Taxation and the Sustainability of Collusion with Asymmetric Costs

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Abstract

This paper explores the sustainability of collusion under either ad valorem or specific taxation in an infinitely repeated duopoly game. I compare ad valorem taxes and specific taxes that generate the same average price or tax revenue in the Nash equilibrium of the stage game. I find that collusion is less sustainable under ad valorem than specific taxation, contrary to prior literature. The difference arises because I consider constant and asymmetric marginal costs.

Keywords: Collusion, Taxation, Firm Asymmetry

JEL Codes: H2, L1, C7

1 Introduction

Taxation typically takes one of two forms. An ad valorem tax is expressed as a percentage of the price. A specific tax is based on the quantity sold. When the effects of taxation are analyzed in the context of a one period static game, ad valorem taxation is often more efficient (Delipalla and Keen 1992; Anderson, De Palma, and Kreider 2001). In a dynamic setting where the possibility of collusion is permitted, Colombo and Labrecciosa (2013) and Azacis and Collie (2018) find that ad valorem taxation facilitates collusion. The critical discount factor, above which collusion is sustainable in a repeated game, is smaller under ad valorem than specific taxation. There is a range of discount factors in which collusion would be sustainable with ad valorem taxation but not sustainable with the comparable specific tax. Both results consider symmetric and increasing marginal costs. The purpose of this note is to establish that ad valorem taxation may result in a larger critical discount factor than specific taxation when firm costs are instead asymmetric and constant.

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Both a homogenous product model of Bertrand competition and a differentiated product linear city model of Bertrand competition are examined. In both models, marginal costs are constant and asymmetric. When taxes are such that average Nash equilibrium prices, in the stage game, are the same under both forms of taxation, ad valorem taxes result in a larger critical discount factor if and only if costs are asymmetric. When taxes are such that tax revenue is equal under both forms of taxation, ad valorem taxes again result in a larger critical discount factor.

The findings arise because an ad valorem tax effectively increases marginal cost multiplicatively, whereas a specific tax increases marginal cost additively. The degree of asymmetry, or difference between the two firm's marginal costs, is unchanged under specific taxation but amplified under ad valorem taxation. Collusion is more difficult to sustain between firms with greater cost asymmetries (Ivaldi et al. 2003). The more efficient firm has a greater incentive to defect, as it can achieve greater profits in both the defection and punishment phase. Ad valorem taxation effectively amplifies differences in marginal costs relevant to pricing, while specific taxation does not, hindering collusion.

Section 2 presents the repeated game framework. Section 3 presents results under homogenous product competition and section 4 presents results under differentiated product competition. Section 5 concludes. All proofs can be found in the appendix.

2 Repeated Game Framework

Consider an infinitely repeated duopoly game where firms compete in prices and are subject to one of two forms of taxation: an ad valorem tax of t_v or a specific tax of t_s. Firms seek to maximize the discounted present value of profit,

$$\sum_{t=0}^{\infty} \delta^t \pi_i(p_{1,t}, p_{2,t}; t_s, t_v) \tag{1}$$

where $\delta \in (0,1)$ is the discount factor, $p_{i,t}$ is the price of firm i at time t, $\pi_i(p_{1,t}p_{2,t}; t_s, t_v)$ is firm i's period t stage game profit when firm 1 sets a price of $p_{1,t}$, firm 2 sets a price of $p_{2,t}$, and taxes t_s and t_v are imposed. Let $H_T = ((p_{1,1}, p_{2,1}), \ldots (p_{1,T}, p_{2,T}))$ denote the history of prices played up to, and including, time T. Strategies σ_i are mappings that specify, for each possible history H_T, a price $p_{i,T+1}$. A pair of strategies (σ_1, σ_2) is a Nash equilibrium if σ_i maximizes the discounted stream of profits (1) for firm i, conditional on the strategy of the rival firm. Attention is restricted to a particular form of strategy functions–grim trigger strategies (Friedman 1971). Each firm plays the collusive price p_C until any firm deviates. When either firm deviates, both play Nash equilibrium prices in perpetuity.

