_2(\bar{p})$ is a strictly concave function of $\bar{p}$, from Lemma 5. Therefore, because $\bar{p}_{V_2m} < \bar{p}_{S_2M}$ from Lemma 5 and because $\bar{p}^* < \bar{p}_{V_2m}$ by assumption, (91) implies:

$$\frac{\partial \widetilde{S}_2(\bar{p}^*)}{\partial \bar{p}} \;>\; \frac{\partial \widetilde{S}_2(\bar{p}_{V_2m})}{\partial \bar{p}} > \frac{\partial \widetilde{S}_2(\bar{p}_{S_2M})}{\partial \bar{p}} \;=\; 0\,. \tag{108}$$

(107) and (108) imply that R's revenue declines and consumer surplus increases as $\bar{p}$ increases above $\bar{p}^*$. Therefore, $\bar{p}^*$ is not the welfare-maximizing value of $\bar{p}$. Hence, by contradiction, $\bar{p}^* \geq \bar{p}_{V_2m}$.

To prove conclusion (i) in the Proposition, define $\widetilde{W}_2(\cdot) \equiv \widetilde{S}_2(\cdot) - d\,\widetilde{V}_2(\cdot)$ and observe that when $\overline{p}_{S_2M} < \overline{p}_2$ and $d > 0$:

$$\left.\frac{\partial \widetilde{W}_2(\bar{p})}{\partial \bar{p}}\right|_{\bar{p}=\overline{p}_{S_2M}} \;=\; -\,d\;\frac{\partial \widetilde{V}_2(\overline{p}_{S_2M})}{\partial \bar{p}} \;<\; -\,d\;\frac{\partial \widetilde{V}_2(\overline{p}_{V_2m})}{\partial \bar{p}} \;=\; 0\,. \tag{109}$$

The inequality in (109) holds because: (i) $\overline{p}_{S_2M} > \overline{p}_{V_2m}$, from Lemma 5; and (ii) $\widetilde{V}_2(\cdot)$ is a strictly convex function of $\overline{p}$, from Lemma 4. (109) implies that $\overline{p}_{S_2M} > \overline{p}^*$ because $\widetilde{W}_2(\cdot)$ is a strictly concave function of $\overline{p}$ (because $\widetilde{S}_2(\cdot)$ is a strictly concave function of $\overline{p}$ and $\widetilde{V}_2(\cdot)$ is a strictly convex function of $\overline{p}$).

To prove conclusion (ii) in the Proposition, observe that when $\overline{p}_{V_2m} > \overline{p}_1$:

$$\left.\frac{\partial \widetilde{W}_2(\bar{p})}{\partial \bar{p}}\right|_{\bar{p}=\bar{p}_{V_2m}} = \frac{\partial \widetilde{S}_2(\bar{p}_{V_2m})}{\partial \bar{p}} > \frac{\partial \widetilde{S}_2(\bar{p}_{S_2M})}{\partial \bar{p}} = 0\,. \tag{110}$$

The inequality in (109) holds because: (i) $\bar{p}_{S_2M} > \bar{p}_{V_2m}$, from Lemma 5; and (ii) $\widetilde{S}_2(\cdot)$ is a strictly concave function of $\bar{p}$, from Lemma 5. (110) implies that $\bar{p}^* > \bar{p}_{V_2m}$ because $\widetilde{W}_2(\cdot)$ is a strictly concave function of $\bar{p}$.

Conclusions (iii) and (iv) in the Proposition follow immediately from (58) because $\bar{p}^* \in (\bar{p}_1, \bar{p}_2)$ is a non-increasing function of d. This is the case because (58) implies that when $\bar{p}^* \in (\bar{p}_1, \bar{p}_2)$:

$$\frac{\partial S(\bar{p}^*)}{\partial \bar{p}} - d\,\frac{\partial \widetilde{V}(\bar{p}^*)}{\partial \bar{p}} = 0 \;\Rightarrow\; \frac{\partial^2 \widetilde{S}(\bar{p}^*)}{\partial (\bar{p})^2}\,\frac{\partial \bar{p}^*}{\partial d} - \frac{\partial \widetilde{V}(\bar{p}^*)}{\partial \bar{p}} - d\,\frac{\partial^2 \widetilde{V}(\bar{p}^*)}{\partial (\bar{p})^2}\,\frac{\partial \bar{p}^*}{\partial d} = 0$$