Firm i's stage game profit under collusion, one shot deviation, and Nash competition are denoted π_i^C, π_i^D

and π_i^N, respectively. For collusion with grim trigger strategies to be an equilibrium, both firms' incentive compatibility constraints must be satisfied. Each firm must find the profit from collusion to be larger than the profit from one period defection followed by Nash equilibrium play. Firm i's incentive compatibility constraint is (omitting price arguments)

$$\sum_{t=0}^{\infty} \delta^t \pi_i^C(t_s, t_v) \geq \pi_i^D(t_s, t_v) + \sum_{t=1}^{\infty} \delta^t \pi_i^N(t_s, t_v).$$

The smallest discount factor which would satisfy both firms' incentive compatibility constraints is (omitting price and tax arguments in profit functions)

$$\delta^*(t_s, t_v) = \max\left\{ \frac{\pi_1^D - \pi_1^C}{\pi_1^D - \pi_1^N}, \frac{\pi_2^D - \pi_2^C}{\pi_2^D - \pi_2^N} \right\}.$$

For any $\delta \geq \delta^*(t_s, t_v)$, collusion is feasible. For any $\delta < \delta^*(t_s, t_v)$, at least one firm will defect and collusion will be unsustainable. I compare the critical discount factor under only specific taxation, $\delta^*(t_s, 0)$, to the critical discount factor under only ad valorem taxation, $\delta^*(0, t_v)$.

In the next two sections, I consider a stage game based on homogenous product competition and a stage game based on differentiated product competition.

3 Homogenous Product Model

Products are homogenous and firms compete in prices. Firm 1 has a marginal cost of c_1 and firm 2 has a marginal cost of $c_2 = c_1 + a$. $a \geq 0$ is a parameter which represents the degree of cost asymmetry between firms. A unit mass of consumers have perfectly elastic demand and a reservation price of r. The following assumption ensures the reservation price is sufficiently high that both firms receive greater profits under collusion than under Nash competition. Additionally, it ensures that equilibrium firm pricing is unaffected by the reservation price.

Assumption 1. $2\frac{c_2+t_s}{1-t_v} < r$

Firm i's profit can be rewritten as

$$\pi_i(p_i, p_{-i}) = \begin{cases} 0 & \text{if } p_i > p_{-i} \\ (1-t_v)(p_i - \tilde{c}_i)\frac{1}{2} & \text{if } p_i = p_{-i} \\ (1-t_v)(p_i - \tilde{c}_i) & \text{if } p_i < p_{-i} \end{cases}$$

where $\tilde{c}_i = \frac{c_i + t_s}{1-t_v}$. Following a standard Bertrand argument, Nash competition results in profits of $\pi_1^N = (1-t_v)(\tilde{c}_2 - \tilde{c}_1) = a$ and $\pi_2^N = 0$. For simplicity, firms are assumed to price uniformly in the collusive phase. This assumption is applicable in a setting where firms are unable or unwilling to coordinate more complicated pricing agreements.[1]

Assumption 2. *In the collusive phase, firms choose a uniform price to maximize joint profit in the stage game. At this price, demand is split evenly.*

As both firms charge the reservation price, firm i's collusive profit is $\pi_i^C = \frac{1}{2}(1-t_v)(r-\tilde{c}_i)$ and firm i's defection profit is $\pi_i^D = (1-t_v)(r-\tilde{c}_i)$.

Lemma 1. *Under assumptions 1 and 2, the critical discount factor under an ad valorem tax of t_v and a specific tax of t_s is*

$$\delta^*(t_v, t_s) = \frac{1}{2 - 2\beta(t_v, t_s)}$$

where $\beta(t_v, t_s) = \frac{a}{r(1-t_v) - c_1 - t_s}$.

When $a = 0$, the critical discount factor, regardless of the form or magnitude of taxation, is $1/2$. This mirrors the result of Azacis and Collie (2018) for the case of Cournot competition.