$$\Rightarrow\; \frac{\partial \bar{p}^*}{\partial d} = \frac{\frac{\partial \widetilde{V}(\bar{p}^*)}{\partial \bar{p}}}{\frac{\partial^2 \widetilde{S}(\bar{p}^*)}{\partial (\bar{p})^2} - d\,\frac{\partial^2 \widetilde{V}(\bar{p}^*)}{\partial (\bar{p})^2}} = \frac{\frac{\partial \widetilde{V}(\bar{p}^*)}{\partial \bar{p}}}{\frac{\partial^2 \widetilde{W}(\bar{p}^*)}{\partial (\bar{p})^2}} \overset{s}{=} -\,\frac{\partial \widetilde{V}(\bar{p}^*)}{\partial \bar{p}}\,. \tag{111}$$

The last conclusion in (111) holds because Lemmas 4 and 5 imply that $\frac{\partial^2 \widetilde{W}(\bar{p}^*)}{\partial (\bar{p})^2} < 0$.

It remains to prove that $\frac{\partial \widetilde{V}_2(\bar{p}^*)}{\partial \bar{p}} \geq 0$. To do so, suppose that $\frac{\partial \widetilde{V}_2(\bar{p}^*)}{\partial \bar{p}} < 0$. Then:

$$\bar{p}^* < \bar{p}_{V_2m}\,. \tag{112}$$

(112) holds because: (i) $\widetilde{V}_2(\bar{p})$ is a strictly convex function of $\bar{p}$, from Lemma 4; and (ii) $\frac{\partial \widetilde{V}_2(\bar{p}_{V_2m})}{\partial \bar{p}} = 0$, from (78). Furthermore, because $\widetilde{S}_2(\bar{p})$ is a strictly concave function of $\bar{p}$, from Lemma 5:

$$\frac{\partial \widetilde{S}_2(\bar{p})}{\partial \bar{p}} > 0 \;\text{ for all } \bar{p} < \bar{p}_{S_2M}\,. \tag{113}$$

Observe that:

$$\bar{p}^* < \bar{p}_{V_2m} < \bar{p}_{S_2M}\,. \tag{114}$$

The first inequality in (114) reflects (112). The second inequality in (114) reflects Lemma 5. (91), (113), and (114) imply:

$$\frac{\partial \widetilde{S}_2(\bar{p}^*)}{\partial \bar{p}} > 0\,. \tag{115}$$

Because $\frac{\partial \widetilde{S}_2(\bar{p}^*)}{\partial \bar{p}} > 0$ (from (115)), $\frac{\partial \widetilde{V}_2(\bar{p}^*)}{\partial \bar{p}} < 0$ (by assumption), and $\bar{p}^* \in (\bar{p}_1, \bar{p}_2)$ (by assumption), consumer surplus increases and R's revenue declines as $\bar{p}$ increases above $\bar{p}^*$. Therefore, $\bar{p}^*$ cannot be the welfare-maximizing value of $\bar{p}$. Hence, by contradiction, $\frac{\partial \widetilde{V}_2(\bar{p}^*)}{\partial \bar{p}} \geq 0$. Consequently, (111) implies that $\frac{\partial \bar{p}^*}{\partial d} \leq 0$. ■

Proof of Lemma 6. The conclusions in the lemma follow directly from Lemma A2. ■

Proof of Proposition 8. (59) implies that consumer surplus is:

$$S \;=\; \frac{b}{2}\, Q^2 + [\, a - \overline{p}\,]\, q_A - b\, Q\, q_A\,. \tag{116}$$

(116) implies that $\overline{p}^*$ is the solution to:

$$\underset{\overline{p}}{\text{Maximize}} \;\; W \;=\; \frac{b}{2}\, Q^2 + [\, a - \overline{p}\,]\, q_A - b\, Q\, q_A - \; d\, \overline{p}\; q_A - d\, a\, q_N + d\, b\, Q\, q_N\,. \tag{117}$$