Following Azacis and Collie (2018), I consider a specific tax of $t_s = \frac{t_v}{1-t_v} c_2$ which would result in the same Nash equilibrium price, in the stage game, as an ad valorem tax of t_v. In the Nash equilibrium of the stage game, an ad valorem tax of t_v and a specific tax of $t_s = \frac{t_v}{1-t_v} c_2$ also result in the same tax revenue raised and the same level of consumer welfare.

Theorem 1. *Under assumptions 1 and 2, $\delta^*(t_v, 0) \geq \delta^*(0, t_s)$ for any pair of taxes (t_s, t_v) that satisfy $t_s = \frac{c_2}{1-t_v} t_v$. When $a > 0$, the inequality holds strictly.*

When firms are asymmetric, ad valorem taxation yields a critical discount factor which is strictly larger than the critical discount factor under the equivalent specific tax. Lemma 1 shows that $\delta(t_v, t_s)$ is an increasing function of $\beta(t_v, t_s)$, which can be written as

$$\beta(t_v, t_s) = \frac{\frac{c_2 - c_1}{1-t_v}}{r - \frac{c_1 + t_s}{1-t_v}}. \tag{2}$$

The denominator of the above expression is unaffected by the form of taxation as $c_1 + t_s = \frac{c_1}{1-t_v}$ holds by assumption. The numerator is increasing in the level of ad valorem taxation, but unaffected by specific taxation. Ad valorem taxation hinders collusion by amplifying the asymmetry in marginal cost between

[1] This assumption has been made previously in both the homogenous product case (Ivaldi et al. 2003) and the differentiated product case (Häckner 1994; Colombo 2009).

firms. An ad valorem tax of t_v is equivalent, for the purposes of collusion, to charging the corresponding specific tax of $t_s = \frac{c_2}{1-t_v} t_v$ and increasing the degree of cost asymmetry from a to $\frac{a}{1-t_v}$. Crucially, it is the multiplicative nature of ad valorem taxes that amplifies asymmetries. Specific taxes, as they are additive, do not have this effect.

Assumption 2 can be relaxed to allow for uneven market sharing in the collusive phase. As the low cost firm has greater incentives to defect from the cartel, cartel members may wish to divide the market such that the low cost firm receives greater demand and profit in the collusive phase, increasing the range of discount factors in which collusion is sustainable.

Assumption 3. *In the collusive phase, firms choose a uniform price to maximize joint profit in the stage game. At this price, demand is split between the two firms so as to maximize the scope for collusion (minimize the critical discount factor).*

If such market sharing is feasible, the market sharing arrangement which maximizes the scope for collusion will involve allocating a market share of $\delta > \frac{1}{2}$ to the low cost firm and a market share of $1 - \delta$ to the high cost firm (Ivaldi et al. 2003). The ability to split the market unevenly, when colluding, increases the scope for collusion, but the critical discount factor is still greater than its value under symmetric costs of $\frac{1}{2}$.

Lemma 2. *Under assumptions 1 and 3, the critical discount factor under an ad valorem tax of t_v and a specific tax of t_s is*

$$\delta^*(t_v, t_s) = \frac{1}{2 - \beta(t_v, t_s)}.$$

The next theorem shows that ad valorem taxation generates a greater critical discount factor than the corresponding specific tax when firms can split the market unevenly.

Theorem 2. *Under assumptions 1 and 3, $\delta^*(t_v, 0) \geq \delta^*(0, t_s)$ for any pair of taxes (t_s, t_v) that satisfy $t_s = \frac{c_2}{1-t_v} t_v$. When $a > 0$, the inequality holds strictly.*

As the critical discount factor is still increasing the degree of cost asymmetry, taxes which exacerbate those asymmetries (ad valorem taxes) hinder collusion relative to those that do not affect asymmetry (specific taxes).