(117) imply that for $\bar{p} \in (\bar{p}_1, \bar{p}_2)$:

$$\begin{aligned} \frac{dW}{d\overline{p}} \;=\; 0 \;\Leftrightarrow\; & \{\, b\,[\, b + k\,]\,[\, b + k_N\,] - b\left[\, 3\, b^2 + 2\, b\left(k + k_N + k^R\right) + k\left(k_N + k^R\right)\right] \\ & \quad - d\, b\left[\, b\left(b + 2\, k^R\right) + k\left(b + k^R\right)\right]\}\, Q \\ & - \;[\, D + b\,(b + k)\,(b + k_N) + d\, D\,]\; q_A + d\, b\,[\, b + k\,]\,[\, b + k_N\,]\; q_N \\ & - \;\{\, 3\, b^2 + 2\, b\left[\, k + k_N + k^R\right] + k\left[\, k_N + k^R\right] \\ & \qquad + \; d\left[\, 3\, b^2 + 2\, b\left(k + k_N + k^R\right) + k\;\left(k_N + k^R\right)\right]\}\, \overline{p} \\ & + \;\{\, 3\, b^2 + 2\, b\left[\, k + k_N + k^R\right] + k\left[\, k_N + k^R\right] \\ & \qquad + \; d\;\left[\, b\left(b + 2\, k^R\right) + k\left(b + k^R\right)\right]\}\; a \;=\; 0\,. \end{aligned} \tag{118}$$

The coefficient on Q in (118) is readily shown to be:

$$-\, b\,[\, 2\, b^2 + b\, k + b\, k_N + 2\, b\, k^R + k\, k^R + b^2 d + 2\, b\, d\, k^R + b\, d\, k + d\, k\, k^R\,] \;<\; 0\,. \tag{119}$$

The coefficient on $-\, q_A$ in (118) is readily shown to be:

$$\begin{aligned} & [\, 1 + d\,]\left\{\, [\, 2\, b + k\,]\left[\, k_N\left(k_A + k^R\right) + k_A\, k^R\right] + b\, k_A\,[\, 3\, b + 2\, k\,] - b^2\,[\, b + k\,]\right\} \\ & \qquad + \; b\,[\, b + k\,]\,[\, b + k_N\,] \;>\; 0\,. \end{aligned} \tag{120}$$

(118) – (120) imply that if $\bar{p}^* \in (\bar{p}_1, \bar{p}_2)$, $\bar{p}^*$ is determined by:

$$G - g\; \overline{p}^* \;=\; 0, \;\text{ where} \tag{121}$$

$$\begin{aligned} G \;\equiv\; & d\, b\,[\, b + k\,]\,[\, b + k_N\,]\; q_N \\ & + \{\, 3\, b^2 + 2\, b\left[\, k + k_N + k^R\right] + k\left[\, k_N + k^R\right] \\ & \qquad + \; d\;\left[\, b\left(b + 2\, k^R\right) + k\left(b + k^R\right)\right]\}\; a \\ & - \; b\,[\, 2\, b^2 + b\, k + b\, k_N + 2\, b\, k^R + k\, k^R + b^2 d \\ & \qquad + \; 2\, b\, d\, k^R + b\, d\, k + d\, k\, k^R\,]\, Q \\ & - \;\{\, [\, 1 + d\,]\, D + b\,[\, b + k\,]\,[\, b + k_N\,]\}\; q_A\,, \;\text{ and} \end{aligned}$$

$$g \equiv \{ 3b^2 + 2b\left[k + k_N + k^R \right] + k\left[k_N + k^R \right]$$
$$+ \; d\left[3b^2 + 2b\left(k + k_N + k^R \right) + k\left(k_N + k^R \right) \right] \} > 0 . \tag{122}$$

To prove that $\frac{d\,\overline{p}^*}{dc_A} > 0$, observe from (122) that $\frac{dg}{d\,\overline{p}} = 0$. Therefore, (121) implies that for parameter x:

$$[\,G_x - \overline{p}^* \, g_x\,]\,dx + [\,G_{\overline{p}} - g\,]\,d\,\overline{p}^* = 0 \;\Rightarrow\; \frac{d\,\overline{p}^*}{dx} = \frac{G_x - \overline{p}\; g_x}{g - G_{\overline{p}}} . \tag{123}$$

(2) and (122) imply that because $D > 0$:

$$G_{c_A} = d\,b\,[\,b + k\,]\,[\,b + k_N\,]\,\frac{dq_N}{dc_A}$$
$$- \; b\,[\,2b^2 + b\,k + b\,k_N + 2\,b\,k^R + k\,k^R + b^2 d + \; 2\,b\,d\,k^R + b\,d\,k + d\,k\,k^R\,]\,\frac{dQ}{dc_A}$$
$$- \; \{\,[\,1 + d\,]\,D + b\,[\,b + k\,]\,[\,b + k_N\,]\,\}\,\frac{dq_A}{dc_A} > 0 . \tag{124}$$

The inequality in (124) holds because Lemma 6 implies that $\frac{dq_A}{dc_A} < 0$, $\frac{dq_N}{dc_A} > 0$, and $\frac{dQ}{dc_A} < 0$.