4 Differentiated Product Model

Consider two firms located at opposite ends of a line segment of unit length, denoted $[0, 1]$. Firm 1 is located at point 0 and firm 2 is located at point 1. As in section 3, Firm 1 has constant marginal cost c_1 and firm 2 has constant marginal cost $c_2 = c_1 + a$. A unit mass of consumers is uniformly distributed throughout the line segment. Each consumer purchases one unit of the good and receives utility r from consumption. To capture product differentiation, a consumer located at point $x \in [0,1]$ incurs a quadratic transportation cost tx^2 when purchasing from firm 1 and a cost of $t(1-x)^2$ when purchasing from firm 2. t represents the level of product differentiation. The utility from purchasing firm 1's product, for a consumer located at point $x \in [0,1]$, is $u_1 = r - tx^2 - p_1$. Utility from purchasing firm 2's product is $u_2 = r - t(1-x)^2 - p_2$. Consumers purchase from the firm which yields the greatest utility. $\hat{x}$ is defined as the location of a consumer who is indifferent between purchasing from firm 1 or firm 2. It follows that $\hat{x}(p_1,p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t}$. $\hat{x}(p_1,p_2)$ is the demand faced by firm 1, and $1 - \hat{x}(p_1,p_2)$ is the demand faced by firm 2.

Assumption 1 is adjusted for the differentiated product setting. This assumption ensures that the entire market is covered both in the Nash equilibrium of the stage game and under collusion. Additionally, it ensures each firm earns greater profits under collusion than Nash competition.

Assumption 4. $r > 4t + \tilde{c}_2$

4.1 Nash Competition

In the one shot Nash equilibrium of the stage game, firms set prices as if they were untaxed and produced at marginal costs $\tilde{c}_1 = \frac{c_1 + t_s}{1 - t_v}$ and $\tilde{c}_2 = \frac{c_2 + t_s}{1 - t_v}$.[2] With the following assumption, I restrict attention to the case of moderate cost asymmetries. This assumption is made so that both firms receive positive market shares in the Nash equilibrium.[3]

Assumption 5. $\tilde{c}_2 - \tilde{c}_1 < 3t$

Using assumption 5, firm 1's equilibrium price is $p_1 = t + \frac{2}{3}\tilde{c}_1 + \frac{1}{3}\tilde{c}_2$ and firm 2's equilibrium price is $p_2 = t + \frac{1}{3}\tilde{c}_1 + \frac{2}{3}\tilde{c}_2$. Equilibrium prices result in Nash equilibrium profits of $\pi_1^N = (1-t_v)\frac{1}{2t}\left(t + \frac{a}{3(1-t_v)}\right)^2$ and $\pi_2^N = (1-t_v)\frac{1}{2t}\left(t - \frac{a}{3(1-t_v)}\right)^2$. Specific taxes do not affect Nash equilibrium firm profit as the incidence of the tax is passed entirely to consumers.

4.2 Collusive Phase

Following assumption 2, firms choose a uniform price to maximize joint profit in the stage game. For sufficiently high prices, consumers in the middle of the Hotelling line are left unserved. Firm profit and consumer demand have two segments, one in which all consumers are served and one in which only consumers sufficiently close to each firm purchase a product.

[2] $\pi_i(p_1,p_2;t_s,t_v) = ((1-t_v)p_i - c_i - t_s)D_i(p_1,p_2) = (1-t_v)\left(p_i - \frac{c_i + t_s}{1-t_v}\right)D_i(p_1,p_2)$

[3] Anderson, De Palma, and Kreider (2001) make a similar assumption in their comparison of the efficiency of ad valorem and unit taxes.

If the collusive price is $p \leq r - \frac{t}{4}$, the whole market is covered and each firm has a demand of $\frac{1}{2}$. When the collusive price is $p > r - \frac{t}{4}$, demand for each firm is $x = \sqrt{\frac{r-p}{t}}$. Joint collusive profit is given by

$$\pi(p) = \begin{cases} (1-t_v)(2p - \tilde{c}_2 - \tilde{c}_1)\sqrt{\frac{r-p}{t}} & \text{if } p > r - \frac{t}{4} \\ (1-t_v)\frac{1}{2}(2p - \tilde{c}_1 - \tilde{c}_2) & \text{if } p \leq r - \frac{t}{4} \end{cases}.$$

It follows immediately that if $p < r - \frac{t}{4}$, the colluding firms wish to increase the price as there is no loss in demand. When $p > r - \frac{t}{4}$, $\frac{\partial \pi(p)}{\partial p} < 0$ by assumption 4. Finally, the collusive price is $p = r - \frac{t}{4}$ and the entire market is served in the collusive phase. Firm i's collusive profit is $\pi_i^C = (1-t_v)(r - \frac{t}{4} - \tilde{c}_i)\frac{1}{2}$.