(2) and (122) imply:

$$G_{\overline{p}} = d\,b\,[\,b + k\,]\,[\,b + k_N\,]\,\frac{dq_N}{d\,\overline{p}}$$
$$- \; b\,[\,2b^2 + b\,k + b\,k_N + 2\,b\,k^R + k\,k^R + b^2 d + 2\,b\,d\,k^R + b\,d\,k + d\,k\,k^R\,]\,\frac{dQ}{d\,\overline{p}}$$
$$- \; \{\,[\,1 + d\,]\,D + b\,[\,b + k\,]\,[\,b + k_N\,]\,\}\,\frac{dq_A}{d\,\overline{p}} < 0 . \tag{125}$$

The inequality in (124) holds because Lemma 3 implies that $\frac{dq_A}{d\,\overline{p}} > 0$, $\frac{dq_N}{d\,\overline{p}} < 0$, and $\frac{dQ}{d\,\overline{p}} > 0$.

(122) implies:

$$g_{c_A} = 0 . \tag{126}$$

(122) – (126) imply that $\frac{d\,\overline{p}^*}{dc_A} = \frac{G_{c_A}}{g - G_{\overline{p}}} > 0$.

The proofs of the remaining conclusions are similar, and so are omitted. ■

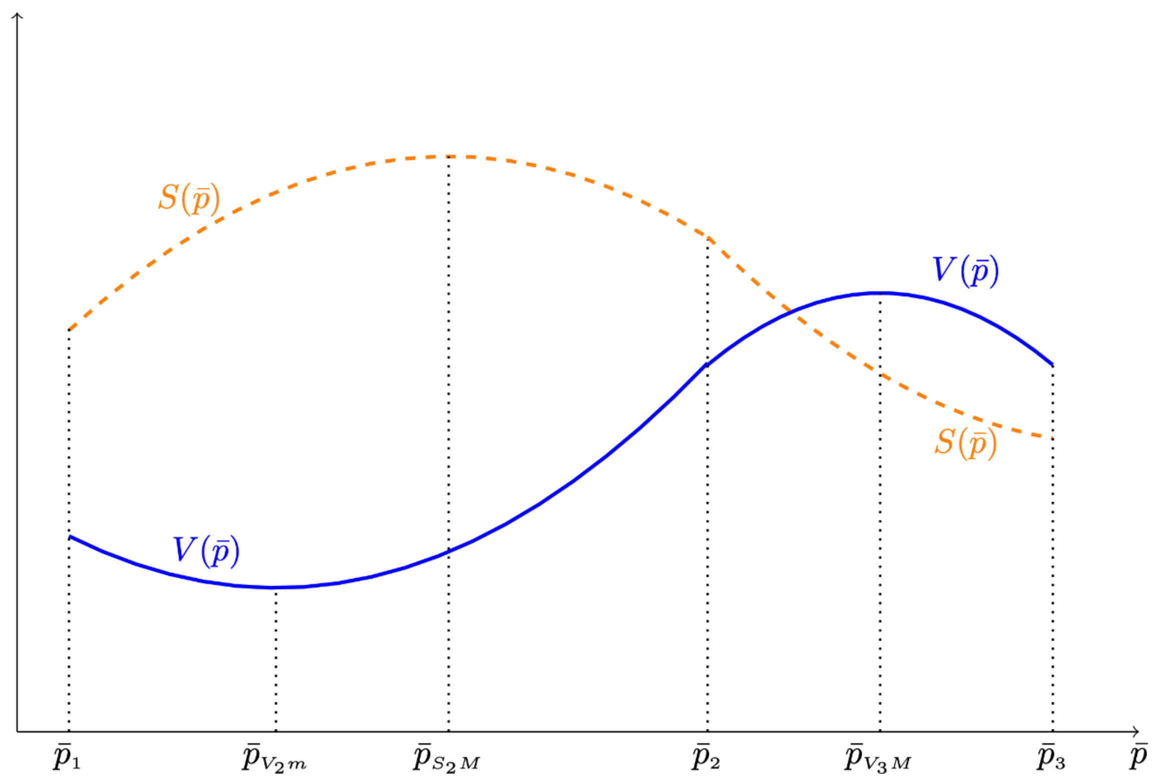

Figure 1. Consumer Surplus $S(\overline{p})$ and Revenue $V(\overline{p})$ when $\overline{p}_1 < \overline{p}_{V_2 m} < \overline{p}_{S_2 M} < \overline{p}_2 < \overline{p}_{V_3 M}$.

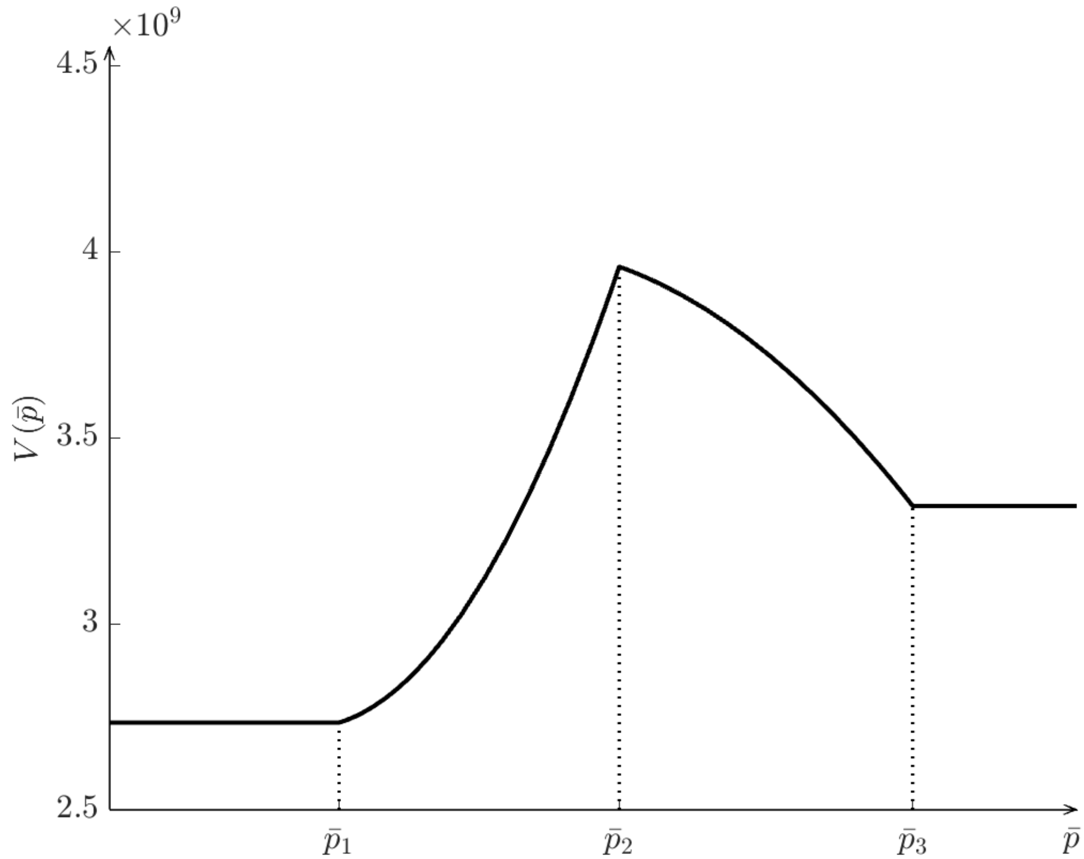

Figure 2. R's Revenue $V(\overline{p})$ in the Baseline Setting.