4.3 Defection

When a firm defects from the collusive agreement, it maximizes profit conditional on its rival continuing to charge the collusive price of $r - \frac{t}{4}$. If the defecting firm decreases its price below $r - \frac{5t}{4}$, all customers wish to purchase from the defecting firm as the difference between the two firms' prices is greater than the transportation cost. If the defecting firm prices above $r - \frac{5t}{4}$ but below the collusive price of $r - \frac{t}{4}$, then the defector's market share will increase, but its rival will still serve some customers who are located close to the rival firm. Lastly, the defector could increase its price above the collusive price. Firm i's defection profit is

$$\pi^D(p) = \begin{cases} (1-t_v)(p - \tilde{c}_i)\sqrt{\frac{r-p}{t}} & \text{if } p > r - \frac{t}{4} \\ (1-t_v)(p_i - \tilde{c}_i)\left(\frac{1}{2} + \frac{r - \frac{t}{4} - p_i}{2t}\right) & \text{if } r - \frac{t}{4} \geq p \geq r - \frac{5t}{4} \\ (1-t_v)(p_i - \tilde{c}_i) & \text{if } p < r - \frac{5t}{4} \end{cases}.$$

Increasing price causes a loss in profits as $\frac{\partial}{\partial p}(p - \tilde{c}_i)\sqrt{\frac{r-p}{t}} < 0$ when $p > r - \frac{t}{4}$ by assumption 4. When $r - \frac{t}{4} \geq p \geq r - \frac{5t}{4}$, the defector wishes to further decrease price, because the gain in profit from larger demand outweighs the loss in profits from a lower price. The defector decreases price until it serves the entire market. Once the entire market is served, there is no benefit to a further reduction in price. $p = r - \frac{5t}{4}$ is the optimal defection price as it is the largest price with which the defector serves the entire market. Firm i's defection profit is $\pi_i^D = (1-t_v)(r - \frac{5t}{4} - \tilde{c}_i)$.

4.4 Critical Discount Factor

As shown in section 2, $\frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i^N}$ is the minimum discount factor necessary for firm i to wish to collude. For any taxes, $\frac{\pi_1^D - \pi_1^C}{\pi_1^D - \pi_1^N} > \frac{\pi_2^D - \pi_2^C}{\pi_2^D - \pi_2^N}$. If firm 1, the low cost firm, wishes to collude, then firm 2 wishes to collude as well. Firm 1 faces greater incentives to deviate because of its lower cost and faces weaker punishments because of its larger Bertrand Nash profit. Substituting expressions for firm 1's profit in each phase, the critical discount factor under an ad valorem tax of t_v and a specific tax of t_s is

$$\begin{aligned}\delta^*(t_v,t_s) &= \frac{\pi_1^D-\pi_1^C}{\pi_1^D-\pi_1^N}\\ &= \frac{(r-\frac{5}{4}t-\tilde{c}_1)-\frac{1}{2}\left(r-\frac{t}{4}-\tilde{c}_1\right)}{(r-\frac{5}{4}t-\tilde{c}_1)-\frac{1}{2t}\left(t+\frac{\tilde{c}_2-\tilde{c}_1}{3}\right)^2}.\end{aligned}$$

Simplifying the above proves the following lemma.

Lemma 3. *Under assumptions 2, 4 and 5, the critical discount factor is*

$$\delta^*(t_s,t_v) = \frac{1}{2-2\gamma(t_v,t_s)}$$

where

$$\gamma(t_v,t_s) = \frac{\frac{1}{1-t_v}\pi_1^N - t}{\frac{2}{1-t_v}\pi_1^C - 2t} = \frac{\frac{1}{2t}\left(t+\frac{a}{3(1-t_v)}\right)^2 - t}{r-\frac{9}{4}t-\frac{c_1+t_s}{1-t_v}}.$$

The critical discount factor is increasing in Nash equilibrium profit and decreasing in collusive profit. The larger the Nash equilibrium profit, the weaker the punishment. A greater critical discount factor is then needed to sustain collusion. A larger collusive profit incentivizes collusion and stabilizes the cartel. The critical discount factor, for any form or level of taxation, is increasing in the degree of cost asymmetry a (Rothschild 1999) because asymmetry increases the Nash equilibrium profits of firm 1. However, the collusive profit of firm 1 is unchanged. The cartel's ability to punish defection by firm 1 is weakened, and collusion is hindered.

4.5 Sustainability of Collusion

The following lemma provides a sufficient condition for an ad valorem tax of t_v to result in a greater critical discount factor than a specific tax of t_s. Let $\pi_i^P(t_v,t_s)$ be firm i's profit in phase P under an ad valorem tax of t_v and a specific tax of t_s.

Lemma 4. *First, assume $a>0$. Under assumptions 2, 4 and 5, if*

$$\left(\frac{1}{1-t_v}\pi_1^N(t_v,0)-\pi_1^N(0,t_s)\right)\left(\frac{3t}{4}+a\right) > 2\left(\pi_1^N(0,0)-t\right)\left(\frac{1}{1-t_v}\pi_1^C(t_v,0)-\pi_1^C(0,t_s)\right),$$

then ad valorem taxation results in a greater discount factor. When $a=0$, ad valorem taxation results in a greater critical discount factor than specific taxation if and only if $t_s-\frac{c_1t_v}{1-t_v}>0$.

Taxation affects the condition in lemma 4 through two terms: the rescaled difference between Nash equilibrium profit under the two forms of taxation and the rescaled difference between collusive profit under the two forms of taxation. If an ad valorem tax of t_v weakens the punishment for defection relative to specific taxation, by increasing Nash equilibrium profit sufficiently to offset any increase in collusive profit, then collusion is hindered.

To compare the critical discount factor under ad valorem and specific taxation, it is necessary to make some restriction on the level of each tax. I consider two such restrictions. The first restriction requires that the ad valorem and specific taxes each result in the same average price in the Nash equilibrium. The second restriction requires that both taxes result in the same tax revenue in the Nash equilibrium. Using Lemma 4, I show that ad valorem taxation results in a greater critical discount factor in both cases.

4.5.1 Equal Average Price

Prior literature has compared ad valorem and specific taxes that result in the same price. In their comparison of the efficiency of ad valorem and specific taxation, Anderson, De Palma, and Kreider (2001) compare taxes that result in the same equilibrium price. Azacis and Collie (2018) compare ad valorem and specific taxes that would result in the same price in the collusive phase. With product differentiation of the chosen form, there does not, in general, exist a specific tax t_s which will result in the same prices as an ad valorem tax t_v in the stage game Nash equilibrium, hindering comparison of the two forms of taxation. Instead, I compare taxes t_s and t_v, which result in the same average price paid by consumers in the Nash equilibrium of the stage game.

Assumption 6. *Let t_v and t_s be positive taxes that result in the same average price in the Nash equilibrium of the stage game.*