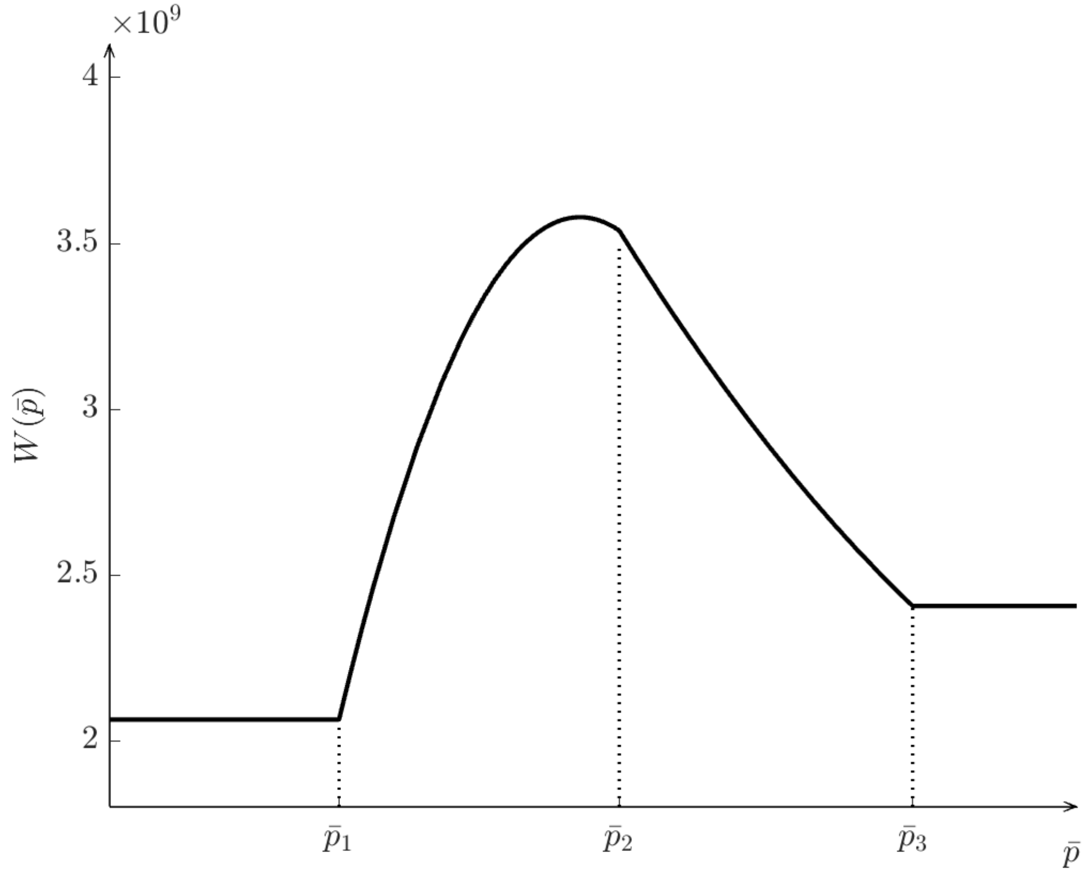

Figure 3. Welfare $W(\overline{p})$ in the Baseline Setting when $d = \frac{1}{2}$.

References

Allaz, Blaise and Jean-Luc Vila, "Cournot Competition, Forward Markets, and Efficiency," *Journal of Economic Theory*, 59(1), February 1993, 1-16.

Baumeister, Christiane, "Pandemic, War, Inflation: Oil Markets at a Crossroads?" NBER Working Paper 31496, July 2023 (https://www.nber.org/papers/w31496).

bp, "Statistical Review of World Energy 2021," 2022 (https://www.bp.com/en/global/corporate/energy-economics/statistical-review-of-world-energy.html).

Caldara, Dario, Michele Cavallo, and Matteo Iacoviello, "Oil Price Elasticities and Oil Price Fluctuations," Board of Governors of the Federal Reserve System International Finance Discussion Paper Number 1173, July 2016 (https://www.federalreserve.gov/econresdata/ifdp/2016/files/ifdp1173.pdf).

Center for Research on Energy and Clean Air, "EU Oil Ban and Price Cap are Costing Russia EUR 160 mn/day, But Further Measures Can Multiply the Impact," CREA Publication, January 11, 2023 (https://energyandcleanair.org/publications).

Ehrhart, Karl-Martin and Ingmar Schlecht, "Introducing a Price Cap on Russian Gas: A Game Theoretic Analysis," ZBW – Leibniz Information Centre for Economics Working Paper, 2022 (http://hdl.handle.net/10419/261345).

Hausmann, Ricardo, "The Case for a Punitive Tax on Russian Oil," *Project Syndicate*, February 26, 2022 (https://www.project-syndicate.org/commentary/case-for-punitive-tax-on-russian-oil-by-ricardo-hausmann-2022-02).

Heussaff, Conall, Lionel Guetta-Jeanrenaud, Ben McWilliams, and Georg Zachmann, "Russian Crude Oil Tracker Dataset," *Bruegel*, June 15, 2023 (https://www.bruegel.org/dataset/russian-crude-oil-tracker).