The average price is

$$\bar{p} = x(p_1, p_2)p_1 + (1 - x(p_1, p_2))p_2.$$

$p_1(t_v) = t + \frac{2c_1}{3(1-t_v)} + \frac{c_2}{3(1-t_v)}$ and $p_2(t_v) = t + \frac{c_1}{3(1-t_v)} + \frac{2c_2}{3(1-t_v)}$ are the prices that would prevail under Nash competition with an ad valorem tax of t_v while $p_1(t_s) = t + \frac{2c_1}{3} + \frac{c_2}{3} + t_s$ and $p_2(t_s) = t + \frac{c_1}{3} + \frac{2c_2}{3} + t_s$ are the prices that would prevail under Nash competition with a specific tax of t_s. Substituting these prices into the above and then equating yields the following condition:

$$t_s = \frac{t_v}{1 - t_v}\left(\frac{c_1 + c_2}{2} - \frac{(c_2 - c_1)^2}{18t} - \frac{(c_2 - c_1)^2}{18t\,(1 - t_v)}\right).$$

In the case of symmetric marginal costs, the above condition reduces to that of section 3.

The following theorem establishes that ad valorem taxation results in a strictly greater critical discount factor when marginal costs are asymmetric. When costs are symmetric, the critical discount factor is the same under both forms of taxation.

Theorem 3. *Under assumptions 2, 4 and 5, $\delta^*(t_s, 0) \leq \delta^*(0, t_v)$ for any taxes t_v and t_s that satisfy assumption 6. When $a = 0$, $\delta^*(t_s, 0) = \delta^*(0, t_v)$. When $a > 0$, the inequality holds strictly.*

4.5.2 Equal Tax Revenue

I next consider taxes t_s and t_v that result in the same per period tax revenue in the Nash equilibrium of the stage game.

Assumption 7. *Let t_v and t_s be positive taxes that generate the same tax revenue in the Nash equilibrium.*

As all customers purchase the good, and there is a unit mass of consumers, specific taxation raises a tax revenue of t_s. Ad valorem taxation raises a revenue, in the Nash equilibrium, of $t_v x(p_1(t_v), p_2(t_v))p_1(t_v) + t_v(1 - x(p_1(t_v), p_2(t_v)))p_2(t_v)$. To raise equal tax revenue under each form of taxation,

$$t_s = t_v x(p_1(t_v), p_2(t_v))p_1(t_v) + t_v(1 - x(p_1(t_v), p_2(t_v)))p_2(t_v)$$

must hold. Substituting equilibrium prices into the above and solving yields

$$t_s = t_v \left(t + \frac{c_2 + c_1}{2(1 - t_v)} - \frac{(c_2 - c_1)^2}{18(1 - t_v)^2 t} \right).$$

For positive taxes that raise the same tax revenue, specific taxation facilitates collusion relative to ad valorem taxation.

Theorem 4. *Under assumptions, 2, 4 and 5, $\delta^*(t_s, 0) < \delta^*(0, t_v)$ for any taxes t_v and t_s that satisfy assumption 7.*

In essence, ad valorem taxation results in a greater critical discount factor than specific taxation, because asymmetries between firms are amplified under ad valorem taxation. When they are taxed, firms set prices as if their marginal costs were $\tilde{c}_1$ and $\tilde{c}_2$. The degree of asymmetry in these costs is $\tilde{c}_2 - \tilde{c}_1 = \frac{c_2 - c_1}{1 - t_v}$. This difference is unaffected by specific taxation, but is increasing in ad valorem taxation. The greater the degree of asymmetry between firms, the greater the incentives for the lower cost firm to defect from the cartel.

5 Conclusion

I have examined the sustainability of collusion under ad valorem taxation and under specific taxation in an infinitely repeated model of Bertrand competition. My study departs from the literature by considering constant, asymmetric marginal costs of production. While prior results (Colombo and Labrecciosa 2013; Azacis and Collie 2018) with symmetric and increasing marginal costs indicate that ad valorem taxation may facilitate collusion relative to specific taxation, ad valorem taxation can hinder collusion relative to specific taxation in the presence of asymmetric and constant marginal costs. Ad valorem taxes result in a larger critical discount factor under both homogeneous and differentiated product competition in the stage game. This result contributes to the classic topic of tax structure in oligopolistic industries. In the case of constant marginal costs, competition in prices, and cost asymmetries, ad valorem taxation may be preferable both in a one period static efficiency analysis and in a dynamic analysis that accounts for the possibility of collusion.

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