Horwich, Jeff, "How the Oil Price Cap Balances Pain for Russia with Protecting U.S. Consumers," February 10, 2023, Federal Reserve Bank of Minneapolis (https://www.minneapolisfed.org%2Farticle%2F2023%2Fhow-the-oil-price-cap-balances-pain-for-russia-with-protecting-us-consumers&psig=AOvVaw3GzPOFvZ5_ZRLLxpmk9ohr&ust=1685284581669017).

Johnson, Simon, Lukasz Rachel, and Catherine Wolfram, "A Theory of Price Caps on Non-Renewable Resources," NBER Working Paper 31347, June 2023a (http://www.nber.org/papers/w31347).

Johnson, Simon, Lukasz Rachel, and Catherine Wolfram, "Design and Implementation of the Price Cap on Russian Oil Exports," *Journal of Comparative Economics*, 2023b (forthcoming).

Kennedy, Craig, "Russian Oil's Achilles Heel," Harvard University's Davis Center for Russian and Eurasian Studies, April 7, 2022 (https://daviscenter.fas.harvard.edu/insights/russian-oils-achilles-heel).

Kubursi, Atif, “Understanding the Rollercoaster Ride of Oil Prices,” *The Conversation*, July 19, 2018 (https://theconversation.com/understanding-the-rollercoaster-ride-of-oil-prices-97848).

Miller, Greg. “Profiting from Sanctions: Shipping Premiums Surge for Russian Oil,” *Freight Waves*, June 12, 2023 (https://www.freightwaves.com/news/how-sanctions-pay-shipping-premiums-surge-for-russian-oil).

Sappington, David and Douglas Turner, “Technical Appendix to Accompany ‘On the Design of Price Caps as Sanctions’,” August 2023 (https://people.clas.ufl.edu/sapping).

Sturm, John, “A Theory of Economic Sanctions as Terms-of-Trade Manipulation,” MIT Discussion Paper, November 4, 2022a (http://sturm-econ.s3.amazonaws.com/Sturm+sanctions+2022.pdf).

Sturm, John, “The Simple Economics of Trade Sanctions on Russia: A Policymaker’s Guide,” MIT Discussion Paper, April 9, 2022b (https://drive.google.com/file/d/1FN07m-B6X_AAJruNrT9_qQoT7JkP8udo/view?usp=sharing).

Sturm, John, “How Should Sanctions Account for Bystander Countries?” *AEA Papers and Proceedings*, 113, May 2023, 39-42.

U.S. Energy Information Administration, “Short-Term Energy Outlook,” May 9, 2023 (https://www.eia.gov/outlooks/steo/).

U.S. Government Accountability Office, “Economic Sanctions: Agencies Face Competing Priorities in Enforcing the U.S. Embargo on Cuba,” GAO Report to Congressional Requesters, November 2007 (https://www.gao.gov/products/gao-08-80).

U.S. Government Accountability Office, “International Trade: Libya Trade Sanctions,” GAO Briefing Report to Congressional Requesters, May 1987 (https://www.gao.gov/products/nsiad-87-132br).

U.S. Government Accountability Office, “Iran Sanctions: New Act Underscores Importance of Comprehensive Assessment of Sanctions’ Effectiveness,” GAO Testimony before the Committee on Oversight and Government Reform, House of Representatives, July 29, 2010 (https://www.gao.gov/ products/gao-10-928t).

U.S. Government Accountability Office, “North Korea Sanctions: United States Has Increased Flexibility to Impose Sanctions, but United Nations Is Impeded by a Lack of Member State Reports,” May 15, 2015 (https://www.gao.gov/products/gao-15-485).

U.S. Government Accountability Office, “U.S. Sanctions Against South Africa,” January 24, 1988 (https://www.gao.gov/products/t-nsiad-88-34).

U.S. Senate and House of Representatives, “Comprehensive Anti-Apartheid Act of 1986,” Public Law 99-440, 100 STAT. 1086, October 2, 1986 (https://www.govtrack.us/congress/bills/99/hr4868/text).

Wachtmeister, Henrik, Johan Gars, and Daniel Spiro, "Quantity Restrictions and Price Discounts on Russian Oil," Uppsula University discussion paper, December 2022 (https://arxiv.org/pdf/2212.00674).

Wolfram, Catherine, Simon Johnson, and Lukasz Rachel, "The Price Cap on Russian Oil Exports, Explained," Harvard Kennedy School's Belfer Center and Mossavar-Rahmani Center Policy Brief, December 2022 (www.belfercenter.org/ENRP).